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Domination in Intuitionistic Fuzzy Graphs of Second Type

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Abstract: In this paper, strong arc, dominating set and domination number in intuitionistic fuzzy graph of second type are discussed. Also some properties of domination in intuitionistic fuzzy graph of second type are studied.

Keywords: Intuitionistic fuzzy graph of second type (IFGST), Strong arc, Dominating set, Domination number, complement of IFGST.

I. INTRODUCTION

In 1965, the notion of fuzzy set was introduced by zadeh [13] as a method of representing uncertainty and vagueness in real life situation. In 1986, Attanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy set. Rosenfeld introduced the notion of fuzzy graphs and several fuzzy analogs of theoretic concepts such as path, cycle and connectedness. Orge and Berge introduced the study of dominating set in graphs. K.T.Attanassov[2] introduce the concept of intuitionistic fuzzy(IF) relations and intuitionistic fuzzy graphs(IFGs). R.Parvathi and M.G.Karunambigai[7] gave a definition for IFG as a special case of IFGs defined by K.T.Attanassov and A.Shanon[10]. Yan Lue [12] defined the cardinality of an IFG. The definitions of order, degree and size gave by A.Nagoor Gani and Shajitha begam.R.Parvathi and G.Tamizhendhi[8] was introduced dominating set, domination number in IFGs. S. Sheik Dhavudh and R. Srinivasan[9] gave the definition of Intuitionistic Fuzzy Graphs of Second Type[IFGST]. Based on the paper [8] and [9], we introduced dominating set and domination number in IFGST.

II. PRILIMINARIES

In this section, some basic definitions relating to IFG and IFGST are given. Also the definition of domination of IFG are studied.

Definition 2.1[5]

An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$, where (i)$V = \{v_1, v_2, \dots, v_n\}$ such that the degree of membership and non - membership of the element $v_i \in V$are defined by $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V, i = 1,2,\dots,n$.

(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le min[\mu_1(v_i), \mu_1(v_j)$, $\gamma_2(v_i, v_j) \le max[\gamma_1(v_i), \gamma_1(vj)$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$.

Here the degree of membership and degree of non-membership of the vertex $v_i$is denote by the triple $(v_i, \mu_{1i}, \gamma_{1i})$. The degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V is defined by the triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$.

Definition 2.2[8]

The cardinality of an $IFG, G = (V, E)$ is defined by $|G| = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2} + \sum_{(v_i, v_j) \in E} \frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)}{2}\right|$ for all $v_i \in V$ and $(v_i, v_j) \in E$.

Definition 2.3[8]

The vertex cardinality of V of an IFG, $G = (V, E)$is defined by $|V| = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right|$for all $v_i \in V$.

Definition 2.4[8]

The edge cardinality of E of an IFG, $G = (V, E)$ is defined by $|E| = \left|\sum_{(v_i, v_j) \in E} \frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)}{2}\right|$ for all$(v_i, v_j) \in E$.

Definition 2.5[8]

Let $G = (V, E)$ be an IFG.The number of vertices (the cardinality of V) is called the order of G and is denoted by $O(G) = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right|$ for all $v_i \in V$.

Definition 2.6[8]

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Let $G=(V,E)$ be an IFG. The number of edges (the cardinality of E) is called the size of G and is denoted by $S(G) = \left|\sum_{(v_i,v_j)\in E} \frac{1+\mu_2(v_i,v_j) - \gamma_2(v_i,v_j)}{2}\right|$ all $(v_i,v_j) \in E$.

Definition 2.7[8]

Let $G=(V,E)$ be an IFG. If v_i, v_j are vertices in G, then $\mu_2^{\infty}(v_i,v_j) = sup\{\mu_2^k(v_i,v_j) | k = 1,2,\dots,n\}$ is called the μ− strength of connectedness between v_i and v_j and $\gamma_2^{\infty}(v_i,v_j) = \inf\{\gamma_2^k(v_i,v_j) | \ k = 1,2,\dots,n\}$ is called the $\gamma -$ strength of connectedness between v_i and v_j. If the path of length k connected by the vertices u,v then $\mu_2^k(u,v)=$ $\sup\{\mu_2(u,v_1) \wedge \mu_2(v_1,v_2) \wedge \mu_2(v_2,v_3) \dots \wedge \mu_2(v_{k-1},$ such that $(u,v_1,v_2 \dots v_{k-1},v \in V)\}$ and $\gamma_2^k(u,v = inf\{\gamma_2(u,v_1) \vee \gamma_2(v_1,v_2) \vee \gamma_2(v_2,v_3) \dots \vee \gamma_2(v_{k-1},v)$ such that $(u,v_1,v_2 \dots v_{k-1},v) \in V\}$

