



Design of Mathematical analysis of Nozzle for High-Powered Solid Rocket Propellant

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Abstract

A high-altitude rocket to be built by SOAR, the Society of Aeronautics and Rocketry. Methods used for analysis came from professional sources via text or informational videos. The equations in focus will look at the temperature/pressure/area-Mach relationships for isentropic flow, pressure expansions in selected materials, and finding other characteristic properties using safe chemical property assumptions using CEARUN. The equations were used under the assumption of an isentropic flow under sea level conditions. If the design and the projected results look promising, they will be implemented soon for machining and eventually a test fire. The Society of Aeronautics and Rocketry (SOAR) at the University of South Florida is a multidisciplinary engineering organization, with a strong interest and focus in aerospace projects and research. One of the SOAR projects under development is a high-powered rocket designed to break the collegiate apogee record. A rocket of this caliber will require stronger build materials, more advanced and powerful means of propulsion and thorough analysis. The first major consideration for a rocket of this magnitude is the size and power of the motor, to which SOAR will use a solid propellant due to available resources and mentor experience. This paper will look specifically to the design and analysis of a solid propellant and optimized rocket nozzle to deliver a student-built launch vehicle to record heights



Methods. The main “ingredients” are heat, oxygen and fuel. The oxygen is presented through oxidizing agents like powdered chemicals, or liquid oxygen. The fuel is presented through combustible and flammable material like metals or special synthetic materials and catalysts. The initial source heat is necessary to start a combustion reaction between all three of these main “ingredients”. Using air breathing ventilation systems to provide the oxidizing agent is not a reliable source for rocket propulsion. It does not

create a large enough combustible reaction to produce a significant and needed change in acceleration and velocity, and a rocket itself does not produce lift like an aircraft thus needing more reliance from a powerful propulsive force versus a sustained lifting force. The thrust, or the ejected propulsive force, comes from the exhaust gases produced by the combustion reaction which are accelerated through a nozzle. Nozzle design and casing considerations will be discussed later. The basic principles of converging-diverging nozzle theory are as follows;

