ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 1, Issue 4, December 2014

Prediction of Free Edge Stresses in the Composite Laminates and It's Validation through Experimental Means

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Abstract: It is well known that interlaminar stresses are developed at free edges in composite laminates where we have material discontinuity. These stresses can lead to delamination and failure of the laminate at loads that are much lower than the failure strength predicted by the classical lamination theory. Within the frame work of linear elasticity, the free-edge effect of a symmetric cross-ply laminate is treated in a closed form analytical way. In order to enhance the accuracy, a higher order plate theory is proposed. The essential characteristic of this analysis is the introduction of a warp deformation mode for the near-edge displacements. For a laminate coupon under uniaxial tension, the displacement field and the accompanying stresses have been determined. In particular, this includes the straight forward calculation of the inter-laminar stress components. In addition to this work, two case studies have been made for E-Glass Epoxy composite laminate and Graphite Epoxy composite laminate and free edge stress values for both the cases have been calculated using the formulated results. To justify the value obtained from our calculation, both the same cases have been analyzed using ANSYS software. Since an additional mode has been included with the existing method, we can clearly state that the results have been improved while using our results.

Index Terms- Inter laminar stresses, free edge stresses, Delamination, warping deformation.

1 .Introduction

The increased applications of composite materials in structural members have simulated interest in the accurate determination of the response characteristics of laminated composites. It is well known that interlaminar stresses are developed at free edges in composite laminates where we have material discontinuity. These stresses can lead to delamination and failure of the laminate at loads that are much lower than the failure strength predicted by the classical lamination theory. Accurate determination of the stress state near the free edge is therefore crucial to correctly describe the laminate behaviour and to prevent its early failure.

2. DERIVATION OF INTERLAMINAR STRESSES

2.1 Higher Order Lamination plate Theory

According to classical laminated plate theory, a uniaxial homogeneous tensile force N_{xx} gives rise to homogeneous strain components ε_x^0 ,ε_y^0 and ε_z^0 with

$$\varepsilon_y^0 = -\gamma_{xy}^{eff}\varepsilon_x^0$$
$$\varepsilon_z^0 = -\gamma_{xz}^{eff}\varepsilon_x^0 \qquad (1)$$

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The effective poisson's ratio γ_{xy}^{eff} is given in terms of the extensional stiffnesses A_{12} and A_{22} as,

$$\gamma_{xy}^{eff} = \frac{A_{12}}{A_{22}} \qquad (2)$$

The above strain components correspond to the following displacements in the x,y and z directions,

$$u = u(x) = \varepsilon_x^0 x$$
$$v = v(y) = -\gamma_{xy}^{eff}\varepsilon_x^0 y$$
$$w = w(z) = -\gamma_{xz}^{eff}\varepsilon_x^0 z \qquad (3)$$

Eqn 3 for v and w may be improved by the addition of a warping deformation mode

In v direction : $p(y) + q(y)\cos\mu z$

In w direction : $r(y)\sin\mu z$

Where, $\mu = \frac{\pi}{h}$

Thus we obtain,

$$\begin{bmatrix}\varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\end{bmatrix} = \begin{bmatrix}-\gamma_{xy}^{eff}\varepsilon_x^0 + \dot p(y) + \dot q(y)\cos\mu z\\ -\gamma_{xz}^{eff}\varepsilon_x^0 + \mu r(y)\cos\mu z\\ -\mu\, q(y)\sin\mu z + \dot r(y)\sin\mu z\end{bmatrix}$$

Hook's law gives the following relationship,

$$\begin{bmatrix}\sigma_y\\ \sigma_z\\ \tau_{yz}\end{bmatrix} = \begin{bmatrix}C_{yy} & C_{yz} & 0\\ C_{yz} & C_{zz} & 0\\ 0 & 0 & C_s\end{bmatrix}\begin{bmatrix}\varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\end{bmatrix}$$

$$= \begin{bmatrix}C_{yy}(-\gamma_{xy}^{eff}\varepsilon_x^0 + \dot p(y) + \dot q(y)\cos\mu z) + C_{yz}(-\gamma_{xz}^{eff}\varepsilon_x^0 + \mu r(y)\cos\mu z)\\ C_{yz}(-\gamma_{xy}^{eff}\varepsilon_x^0 + \dot p(y) + \dot q(y)\cos\mu z) + C_{zz}(-\gamma_{xz}^{eff}\varepsilon_x^0 + \mu r(y)\cos\mu z)\\ C_s(-\mu\, q(y)\sin\mu z + \dot r(y)\sin\mu z)\end{bmatrix}$$

2.2 Determination of displacements

In general, it is not possible to determine the unknown functions p(y), q(y) and r(y). However, equilibrium can be satisfied in an average sense for appropriate stress resultants after integrating through the laminate thickness.