Definition 2.8[7]

Let $G=(V,E)$ be an IFG. The complement of G is an IFG, $\bar{G} = (\bar{V},\bar{E})$ where

(i) $\bar{V} = V$

(ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$ for all $i = 1,2,\dots,n$.

(iii) $\overline{\mu_{2ij}} = min(\mu_{1i},\mu_{1j}) - \mu_{2ij}$ and $\overline{\gamma_{2ij}} = max(\gamma_{1i},\gamma_{1j}) - \gamma_{2ij}$ for all $i,j = 1,2,\dots,n$.

Definition 2.9[9]

An Intuitionistic Fuzzy Graph of second type($IFGST$) is of the form $G=(V,E)$, where (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degree of membership and non - membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i)^2 + \gamma_1(v_i)^2 \le 1$, for every $v_i \in V, i = 1,2,\dots,n$.

(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that $\mu_2(v_i,v_j) \le min[\mu_1(v_i)^2, \mu_1(v_j)^2]$, $\gamma_2(v_i,v_j) \le max[\gamma_1(v_i)^2, \gamma_1(v_j)^2]$ and $0 \le \mu_2(v_i,v_j)^2 + \gamma_2(v_i,v_j)^2 \le 1$, for every $(v_i,v_j) \in E$, $(i,j = 1,2,\dots,n)$.

III.MAIN RESULTS

In this section we defined domination in IFGST and also some properties of domination in IFGST.

Definition 3.1

Let $G=(V,E)$ be an IFGST and let $(u,v) \in E$ be an arc in G. If $\mu_2(u,v) \ge \mu_2^{\infty}(u,v)$ and $\gamma_2(u,v) \ge \gamma_2^{\infty}(u,v)$ then arc is called strong arc.

Example 3.1.

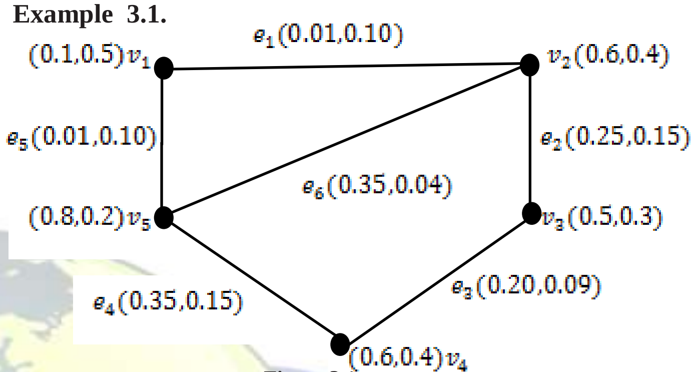

Figure 3.1

In the above figure, for edge e_1

$\mu_2(v_1,v_2) = 0.01, \gamma_2(v_1,v_2) = 0.10,$

$\mu_2^{\infty}(v_1,v_2) = sup\{(0.00 \wedge 0.35), (0.00 \wedge 0.35 \wedge 0.20 \wedge 0.25)\}$

$= sup\{0.00, 0.00\} = 0.00$

$\mu_2(v_1,v_2) \ge \mu_2^{\infty}(v_1,v_2)$

$\gamma_2^{\infty}(v_1,v_2) = inf\{(0.03 \vee 0.04), (0.03 \vee 0.15 \vee 0.09 \vee 0.15)\}$

$= inf\{0.04, 0.15\} = 0.04$

$\gamma_2(v_1,v_2) \ge \gamma_2^{\infty}(v_1,v_2)$

$\therefore e_1$ is a strong arc.

Similarly, e_2 and e_4 are strong arc.

Definition 3.2

Let $G=(V,E)$ be an IFGST and let any two vertices $u,v \in V$. We say that u dominates v in G if there exists a strong arc between them.

Example 3.2

In figure 3.1, v_2 dominates v_3 and v_1, v_4 dominates v_5 and vice versa.