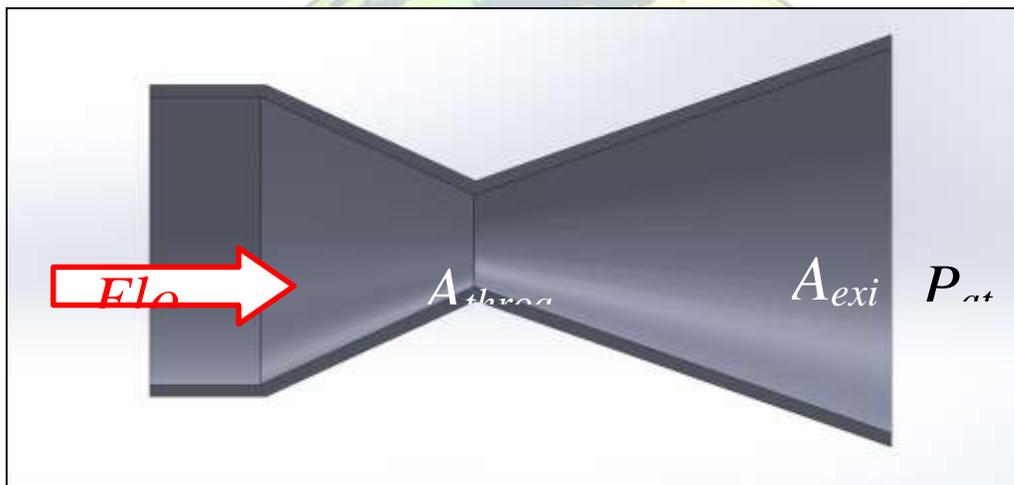


Figure 1: A simple diagram of a converging-diverging nozzle



1. The converging section lies between the throat of the nozzle, A_{throat} , and the combustion chamber. This area is characterized by a mass flow of subsonic speed. For subsonic speeds, as area decreases then velocity of the fluid increases.
2. The throat of the nozzle is intended to create a sonic state. Here, the velocity of the fluid is equal to the speed of sound and thus represents an effectively “choked” nozzle.
3. The diverging section of the nozzle lies between the throat and the exit, A_{exit} . In this section, the fluid undergoes supersonic flow. For a supersonic flow, as the area increases so does the velocity of the fluid. This results in a Mach number greater than 1 at the nozzle exit.
4. For an effective nozzle, the exit pressure of the nozzle, P_{exit} , must equal the atmospheric pressure, P_{atm} , adjacent to the nozzle exit. In the case where $P_{exit} > P_{atm}$, the flow experiences over-expanded flow and under-expanded flow for when $P_{exit} < P_{atm}$. These irregular flows decrease the overall efficiency of the propulsion system but are unavoidable. Mechanisms and subsystems such

as extending nozzles are designed to ensure for the exit pressure of the exhaust gases to be equivalent to the atmospheric pressure.

Solid propellants, or motors, are preferable for high-altitude flights due to their ease of design and manufacturing methods. Typical motors use a mixture of powdered chemicals and liquid agents to bond and cure these combinations into reactive fuels. Ammonium Perchlorate, NH_4O_4 , is the most common and among the most reliable and readily available oxidizers on the market. There are varied sizes for purchase, ranging from 90 microns in diameter to 400 and greater. Metal powders and additives such as magnesium, aluminum and beryllium act as both a catalyst and a fuel. Other elements of motors such as binders and plasticizers bound all these ingredients together. Motors are commonly stored in metal tubular casing divided into a desired number of segments called grains, where they will burn upon ignition and expel their gases out of an open end fixed with a nozzle. There are many possible combinations to formulate and test, but there are restrictions given from the launch vehicle system based on materials selection. Two equations can be used to approximate the expansion of a motor casing, for both length and diameter, during a propellant burn due to chamber pressure:



Examine the chart below to note the differences between thicknesses, casing length and elongation;

6061-T6 Aluminum, 1/4" thickness		
<u>Length</u>	<u>ΔL</u>	<u>Max psi</u>
15"	1.89"	588.235
30"	2.88"	588.235
45"	3.72"	588.235
60"	4.96"	588.235
72"	8.72"	588.235
78"	9.12"	588.235
82"	8.94"	588.235
6061-T6 Aluminum, 1/2" thickness		
<u>Length</u>	<u>ΔL</u>	<u>Max psi</u>
15"	1.89"	1176.471
30"	2.88"	1176.471
45"	3.72"	1176.471
60"	4.96"	1176.471
72"	8.72"	1176.471
78"	9.12"	1176.471
82"	8.94"	1176.471

Figure 2: A visual for the relationship between elongation at break and max internal pressure



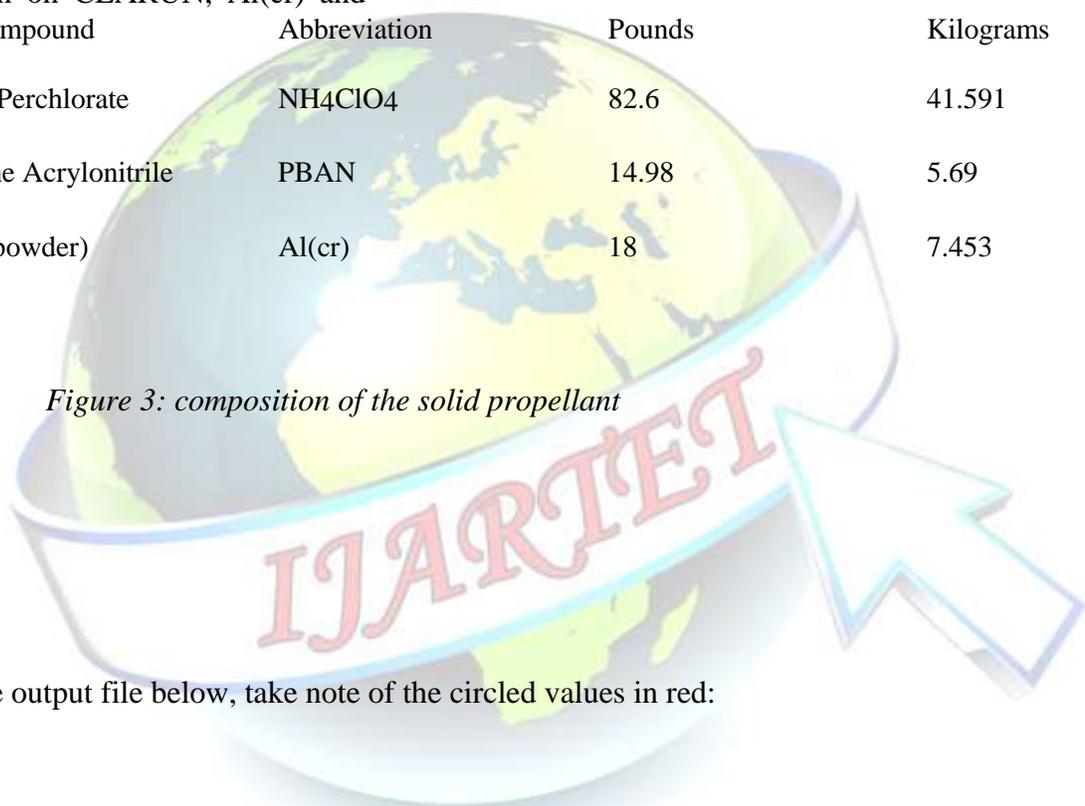
he exact propellant mixture is not yet known, NH_4ClO_4 will serve as the main oxidizing compound. There are general recipes found online for high-performance applications including military missiles and the SRBs used for the Space Shuttle. Solid propellants, especially at the collegiate and amateur level, rarely burn for more than half a minute due to limited resources and obvious size restrictions.