(i) $$\int_{-h/2}^{h/2}\left[\sigma_{y,y} + \tau_{yz,z}\right]dz = 0$$

(ii $$\int_{-h/2}^{h/2}\left[\sigma_{y,y} + \tau_{yz,z}\right]\cos\mu z\,dz = 0$$

(iii) $$\int_{-h/2}^{h/2}\left[\tau_{yz,y} + \sigma_{z,z}\right]\sin\mu z\,dz = 0$$

(5)

After the integration, we obtain

$$\begin{pmatrix} C_{yy}^{c_0} & C_{yy}^{c_1} & 0 \\ C_{yy}^{c_1} & C_{yy}^{c_2} & 0 \\ 0 & 0 & C_s^{s_2}\end{pmatrix}\begin{bmatrix}\ddot p\\ \ddot q\\ \ddot r\end{bmatrix} + \begin{pmatrix}0 & 0 & (C_{yz}^{c_1}+C_s^{c_1})\\ 0 & 0 & (C_{yz}^{c_2}+C_s^{c_2})\\ 0 & -(C_{yz}^{s_2}+C_s^{s_2}) & 0\end{pmatrix}\mu\begin{bmatrix}\dot p\\ \dot q\\ \dot r\end{bmatrix} + \mu^2\begin{pmatrix}0 & -C_s^{c_1} & 0\\ 0 & -C_s^{c_2} & 0\\ 0 & 0 & -C_{zz}^{s_2}\end{pmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix} \qquad (6)$$

The general solution is expressed in the following standard form:

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$$\begin{bmatrix} p \\ q \\ r \end{bmatrix}(y) = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} e^{-\lambda \mu y} \tag{7}$$

Where, P, Q, R and λ are undetermined constants.

$$\lambda^2 \begin{pmatrix} C_{yy}^{c_0} & C_{yy}^{c_1} & 0 \\ C_{yy}^{c_1} & C_{yy}^{c_2} & 0 \\ 0 & (C_{yz}^{s_2} + C_s^{s_2}) & C_s^{s_2} \end{pmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} 0 & C_s^{c_1} & (C_{yz}^{c_1} + C_s^{c_1}) \\ 0 & C_s^{c_2} & (C_{yz}^{c_2} + C_s^{c_2}) \\ 0 & 0 & C_{zz}^{s_2} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \tag{8}$$

Therefore, the general homogeneous solution of equation 21 takes the following form:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix}(y) = k_1 \begin{bmatrix} P_1 \\ 0 \\ 0 \end{bmatrix} y + k_2 \begin{bmatrix} P_2 \\ Q_2 \\ R_2 \end{bmatrix} e^{-\lambda_2 \mu y} + k_3 \begin{bmatrix} P_3 \\ Q_3 \\ R_3 \end{bmatrix} e^{-\lambda_3 \mu y} \tag{9}$$

Where, k_1, k_2 and k_3 are free constants.

The free-edge boundary conditions are integrated in the following way (for y=0):

$$\int_{-h/2}^{h/2} \sigma_y dz = 0$$
$$\int_{-h/2}^{h/2} \sigma_y \cos \mu z dz = 0$$
$$\int_{-h/2}^{h/2} \tau_{yz} \sin \mu z dz = 0 \tag{10}$$