Definition 3.3

Let $G=(V,E)$ be an IFGST. A subset S of V is said to be a dominating set in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

Example 3.3

In figure 3.1, dominating sets of G are $D_1 = \{v_1, v_3, v_5\}$, $D_2 = \{v_1, v_3, v_4\}$, $D_3 = \{v_2, v_4\}$ and $D_4 = \{v_2, v_5\}$.

Definition 3.4

Let $G=(V,E)$ be an IFGST. Let S be the dominating set of G. The dominating set S is called minimal dominating set, if no proper subset of G is dominating set.

Example 3.4

In figure 3.1 (or in Example 3.3), the minimal

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dominating set are $D_3 = \{v_2, v_4\}$ and $D_4 = \{v_2, v_5\}$.

Definition 3.5

Let $G = (V, E)$ be an IFGST. The lower domination number of G is defined as the minimum cordinality among all minimal dominating set of G and it is denoted by $d(G)$.

The upper domination number of G is defined as the minimum cordinality among all minimal dominating set of G and it is denoted by $D(G)$.

Example 3.5

In figure 3.1 (in Example 3.4), $d(G) = 1.2, ie., |D_3|$; $D(G) = 1.4, ie., |D_4|$.

Definition 3.6

Let $G = (V, E)$ be an IFGST. A vertex $v \in V$ is called an isolated vertex if $\mu_2(u, v) = 0$ and $\gamma_2(u, v) = 0\ \forall\ u, v \in V$.

Example 3.6.

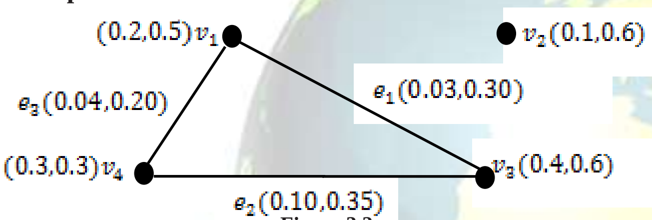

Figure 3.2

In the above Figure 3.2, v_2 is an isolated vertex of G.

Note : An isolated vertex does not dominate any other vertex in IFGST.

Definition 3.7

Let $G = (V, E)$ be an IFGST.The complement of G is on IFGST, $\bar{G} = (\bar{V}, \bar{E})$ here

(i) $\bar{V} = V$

(ii) $\bar{\mu}_{1i} = \mu_{1i}$ and $\bar{\gamma}_{1i} = \gamma_{1i}$ for all $i = 1,2, \ldots, n$.

(iii) $\bar{\mu}_{2ij} = \min({\mu_{1i}}^2, {\mu_{1j}}^2) - \mu_{2ij}$ and

$\bar{\gamma}_{2ij} = \max({\gamma_{1i}}^2, {\gamma_{1j}}^2) - \gamma_{2ij}$

for all $i = 1,2, \ldots, n$.

Example 3.7.

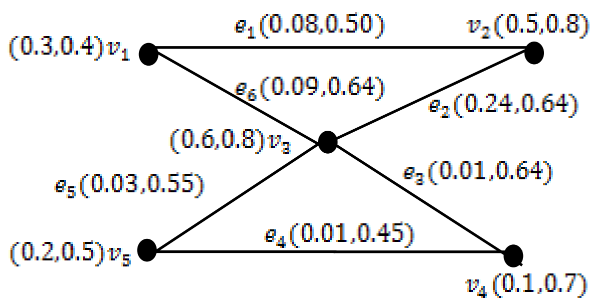

Figure 3.3 G

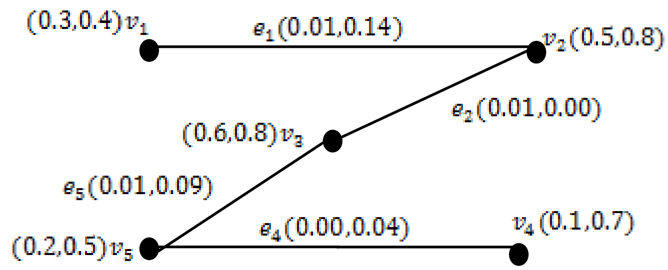

Figure 3.4 $\bar{G}$

Theorem 3.1.