A common fuel and binder are available for selection on CEARUN, Al(cr) and

PBAN. PBAN, polybutadiene acrylonitrile, is a copolymer used as a binding agent that is slower to cure but less toxic and lower cost than HTPB, hydroxyl-terminated polybutadiene. Using the following composition of Al(cr), PBAN and NH_4ClO_4 , and assuming these three compounds take up the full 100lb estimate (this would not be the case for the actual motor composition), convert it into kilograms

Compound	Abbreviation	Pounds	Kilograms
Ammonium Perchlorate	NH_4ClO_4	82.6	41.591
Polybutadiene Acrylonitrile	PBAN	14.98	5.69
Aluminum (powder)	Al(cr)	18	7.453

Figure 3: composition of the solid propellant



Viewing the output file below, take note of the circled values in red:



THEORETICAL ROCKET PERFORMANCE ASSUMING EQUILIBRIUM
 COMPOSITION DURING EXPANSION FROM INFINITE AREA COMBUSTOR

Pin = 588.2 PSIA
 CASE = Ares4184

REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
PBAN	0.1233000	-63220.000	298.150
NH4CLO4(I)	0.7128000	-295767.000	298.150
AL(cr)	0.1639000	0.000	298.150

O/F= 0.00000 %FUEL=100.000000 R, EQ. RATIO= 1.571894 PHI, EQ. RATIO= 0.000000

	CHAMBER	THROAT	EXIT
PinF/P	1.0000	1.7285	40.016
P, BAR	40.557	23.464	1.0135
T, K	3490.65	3310.39	2327.00
RHO, KG/CU M	4.0934	2.5202	1.6037
H, KJ/KG	-1872.37	-2398.09	-4836.60
U, KJ/KG	-2863.18	-3329.13	-5468.60
G, KJ/KG	-35063.0	-33874.7	-26962.7
S, KJ/(KG)(K)	9.5084	9.5084	9.5084
M, (1/n)	29.292	29.563	30.614
MW, MOL WT	27.071	27.238	28.011
(dLV/dLP)t	-1.02568	-1.02057	-1.00236
(dLV/dLT)p	1.4496	1.3744	0.0000
Cp, KJ/(KG)(K)	4.3020	3.9333	0.0000
GAMMA _s	1.1273	1.1293	0.9976
SON VEL, M/SEC	1056.9	1025.4	794.0
MACH NUMBER	0.000	1.000	3.066

PERFORMANCE PARAMETERS

Ae/At	1.0000	6.6182
CSTAR, M/SEC	1569.4	1569.4
CF	0.6534	1.5514
Ivac, M/SEC	1933.4	2694.4
Isp, M/SEC	1025.4	2434.8

Figure 4: The CEARUN chart, with the red circled values being the ones focused on in this paper

Without inputting sub or supersonic area ratios or chamber temperature, CEARUN will give a reliable estimate for these values. The circled values will be needed for computing the mass flow rate later.

