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Anti Q-Fuzzy M-Semigroup

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Abstract: The purpose of this paper is to introduce the concept of anti Q-fuzzy M-semigroup by means of anti Q-fuzzy set theory and M-semigroup theory.

Keywords: M-semigroup, fuzzy M-semigroup, anti fuzzy M-semigroup, anti fuzzy M-semigroup and anti Q-fuzzy M-semigroup.

I. Introduction

L.A.Zadeh [12] in his pioneering paper, initiated the concept of fuzzy set. Kuroki [5,6,7] have studied fuzzy semigroups, fuzzy left(right) ideals, fuzzy bi-ideal and fuzzy interior ideals in semigroups. L.Lakshmanan [8] analysed the complete structure of M-semigroups. Following him, AL.Narayanan and AR.Meenakshi [9] introduced the notion of fuzzy M-subsemigroup as a generalization of M-semigroup. Recently,S.Vijayabalaji and S.Sivaramakrishnan [11] introduced the notion of anti fuzzy M-semigroup.

K.T.Atanassov [2] initiated the concept of intuitionistic Q-fuzzy set of as a generalization of the notion of fuzzy set. K.H.Kim [4] introduced the concept of intuitionistic Q-fuzzy ideals of semigroups.

In this paper, we attempt a new algebraic structure of an anti Q-fuzzy M-semigroup and study some of their related properties.

II. Preliminaries

In this section, we recall some basic definitions used in this paper.

Definition 2.1[8]. Asemigroup M is called an M-semigroup if the following conditions are satisfied.

(i) there exists at least one left identity $e \in M$ such that ex=x, for all $x \in M$,

(ii) for every $x \in M$, there is a unique left identity, say e_x such that $xe_x=x$, that is e_x is a two –sided identity for x.

Definition 2.2 [11]. Let M be an M-semigroup. Let $\gamma: M \to [0,1]$ be a fuzzy set. Then (M,γ) is called an anti fuzzyM-semigroup if

(i) $\gamma(xy) \le \max\{\gamma(x),\gamma(y)\}$,for every $x,y \in M$,

(ii) $\gamma(e)=0$,forevery leftidentity e in M.

Example2.3[11]. Let M={e,f,a,b} be an M-semigroup with the following operation.

TABLE 2.1

.	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	e	f
b	a	b	e	f

Define the fuzzy set $\gamma: M \to [0,1]$ by

$$\gamma(x) = \begin{cases} 0, & if\ x = e, f \\ \alpha, & otherwise,\ 0 < \alpha \le 1. \end{cases}$$

Then (M,γ) is an antifuzzy M-semigroup.

Definition2.4[2]. Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset $\mu : X \times Q \to [0,1]$.

Example 2.5[2]. Let X = {a, b, c} be a set and Q = {p}. Then μ = {<(a,p),0.4>, <(b,p),0.2>, <(c,p),0.6>} is a Q-fuzzy subset of X.

Definition 2.5[10]. Let (G, .) be a group and Q be a non-empty set. A Q-fuzzy subset A of G is said to be an anti Q-

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fuzzy subgroup of G if the following conditions are satisfied:

(i) $A(xy,q) \le \max\{A(x,q),A(y,q)\}$,

(ii) $A(x^{-1},q) = A(x,q)$,for all x and y in G and q in Q.

III. ANTI Q-FUZZY M-SEMIGROUP

In this section, we introduce the notion of anti Q-fuzzy M-semigroup.

Definition 3.1. Let M be an M-semigroup and Q be a non-empty set. A Q-fuzzy subset of M is said to be an anti Q-fuzzy M-semigroup of M if the following conditions are satisfied:

(i) $\gamma(xy,q) \le \max\{\gamma(x,q),\gamma(y,q)\}$

(ii) $\gamma(e,q) = 0$, for every left identity e in M.

Example 3.2. Let M = {e, f, a, b} be an M-semigroup with the following operation.