The result of the above integration will be,

$$\begin{bmatrix} P_1 C_{yy}^{c_0} & (-\lambda_2 \mu P_2 C_{yy}^{c_0} - \lambda_2 \mu Q_2 C_{yy}^{c_1} + \mu R_2 C_{yz}^{c_1}) & (-\lambda_3 \mu P_3 C_{yy}^{c_0} - \lambda_3 \mu Q_3 C_{yy}^{c_1} + \mu R_3 C_{yz}^{c_1}) \\ P_1 C_{yy}^{c_1} & (-\lambda_2 \mu P_2 C_{yy}^{c_1} - \lambda_2 \mu Q_2 C_{yy}^{c_2} + \mu R_2 C_{yz}^{c_2}) & (-\lambda_3 \mu P_3 C_{yy}^{c_1} - \lambda_3 \mu Q_3 C_{yy}^{c_2} + \mu R_3 C_{yz}^{c_2}) \end{bmatrix} *$$

$$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \varepsilon_x^0 \begin{bmatrix} -\nu_{xy}^{eff} C_{yy}^{c_0} - \nu_{xz}^{eff} C_{yz}^{c_0} \\ -\nu_{xy}^{eff} C_{yy}^{c_1} - \nu_{xz}^{eff} C_{yz}^{c_1} \end{bmatrix} \tag{11}$$

3. CASE STUDIES

3.1 ORTHOTROPIC E-GLASS EPOXY COMPOSITE

For the first case study, as an example, a $[0^0/90^0/0^0/90^0]$ composite laminate is considered. The following properties are taken from the Inter Net. To show the improved results of our new proposed method, the interlaminar stress values are calculated for the E-Glass Epoxy composite laminate for the given load N_{xx} = 1000 KN/m.

Properties:

V_f= 0.65 ; V_m= 0.35

E_f= 70 GPa ; E_m= 2.5 GPa

υ_f= 0.22 ; υ_m= 0.3

Using classical lamination theory, it is obtained that,

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$\varepsilon_x^0 = 3.125 \times 10^{-3}$

$\upsilon_{xy}^{eff} = \frac{A_{12}}{A_{22}} = \frac{0.1961}{0.4524} = 0.4335$

$C_{yy}^{c_0} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yy} dz = 0.1401$

$C_{yy}^{c_1} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yy} \cos \mu z = 0.0892$

$C_{yy}^{c_2} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yy} \cos^2 \mu z dz = 0.0701$

$C_{yz}^{c_0} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yz} dz = 0.0602$

$C_{yz}^{c_1} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yz} \cos \mu z dz = 0.0383$

$C_{yz}^{c_2} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yz} \cos^2 \mu z dz = 0.0301$

$C_{yz}^{s_2} = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{yz} \sin^2 \mu z dz = 0.0301$

To find the Eigen values,

$$\begin{bmatrix} 0 & C_s^{c_1} & C_s^{c_1} + C_{yz}^{c_1} \\ 0 & C_s^{c_2} & C_s^{c_2} + C_{yz}^{c_2} \\ 0 & 0 & C_{zz}^{s_2} \end{bmatrix} = \begin{bmatrix} 0 & 0.0254 & 0.0637 \\ 0 & 0.02 & 0.0501 \\ 0 & 0 & 0.0701 \end{bmatrix}$$

$S_1 = 0.0901$

$S_2 = 1.402 \times 10^{-3}$

$S_3 = 0$

Characteristic equation:

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda = 0, 0.0701, 0.02$

Corresponding Eigen vectors are:

$$\begin{bmatrix} P_1 \\ Q_1 \\ R_1 \end{bmatrix} = \begin{bmatrix} -1.46 \times 10^{-6} \\ 0 \\ 0 \end{bmatrix}; \begin{bmatrix} P_2 \\ Q_2 \\ R_2 \end{bmatrix} = \begin{bmatrix} 4.4630 \times 10^{-3} \\ -3.5192 \times 10^{-3} \\ 3.5793 \times 10^{-3} \end{bmatrix}; \begin{bmatrix} P_3 \\ Q_3 \\ R_3 \end{bmatrix} = \begin{bmatrix} 1.2743 \times 10^{-3} \\ 1.0047 \times 10^{-3} \\ 1.4699 \times 10^{-6} \end{bmatrix}$$