BurnSim3 is a very helpful program for examining grain geometries for general

propellant compositions, and approximating key characteristics of such propellants. There are two propellant types available for the user, but in order to simulate as accurately as possible, the propellant of choice listed above (Al, PBAN and NH₄CIO) will be created within the program. Chapters 12 and 13 of Rocket



Maximum chamber pressure and so forth.

However, just like the burning rate equation, this relationship is not

$$Kn = \frac{A_p}{A_t} \quad (2)$$

Using the dimensions from the grain geometry in Figure 6 we obtain:

$$\frac{494.62}{15 \text{ in}^2} = 223$$

$$2.217 \text{ in}^2$$

It is possible to calculate the Mach exit speed based upon chamber and exit

The left side of the equation contains p_e , exit or ambient pressure at sea level which is 14.7 psi, and p_c , chamber pressure which is 588.235 psi. Any unit for pressure is acceptable here since it represents only a ratio and Mach number, M , and specific heat ratio, γ , is dimensionless. Recalling from the circled values produced by CEARUN, the Mach can be estimated based on the parameters for system pressure. The roots of the equation can be used to find M , although values greater than 10 would be discarded, for

proportionally linear. The equation for Kn is as follows:

A_t conditions of temperature and pressure. When prompted for specific heat ratio, γ , we use the value found within the chamber state. This value is 1.1273, given from Figure 4.

As mentioned above, the Mach speed of the exhaust gases from the flow of propellant can be evaluated from knowing the chamber pressure and the ambient pressure, or desirable nozzle exit pressure.

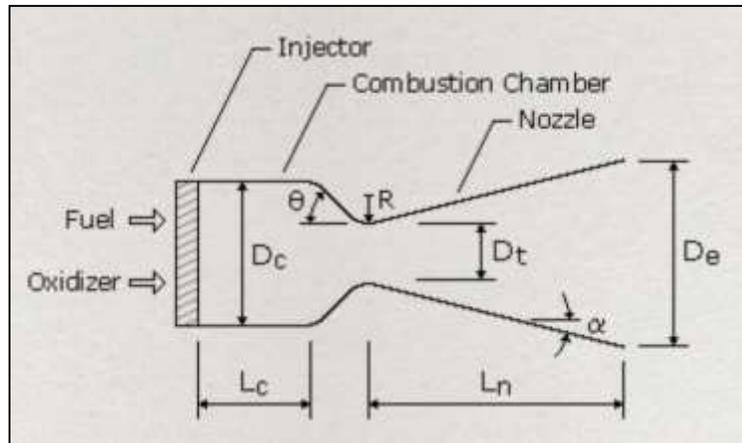


Figure 7: A diagram of a converging-diverging nozzle with considerations to half angles near the





Conclusion

If the calculations and mathematical methods have been applied correctly, then with a solid propellant composed of roughly 72% Ammonium Perchlorate, 16% Aluminum and 12% PBAN reaching a max contained combustion pressure of 588.235 psi, there will be an isentropic and resulting supersonic flow in the nozzle with the dimensions specified in Figure 12. There is a suspected exit Mach number of 2.85, and a flow rate of 8.1 lbs/s but these were dependent upon the thermal and chemical properties. The real propellant will have several more ingredients and its properties cannot be known until an effective test and data are received and interpreted. The area of the throat and exit, and their subsequent expansion ratio are subject to change and likely will reflect the differences in chamber pressure, Temperature and propellant flow rate. A recommendation to anyone else performing this analysis is to have thermal properties data on hand, and to compare between types and compositions of propellants. Propellants with lower combustibility or slower combustion rates can contribute to a lower internal pressure and thus change every dimension of the nozzle. Another recommendation would be to invest in BurnSim3 to use, for it is hard to formulate the throat diameter and area using only pressure and temperature. The area expansion ratio can be found, but many equations are dependent on the

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