TABLE 3.1

.	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	e	f
b	a	b	e	f

We take Q = {q}.
Define a Q-fuzzy subset γ of M is a function
γ: M x Q $\rightarrow$[0,1] by

$$\gamma(x,q) = \begin{cases} 0 \text{ , if } x = e, f \\ \alpha, \text{ otherwise}, 0 < \alpha \le 1. \end{cases}$$

Then $\gamma(x,q)$ is an anti Q-fuzzy M-semigroup.

Definition 3.3. If γ and δ are two anti Q-fuzzy M-semigroups of M, then their union $\gamma \cup \delta$ is an anti Q-fuzzy M-semigroup of M.

Proof. Let x and y in M and q in Q.

(i) $(\gamma\cup\delta)(xy,q) = \max\{\gamma(xy,q),\delta(xy,q)\}$

$\le \max\{\max[\gamma(x,q), \gamma(y,q)], \max[\delta (x,q), \delta(y,q)]\}$

$\le \max\{\max[\gamma(x,q), \delta (x,q)], \max[\gamma(y,q), \delta (y,q)]\}$

$= \max \{(\gamma\cup\delta)(x,q), (\gamma\cup\delta)(y,q)\}$

Therefore,

$(\gamma\cup\delta)(xy,q) \le \max \{(\gamma\cup\delta)(x,q), (\gamma\cup\delta)(y,q)\}$

(ii) $(\gamma\cup\delta)(e,q) = \max\{\gamma(e,q),\delta(e,q)\}$

$= \max\{0,0\}$

$= 0$, for every left identity e in M.

Hence $\gamma\cup\delta$ is an anti Q-fuzzy M-semigroup of M.

Theorem 3.4. The union of a family of anti Q-fuzzy M-semigroups of M is an anti Q-fuzzy M-semigroup of M.

Proof. Straightforward.

The intersection of two anti Q-fuzzy M-semigroups need not be an anti Q-fuzzy M-semigroup is illustratre below.

Example.3.5. Let M= {e,a,b,ab} be an M-semigroup, where $a^2 = e = b^2 = (ab)^2$ and ab=ba.

TABLE 3.2

.	a	b	ab	e
a	e	ab	b	a
b	ab	e	a	b
ab	b	a	e	ab
e	a	b	ab	e

Choose numbers $t_i \in [0,1]$, $0 \le i \le 5$ such that $0= t_0<t_1<t_2<t_3<t_4<t_5$.

We take Q = {q}.

Define the fuzzy sets γ_1 (x,q), γ_2(x,q):M$\rightarrow$[0,1] as follows:

$$\gamma_1(x,q) = \begin{cases} 0, \text{ if } x = e \\ t_1, \text{ if } x = a \\ t_5 \text{ , if } x = b, ab \end{cases}$$

$$\gamma_2(x,q) = \begin{cases} 0, \text{ if } x = e \\ t_4, \text{ if } x = a, ab \\ t_2 \text{ , if } x = b \end{cases}$$

Observe that γ_1 (x,q), γ_2 (x,q) are anti Q-fuzzy M-semiroups.

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Define $(\gamma_1 \cap \gamma_2)(x,q) = \begin{cases} 0, if\ x = e \\ t_1, if\ x = a \\ t_2, if\ x = b \\ t_4, if\ x = ab \end{cases}$ (1)

But $(\gamma_1 \cap \gamma_2)(ab,q) \leq \max\{(\gamma_1 \cap \gamma_2)(a,q), (\gamma_1 \cap \gamma_2)(b,q)\}$
$= \max\{t_1, t_2\} = t_2$

Therefore, $(\gamma_1 \cap \gamma_2)(ab,q) \leq t_2$

However, $(\gamma_1 \cap \gamma_2)(ab,q) = t_4$, by (1), so $t_4 \leq t_2$.

This is absurd.

$\gamma_1(x,q)$, $\gamma_2(x,q)$ are anti Q-fuzzy M-semiroups, whereas $(\gamma_1 \cap \gamma_2)(x,q)$ is not an anti Q-fuzzy M-semigroup.

Definition 3.6. Let γ and δ be any two anti Q-fuzzy M-semigroups of $M_1$and M_2 respectively, then anti product of Q-fuzzy M-semigroup is defined by

$(\gamma \times \delta)((x, y), q) = \max\{\gamma(x, q); \delta(y, q)$g for all $x \in M_1$, $y \in M_2$ and $q \in Q$.

