



Design Of Mathematical Analysis Of Nozzle For High-Powered Solid Rocket Propellant

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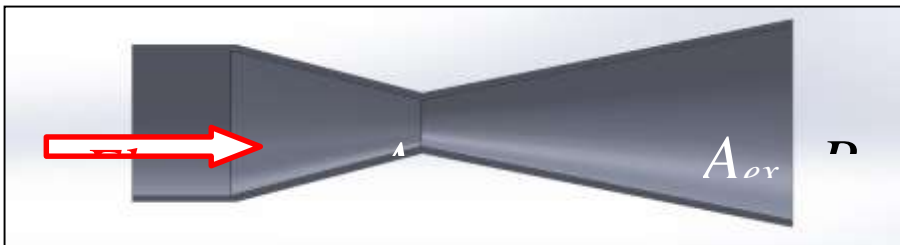
Abstract

A high-altitude rocket to be built by SOAR, the Society of Aeronautics and Rocketry. Methods used for analysis came from professional sources via text or informational videos. The equations in focus will look at the temperature/pressure/area-Mach relationships for isentropic flow, pressure expansions in selected materials, and finding other characteristic properties using safe chemical property assumptions using CEARUN. The equations were used under the assumption of an isentropic flow under sea level conditions. If the design and the projected results look promising, they will be implemented soon for machining and eventually a test fire. The Society of Aeronautics and Rocketry (SOAR) at the University of South Florida is a multidisciplinary engineering organization, with a strong interest and focus in aerospace projects and research. One of the SOAR projects under development is a high-powered rocket designed to break the collegiate apogee record. A rocket of this caliber will require stronger build materials, more advanced and powerful means of propulsion and thorough analysis. The first major consideration for a rocket of this magnitude is the size and power of the motor, to which SOAR will use a solid propellant due to available resources and mentor experience. This paper will look specifically to the design and analysis of a solid propellant and optimized rocket nozzle to deliver a student-built launch vehicle to record heights

Problem Statement

A strong interest and focus in aerospace projects research. One of the SOAR projects under development is a high-powered rocket designed to break the collegiate apogee record. A rocket of this caliber will require stronger build materials, more advanced and powerful means of propulsion and thorough analysis. The first major consideration for a rocket of this magnitude is the size and power of the motor, to which SOAR will use a solid propellant due to available resources and mentor experience. This paper will look specifically to the design and analysis of a solid propellant and optimized rocket nozzle to deliver a student-built launch vehicle to record heights. The research and effort into this project is to present analytic methods and approaches to effectively selecting solid propellant propulsion systems for collegiate and amateur rocketeers. This is a major step towards not only facilitating learning of more advanced engineering projects, but also presenting research opportunities and collaborative efforts with professionals and other students. The goal of this project is to advance research for the high-altitude program in SOAR and aid in producing an important subsystem for the rocket.

change in acceleration and velocity, and a rocket itself does not produce lift like an aircraft thus needing more reliance from a powerful propulsive force versus a sustained lifting force. The thrust, or the ejected propulsive force, comes from the exhaust gases produced by the combustion reaction which are accelerated through a nozzle. Nozzle design and casing considerations will be discussed later. The basic principles of converging-diverging nozzle theory are as follows;



Mathematical Description

To manufacture and successfully launch a high-altitude rocket, a mission goal and an altitude goal must be set to aid preliminary designs. Payloads and scientific experiments must be designed properly to avoid potential system and launch vehicle failures, and to optimize both physical space and weight for the rocket. Before we discuss how to design an effective solid propellant propulsion system, it is important to know the general operational and chemical procedures of traditional r

Methods. The main “ingredients” are heat, oxygen and fuel. The oxygen is presented through oxidizing agents like powdered

chemicals, or liquid oxygen. The fuel is presented through combustible and flammable material like metals or special synthetic materials and catalysts. The initial source heat is necessary to start a combustion reaction between all three of these main “ingredients”. Using air breathing ventilation systems to provide the oxidizing agent is not a reliable source for rocket propulsion. It does not create a large enough combustible reaction to produce a significant and needed