Table I

RESULT TABLE OF FREE EDGE STRESS EVALUATION

FOR E-GLASS EPOXY COMPOSITEFOR Nxx = 1000 kN/m

Distance from the centre, @y	$\tau^{0^0}/_{90^0}$ **(GPa)**	$\sigma^{0^0}/_{90^0}$ **(GPa)**
y=0	0	-1.189×10^{-4}
y=0.001 m	-5.5449×10^{-7}	-1.1848×10^{-4}
y=0.002 m	-1.0952×10^{-6}	-1.1806×10^{-4}
y=0.003 m	-2.0146×10^{-6}	-1.1763×10^{-4}
y=0.004 m	-2.3411×10^{-6}	-1.1721×10^{-4}
y=0.005 m	-2.9018×10^{-6}	-1.1680×10^{-4}
y=0.006 m	-3.4501×10^{-6}	-1.1637×10^{-4}
y=0.007 m	-3.9923×10^{-6}	-1.1596×10^{-4}

y=0.008 m	-4.5221×10^{-6}	-1.1554×10^{-4}
y=0.009 m	-5.1396×10^{-6}	-1.1513×10^{-4}
y=0.010 m	-5.5510×10^{-6}	-1.1471×10^{-4}

3.2 CASE STUDY 2

ORTHOTROPIC GRAPHITE EPOXY COMPOSITE

In the same way as followed above, the interlaminar stresses and the free edge stresses are evaluated in the case of Graphite Epoxy laminate too.

4. FINITE ELEMENT ANALYSIS

E-Glass Epoxy composite laminate is analyzed in ANSYS. The following properties have been given as the material properties. The composite laminate with the dimensions length = 100 mm, breadth = 20 mm and height = 16 mm have been chosen for the analysis.

E_{11} = 46.375 GPa

E_{22} = 6.6986 GPa

E_{33} = 6.6986 GPa

υ_{12} = 0.248

υ_{23} = 0.342

υ_{13} = 0.248

G_{12} = 2.5862 GPa

G_{23} = 2.4958 GPa

G_{13} = 2.5862 GPa

Length, l = 100 mm = 0.1 m

Breadth, w = 20 mm = 0.02 m

Height / thickness, h = 16 mm = 0.016 m

Number of lamina, n = 4

N_{xx} = 1000 kN/m = 1×10^{-3} GN/m

With the above values, E-Glass Epoxy composite laminate has been analyzed using ANSYS software. For the given load, the stress distribution along the composite laminate has been successfully got, as shown in fig.1. On each nodal point, the stress (both normal and shear stress) values have been taken from the Nodal solutions as shown in Fig.2.

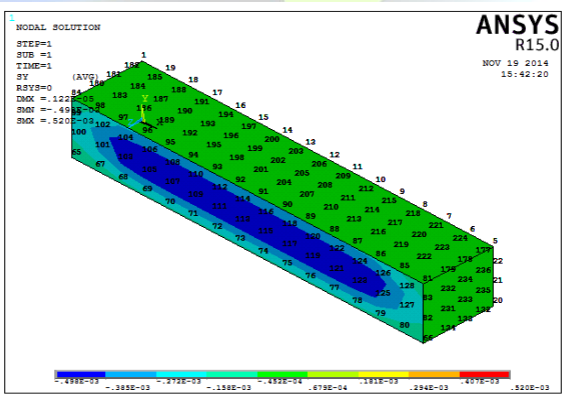

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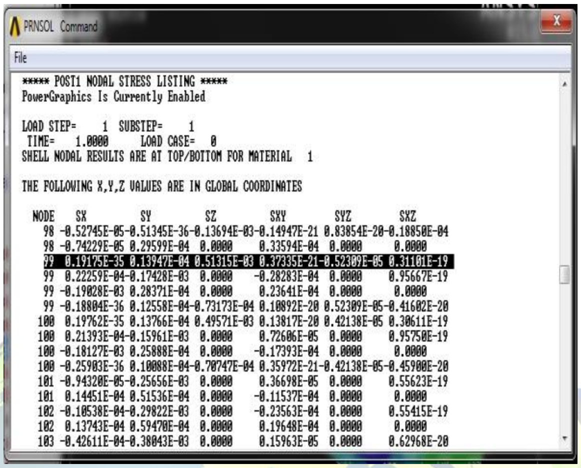
Fig.1.Stress distribution on E-Glass Epoxy composite laminate