A dominating set D of an intuitionistic fuzzy graph of second type G is minimal if and only if for each $d \in D$ one of the following conditions is satisfied.

(i) d is not a strong neighbor of any vertex in D.

(ii) There is a vertex $v \in V - D$ such that $N(v) \cap D = d$.

Proof :

Let $G = (V, E)$ be an $IFGST$ and let a minimal dominating set of G is D. Then for every vertex $d \in D$, we find that $D - \{d\}$ is not a dominating set and hence there exists $v \in V - (D - \{d\})$ which is not dominated by any vertex in $D - \{d\}$. If $v = d$, we get v is not a strong neighbor of any vertex in D and $v = d$, v is not dominated by $D - \{v\}$ but is dominated by D, then the vertex v is strong neighbor only to d in D. Therefore, $N(v) \cap D = \{d\}$.

Conversely, assume that D is a dominating set and for each vertex $d \in D$, one of the above conditions holds. Suppose D is not a minimal dominating set, then there exists a vertex $d \in D$, $D - \{d\}$ is a dominating set. Hence d is a strong neighbor to atleast one vertex in $D - \{d\}$, the condition (i) does not hold. If $D - \{d\}$ is a dominating set then every vertex in $V - D$ is a strong neighbor to atleast one vertex in $D - \{d\}$, the condition(ii) does not hold, which is a contradiction our assumption that atleast one of these conditions holds. Therefore D is a minimal dominating set.

Theorem 3.2.

Let $G = (V, E)$ be an $IFGST$ without isolated vertices and D is a minimal dominating set. Then $V - D$ is a dominating set of G.

Proof :

Let $G = (V, E)$ be an $IFGST$ and let a minimal dominating set of G is D. Let v be any vertex of D. Given that G has no isolated vertices, there is a vertex $d \in N(v)$. v must be dominated by atleast one vertex in $D - \{v\}$, that is $D - \{v\}$ is a dominating set. By theorem 3.1, it follows that $d \in V - D$. Therefore each vertex in D is dominated by atleast one vertex in $V - D$ and $V - D$ is a dominating set.

Theorem 3.3.

For any $IFGST$, $d(G) + d(\bar{G}) \leq 2O(G)$, where $d(G)$ is the lower domination number of G and equality holds if and only if $0 < \mu_2(v_i, v_j) < \mu_2^{\infty}(v_i, v_j)$ and $0 < \gamma_2(v_i, v_j) < \gamma_2^{\infty}(v_i, v_j)$ for all $v_i, v_j \in V$.

Proof :

When $d(G) + d(\bar{G}) < 2\,O(G)$ the result is obvious. If $d(G) = O(G)$ if and only if $\mu_2(v_i, v_j) < \mu_2^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) < \gamma_2^{\infty}(v_i, v_j)$ for all $v_i, v_j \in V$. d(G) = 2 O(G) if and only if $\mu_2(v_i, v_j) - \mu_2(v_i, v_j) < \mu_2^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) - \gamma_2(v_i, v_j) < \gamma_2^{\infty}(v_i, v_j)$ for all $v_i, v_j \in V$ which implies $\mu_2(v_i, v_j) > 0$ and $\gamma_2(v_i, v_j) > 0$. Therefore, $d(G) + d(\bar{G}) \leq 2O(G)$.

Corollary 3.4.

Let $G = (V, E)$ be any $IFGST$ with no isolated vertices, then $d(G) \leq O(G)/2$.

Corollary 3.5.

Let $G = (V, E)$ be an $IFGST$ such that both G and $\bar{G}$ have no isolated vertices. Then $d(G) + d(\bar{G}) \leq O(G)$, where $d(\bar{G})$ is the lower domination number of $\bar{G}$. Further equality holds if and only if $d(G) = d(\bar{G}) = O(G)$.

Theorem 3.6.

$d(G) \leq O(G) - \Delta_N$

Proof :

Let $G = (V, E)$ be an $IFGST$ and let $v \in V$ in G. Assume that $d_N(v) = \Delta_N$. Then $V - N(v)$ is a dominating set of G, therefore $d(G) \leq |V - N(v)| = O(G) - \Delta_N$.

IV. CONCLUSION

Here we have defined strong arc, dominating set and domination number in IFGST . Also some theorems related to the concepts are discussed. In future some more theorems and application of IFGST are studied.

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