1. The converging section lies between the throat of the nozzle, A_{throat} , and the combustion chamber. This area is characterized by a mass flow of subsonic speed. For subsonic speeds, as area decreases then velocity of the fluid increases.
2. The throat of the nozzle is intended to create a sonic state. Here, the velocity of the fluid is equal to the speed of sound and thus represents an effectively “choked” nozzle.
3. The diverging section of the nozzle lies between the throat and the exit, A_{exit} . In this section, the fluid undergoes supersonic flow. For a supersonic flow, as the area increases so does the velocity of the fluid. This results in a Mach number greater than 1 at the nozzle exit.
4. For an effective nozzle, the exit pressure of the nozzle, P_{exit} , must equal the atmospheric pressure, P_{atm} , adjacent to the nozzle exit. In the case where $P_{exit} > P_{atm}$, the flow experiences over-expanded flow and under-expanded flow for when $P_{exit} < P_{atm}$. These irregular flows decrease the overall efficiency of

propulsion system but are manageable. Mechanisms and designs such as extending nozzles are designed to ensure for the exit pressure of the exhaust gases to be equal to the atmospheric pressure.

Solid propellants, or motors, are preferable for high-altitude flights due to their ease of design and manufacturing methods. Typical motors use a mixture of powdered chemicals and liquid agents to bond and cure these combinations into reactive fuels. Ammonium Perchlorate, NH_4O_4 , is the most common and among the most reliable and readily available oxidizers on the market. There are varied sizes for purchase, ranging from 90 microns in diameter to 400 and greater. Metal powders and additives such as magnesium, aluminum and beryllium act as both a catalyst and a fuel. Other elements of motors such as binders and plasticizers

bound all these ingredients together. Motors are commonly stored in metal tubular casing divided into a desired number of segments called grains, where they will burn upon ignition and expel their gases out of an open end fixed with a nozzle. There are many possible combinations to formulate and test, but there are restrictions given from the launch vehicle system based on materials selection. Two equations can be used to approximate the expansion of a motor casing, for both length and diameter, during a propellant burn due to chamber pressure:

where ΔL is the expansion longitudinally in inches, ΔD is the expansion diametrically, p is the chamber pressure in pounds/in², L is the length of the case in inches, D is the inner diameter of the case in inches, E is Young's Modulus of Elasticity measured in kilopound/in² (or 1000 lb/in²), d is the casing thickness in inches, and ν is Poisson's ratio specific to the casing material.

The Ares team and the Society of Aeronautics and Rocketry have purchased 10-foot-long 6 ID x 6.5" OD tubing composed of 6061-T6 Aluminum. This type of aluminum is incredibly strong, relatively lightweight and is commonly used as motor casings for commercially produced amateur rocket motors. A minimum diameter rocket is a type of rocket which uses the walls of the rocket airframe as the casing for the motor, whereas traditional collegiate and amateur building methods use a motor mount to fit a motor which has a smaller diameter than the airframe. High-altitude rockets are incredibly efficient when they are built minimum diameter, therefore maximizing the potential amount of solid propellant.

There are several material characteristics of metals to pay attention to: Young's Modulus of Elasticity, thickness, and Elongation at Break. Elongation at Break is a characteristic in metals that defines the total longitudinal stretch allowed before the material breaks, and is given as a percentage of the original length. For 6061-T6 Aluminum, the elongation at break is 12%, the Young's Modulus is 10,000 ksi, Poisson's Ratio is 0.33 and the thickness is 1/4" in this particular case.

An interesting trend develops based on these numbers. The maximum internal psi before fracture or breakage is the same for a certain uniform thickness despite the length of the motor casing. To safely ensure a higher chamber pressure is achieved without failure, increase the thicknesses of the case walls and or finding a material with a higher Young's Modulus or lower Poisson's ratio.

The exact propellant mixture is not yet known, NH₄ClO₄ will serve as the main oxidizing compound. There are general recipes found online for high-performance applications including military missiles and the SRBs used for the Space Shuttle. Solid propellants, especially at the collegiate and

amateur level, rarely burn for more than half a minute due to limited resources and obvious size restrictions.