PRNSOL Command

File

***** POST1 NODAL STRESS LISTING *****
PowerGraphics Is Currently Enabled

LOAD STEP= 1 SUBSTEP= 1
TIME= 1.0000 LOAD CASE= 0
SHELL NODAL RESULTS ARE AT TOP/BOTTOM FOR MATERIAL 1

THE FOLLOWING X,Y,Z VALUES ARE IN GLOBAL COORDINATES

NODE	SX	SY	SZ	SXY	SYZ	SXZ
98	-0.52745E-05	-0.51345E-36	-0.13694E-03	-0.14947E-21	0.83854E-20	-0.18850E-04
98	-0.74229E-05	0.29599E-04	0.0000	0.33594E-04	0.0000	0.0000
99	0.19175E-35	0.13947E-04	0.51315E-03	0.37335E-21	-0.52309E-05	0.31101E-19
99	0.22259E-04	-0.17428E-03	0.0000	-0.28283E-04	0.0000	0.95667E-19
99	-0.19028E-03	0.28371E-04	0.0000	0.23641E-04	0.0000	0.0000
99	-0.18804E-36	0.12558E-04	-0.73173E-04	0.10892E-20	0.52309E-05	-0.41602E-20
100	0.19762E-35	0.13766E-04	0.49571E-03	0.13817E-20	0.42138E-05	0.30611E-19
100	0.21393E-04	-0.15961E-03	0.0000	0.72606E-05	0.0000	0.95750E-19
100	-0.18127E-03	0.25888E-04	0.0000	-0.17393E-04	0.0000	0.0000
100	-0.25903E-36	0.10088E-04	-0.70747E-04	0.35972E-21	-0.42138E-05	-0.45900E-20
101	-0.94320E-05	-0.25656E-03	0.0000	0.36698E-05	0.0000	0.55623E-19
101	0.14451E-04	0.51536E-04	0.0000	-0.11537E-04	0.0000	0.0000
102	-0.10538E-04	-0.29822E-03	0.0000	-0.23563E-04	0.0000	0.55415E-19
102	0.13743E-04	0.59470E-04	0.0000	0.19648E-04	0.0000	0.0000
103	-0.42611E-04	-0.38043E-03	0.0000	0.15963E-05	0.0000	0.62968E-20

Fig.2. Stress values at the free edge of E-Glass Epoxy composite Laminate

5. CONCLUSION

In this work, a new method of accurate evaluation of free edge stresses in the composite laminates has been formulated. Using classical lamination plate theory also, one can determine the stress values acting in the composite laminates while subjected to loading conditions. But, classical lamination plate theory and all other existing theoretical methods are not sufficient to evaluate the interlaminar stresses at the free edges accurately, because, all those methods have been formulated with the negligence of some main minor modes for simplicity. Getting accurate value of interlaminar stresses is very important in composite laminates while they are being subjected to loading in a particular application.

Therefore, Higher order lamination theory has been derived in this work with the consideration of warping deformation mode, because, the effect of warping mode plays an important role in the phenomenon of delamination. After the formulae have been derived, for two case studies (E-Glasss Epoxy composite laminate and Graphite Epoxy composite laminate), the values of interlaminar stresses and free edge stress have been evaluated using the derived formulae and the values have been tabulated.

Using ANSYS software, the interlaminar stresses in both the case studies (E-Glass Epoxy and Graphite Epoxy) have been evaluated. The results obtained using ANSYS software justified the results obtained from the Higher order theory. As classical lamination theory was used to derive the Higher order lamination theory including warping deformation mode, the results obtained from this method will surely give the improved and accurate results than all other existing theoretical methods. With the values obtained from the classical lamination theory, we can come to a conclusion about our proposed method. That work is going to be carried out in the upcoming phase.

Future work for phase II

An experimental analysis has to be made for comparing and validating the results with the proposed method. Therefore, it has been planned to carry out an experimental test to determine the interlaminar stress values acting in the composite laminates using Universal Testing Machine (UTM) in the upcoming phase of this project work. In addition to that, stress values will be calculated by means of classical lamination theory also for comparison.

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