Theorem 3.7. If γ and δ are anti Q-fuzzy M-semigroups of M_1 and M_2 respectively, then anti product $\gamma \times \delta$ is an anti Q-fuzzy M-semigroup of $M_1 x\ M_2$.

Proof. Let x_1 and x_2 be in M_1 and y_1 and y_2 be in M_2.

Then (x_1,y_1) and (x_2, y_2) are in $M_1 x\ M_2$ and $q \in Q$.

Now,(*i*)$(\gamma\times\delta)[(x_1,y_1)(x_2,y_2),q]=(\gamma\times\delta)[(x_1x_2,y_1y_2),q]$

$= \max\{\gamma(x_1x_2,\ q),\delta(y_1y_2,\ q)\}$

$\leq \max\{\max[\gamma(x_1,q),\gamma(x_2,q)],\max[\delta(y_1,q),\delta(y_2,q)]\}$

$= \max\{\max[\gamma(x_1,q),\delta(y_1,q)],\max[\gamma(x_2,q),\delta(y_2,q)]\}$

$= \max\{(\gamma\times\delta)((x_1,y_1),q),(\gamma\times\delta)((x_2,y_2),q)\}$

Therefore,

$(\gamma\times\delta)[(x_1,y_1)(x_2,y_2),q] \leq \max\{(\gamma\times\delta)((x_1,y_1),q),(\gamma\times\delta)((x_2,y_2),q)\}$

(ii)Let e and e' be two left identity of M_1 and M_2 respectively.

$(\gamma\times\delta)[(e,e'),q] = \max\{\gamma(e,q),\delta(e',q)\}$

$= \max\{0,0\}=0$

$\Rightarrow (\gamma\times\delta)[(e, e'),q] = 0$

Hence $\gamma\times\delta$ is an anti *Q*-fuzzy *M*-semigroup of $M_1 \times M_2$.

Theorem 3.8. Let M be an M-semigroup and Q be a non-empty set. Then γ is an anti Q-fuzzy M-semigroup in M if and only if γ^C is a Q-fuzzy M-semigroup in M.

Proof. Let γ be an anti Q -fuzzy M-semigroup in M. We have for all $x,y \in M$ and $q \in Q$.

$\gamma^c(xy,q)=1-\gamma(xy,q)$

$\geq 1-\max\{\gamma(x,q),\gamma(y,q)\}$

$= \min\{1-\gamma(x,q),1-\gamma(y,q)\}$

$= \min\{\gamma^c(x,q),\gamma^c(y,q)\}$

$\Rightarrow \gamma^c(xy,q) \geq \min\{\gamma^c(x,q),\gamma^c(y,q)\}$,and

$\gamma^c(e,q) = 1-\gamma(e,q)=1-0$

$\Rightarrow \gamma^c(e,q)=1$,where e is a left identity in M

Hence γ^C is also a Q-fuzzy M-semigroup in M.

Conversely, let γ^C be a Q-fuzzy M-semigroup in M.

To Prove γ is an anti Q-fuzzy M-semigroup in M, we have

$\gamma(xy,q) = 1-\gamma^c(xy,q)\leq 1-\min\{\gamma^c(x,q),\gamma^c(y,q)\}$

$= \max\{1-\gamma^c(x,q),1-\gamma^c(y,q)\}$

$= \max\{\gamma(x,q),\gamma(y,q)\}$

$\Rightarrow \gamma(xy,q)\leq \max\{\gamma(x,q),\gamma(y,q)\}$,and

$\gamma(e,q)=1-\gamma^c(e,q)=1-1=0.$

$\Rightarrow \gamma(e,q)=0$,where*e*isaleft identityin*M*

Hence γ is an anti *Q*-fuzzy*M*-semigroup in *M*.

IV. Conclusion

In this paper, the concept of anti Q-fuzzy M-semigroup by means of anti Q-fuzzy set theory and M-semigroup theory is defined and some of their properties are discussed.

In further research, the following topics will be discussed

(i) Anti Q-fuzzy linear space,
(ii) Anti Q-fuzzy soft linear space.

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