A common fuel and binder are available for selection on CEARUN, Al(cr) and PBAN. PBAN, polybutadiene acrylonitrile, is a copolymer used as a binding agent that is slower to cure but less toxic and lower cost than HTPB,

THEORETICAL ROCKET		EQUILIBRIUM	
COMPOSITION DURING EX		AREA COMBUSTOR	
Pin = 588.2 PSIA			
CASE = Ares4184			
REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
PBAN	0.1233000	-63220.000	298.150
NH4ClO4(I)	0.7128000	-295767.000	298.150
AL(cr)	0.1639000	0.000	298.150
O/F= 0.00000		%FUEL=100.000000	
		R,EQ.RATIO= 1.571894	
		PHI,EQ.RATIO= 0.000000	
	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.7285	40.016
P, BAR	40.557	23.464	1.0135
T, K	3490.65	3310.39	2327.00
RHO, KG/CU M	4.0934	2.5202	1.6037
H, KJ/KG	-1872.37	-2398.09	-4836.60
U, KJ/KG	-2863.18	-3329.13	-5468.60
G, KJ/KG	-35063.0	-33874.7	-26962.7
S, KJ/(KG)(K)	9.5084	9.5084	9.5084
M, (1/n)	29.292	29.563	30.614
MW, MOL WT	27.071	27.238	28.011
(dLV/dLP)t	-1.02568	-1.02057	-1.00236
(dLV/dLT)p	1.4496	1.3744	0.0000
Cp, KJ/(KG)(K)	4.3020	3.9333	0.0000
GAMMA _s	1.1273	1.1293	0.9976
SON VEL, M/SEC	1056.9	1025.4	794.0
MACH NUMBER	0.000	1.000	3.066
PERFORMANCE PARAMETERS			
Ae/At	1.0000	6.6182	
CSTAR, M/SEC	1569.4	1569.4	
CF	0.6534	1.5514	
Ivac, M/SEC	1933.4	2694.4	
Isp, M/SEC	1025.4	2434.8	

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Rocket

Propulsion Elements, there is a range given for the burning rate from 0.25 to 1 at 1000 psia. A linear or

proportional relationship cannot be made to compare 1000 psia to the max chamber pressure of 588.235 psi, for the behavior of solid propellants and their reactions represents more exponential forms. The graph is a log-log plot and shows the number 0.28, which roughly represents the burning rate at 588.235 psia for an ambient temperature around 70 degrees Fahrenheit:

Now, use a as the Burning Rate Coefficient for the "PBAN / AP / Al" custom propellant in BurnSim3 along with the other characteristics to approximate thrust and chamber pressure values for the propellant of choice.

Nozzle & Thrust		Propellants		Startup	
Name: PBAN / AP / AI					
Standard Properties		Pressure Varied Properties		Notes	
C* :	7784.4	ft / sec	S. Heat Ratio	1.1273	
Char. ISP:	242	sec	Mol. Mass	0	
BR Coef (a):	.04695				
BR Exp (n):	.21				
Density :	0.06074	lb / in. ^3			
New		Edit		Delete	

and so forth.
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ally linear. The

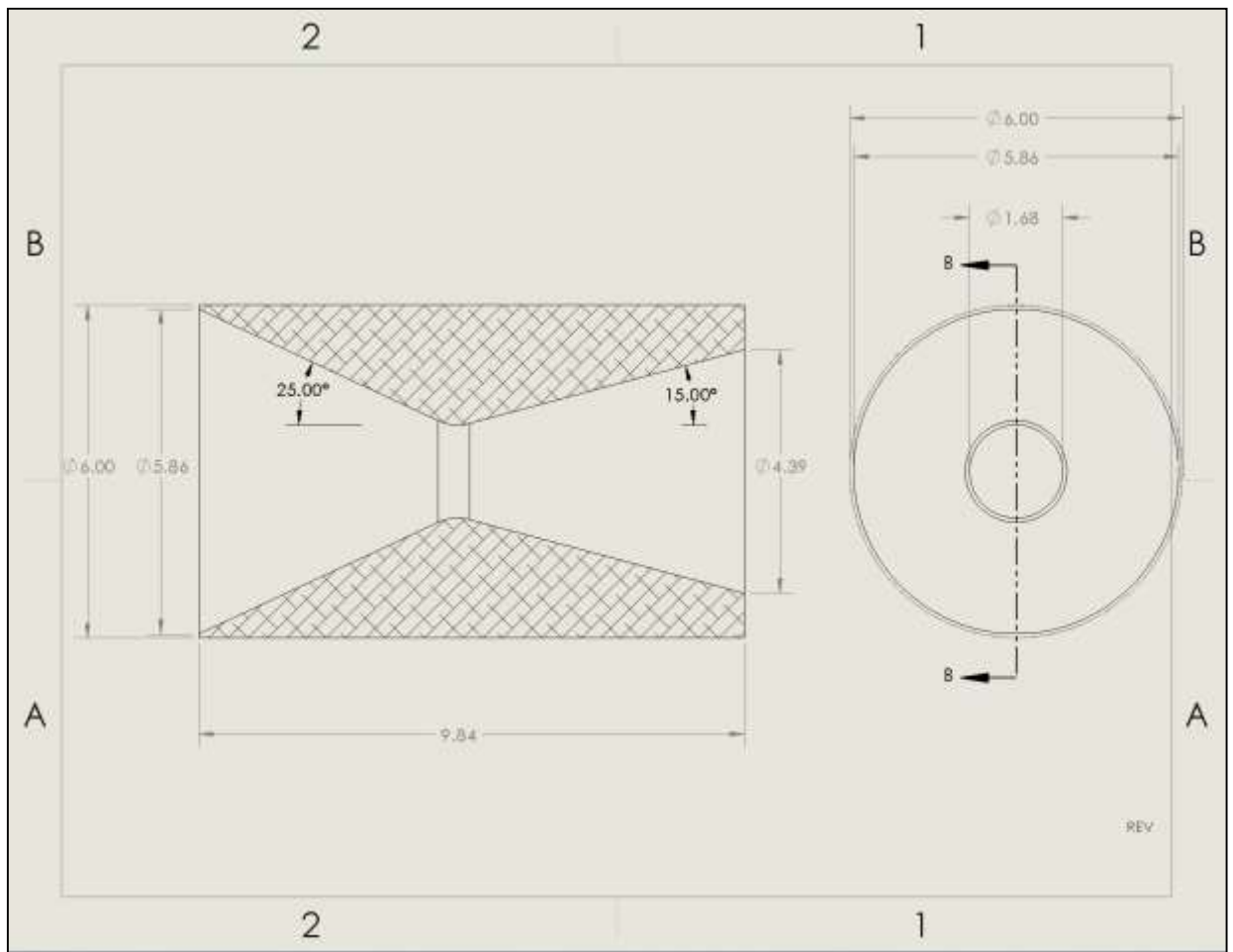


Figure 9: The first complete design of the Ares 1-S nozzle.

Conclusion

If the calculations and mathematical methods have been applied correctly, then with a solid propellant composed of roughly 72% Ammonium Perchlorate, 16% Aluminum and 12% PBAN reaching a max contained combustion pressure of 588.235 psi, there will be an isentropic and resulting supersonic flow in the nozzle with the dimensions specified in Figure 12. There is a suspected exit Mach number of 2.85, and a flow rate of 8.1 lbs/s but these were dependent upon the thermal and chemical properties. The real propellant will have several more ingredients and its properties cannot be known until an effective test and data are received and interpreted. The area of the throat and exit, and their subsequent expansion ratio are subject to change and likely will reflect the differences in

chamber pressure, Temperature and propellant flow rate. A recommendation to anyone else performing this analysis is to have thermal properties data on hand, and to compare between types and compositions of propellants. Propellants with lower combustibility or slower combustion rates can contribute to a lower internal pressure and thus change every dimension of the nozzle. Another recommendation would be to invest in BurnSim3 we use, for it is hard to formulate the throat diameter and area using only pressure and temperature. The area expansion ratio can be found, but many equations are dependent on the throat, and exit, of the nozzle. BurnSim3 also takes into consideration the propellant properties and can estimate the average thrust, total impulse, burn time and many other important motor features that were not covered in this paper.

