



Morphology on Intuitionistic Fuzzy Soft Graphs

Abraham Jacob¹, Dr.P.B.Ramkumar²

Research Scholar, Rajagiri School of Engineering & Technology, Kakkanadu, Kerala, India¹
Department of Mathematics, Rajagiri School of Engineering & Technology, Kakkanadu, Kerala, India²
dearavarana@gmail.com¹ ramkumar_pbrajagiritech.edu.in²

Abstract- Mathematical Morphology is the study of the form and structure of objects based on set theory, lattice theory and topology. Dilation and erosion are the basic operators in Mathematical Morphology. Fuzzy Graph represents relationships that involve uncertainty. We introduce P_n -adjacency of vertices, P_n -adjacency of edges, P_n -adjacency of dilation, and P_n -adjacency of erosion on Intuitionistic Fuzzy Soft Graph (IFSG). We also present an algorithm for P_n -adjacency of dilation on IFSG.

Keywords- Dilation and erosion based on Lattice Structure, Intuitionistic Fuzzy Soft Graph, P_n -adjacency of vertices, P_n -adjacency of edges, P_n -adjacency of dilation, and P_n -adjacency of erosion on Intuitionistic Fuzzy Soft Graph (IFSG)

I. INTRODUCTION

Traditional Mathematical Morphology is a study of forms and structures based on Set theory, Lattice Theory and Topology. It is used to as a powerful tool for image analysis. The book by Jean Serra [20] "Image Analysis and Mathematical Morphology" made Mathematical Morphology popular. Works on Traditional Mathematical Morphology has been extended to many field, especially in Graph Theory [7], [13].

After introduction of Graph Theory by Euler in 1736, Kaffmann [11] put forward the first definition of Fuzzy Graph in 1973. But, Rosenfeld [19] introduced another definition of Fuzzy Graph which is similar to the graph theoretic concepts. In 1999, Atanassov [1] introduced Intuitionistic Fuzzy Graph. Karunambigai and Parvathy [9] also did some extension to Atanassov's Intuitionistic Fuzzy Graph. Thumbakara and George [26] put forward the idea of Soft Graph. Mohita and Samanta [22] first presented the notion of Fuzzy Soft Graph. Sunitha [24] introduced the Intuitionistic Fuzzy Soft Graph. Shyla and Mathew Varkey [21] also did their work in Intuitionistic Fuzzy Soft Graph.

In this paper, we propose definitions of basic morphological operations such as dilation and erosion in Intuitionistic Fuzzy Soft Graph in section III using the notions called P_n -adjacency of vertices and P_n -adjacency of edges those we introduce in section 2. Definitions in section II make this journey to our destination smooth. In section IV, we proved that morphological operators defined in section III form an adjunction. We also give an P_n -adjacency algorithm

to find P_n -adjacency dilation and P_n -adjacency erosion in IFSG in section V.

II. PRELIMINARIES

Before defining P_n -adjacency vertices and P_n -adjacency edges, we recall definitions of intuitionistic fuzzy soft graph and elementary morphological operators on a lattice.

Definition II.1

Let $G^* = \{x_1, x_2, x_3, \dots, x_n\}$ be a non-empty set, P be a parameter set, and $A \subseteq P$, $a \in A$, $E \subseteq G^* \times G^*$. Let $F(G^*)$ be collection of all fuzzy subsets of G^* and $F(G^* \times G^*)$ be collection of all fuzzy subsets of $G^* \times G^*$. Also let

(1) $\mu_1, \gamma_1: A \rightarrow F(G^*)$ such that

$$a \mapsto \mu_1(a) = \mu_{1a}(\text{say})$$

and $a \mapsto \gamma_1(a) = \gamma_{1a}(\text{say})$

where $\mu_{1a}: G^* \rightarrow [0, 1]$ such that

$x_i \mapsto \mu_{1a}(x_i)$ and

$\gamma_{1a}: G^* \rightarrow [0, 1]$ such that $x_i \mapsto \gamma_{1a}(x_i)$

(A, μ_1, γ_1) is intuitionistic fuzzy soft vertex if

$$0 \leq \mu_{1a} + \gamma_{1a} \leq 1, \forall x_i \in V \text{ and } a \in A.$$



Table 1

μ_{1a}	v_1	v_2	v_3
a_1	.8	.6	0
a_2	.9	.65	.5
a_3	.2	.75	.8

γ_{1a}	v_1	v_2	v_3
a_1	.1	.2	0
a_2	.05	.25	.4
a_3	.6	.1	.15

Table 2

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.15	0.25	0.1
a_3	0.01	0.1	0.2

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.2	0.1	0.2
a_3	0.5	0.2	0.01

(2) $\mu_2, \gamma_2: A \rightarrow F(G^* \times G^*)$ such that
 $a \mapsto \mu_2(a) = \mu_{2a}(say)$ and
 $a \mapsto \gamma_2(a) = \gamma_{2a}(say)$
 where $\mu_{2a}: G^* \times G^* \rightarrow [0,1]$ such that

$$(x_i, x_j) \mapsto \mu_{2a}(x_i, x_j)$$

$$\gamma_{2a}: G^* \times G^* \rightarrow [0,1] \text{ such that}$$

$$(x_i, x_j) \mapsto \gamma_{2a}(x_i, x_j)$$

(A, μ_2, γ_2) is an intuitionistic fuzzy soft edge if
 $a \in A,$

$$\forall (x_i, x_j) \in E \text{ and } \forall i, j = 1, 2, \dots, n$$

$$(1) \mu_{2a}(x_i, x_j) \leq \mu_{1a}(x_i) \wedge \mu_{1a}(x_j)$$

$$(2) \gamma_{2a}(x_i, x_j) \leq \gamma_{1a}(x_i) \vee \gamma_{1a}(x_j)$$

$$(3) 0 \leq \mu_{2a}(x_i, x_j) + \gamma_{2a}(x_i, x_j) \leq 1$$

Then $A(\mu_{1a}, \gamma_{1a}, \mu_{2a}, \gamma_{2a})$ is called intuitionistic fuzzy soft graph (IFSG) is denoted by $G_{A, G^*, E}$.

Let P be a parameter. Let $A, B \subseteq P, a \in A$. Let $G^i = (A, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i)$ and $G^j = (A, \mu_{1a}^j, \gamma_{1a}^j, \mu_{2a}^j, \gamma_{2a}^j)$ be IFSG's. Then G^i is an intuitionistic fuzzy soft sub graph of G^j if

$$(1) A \subseteq B$$

$$(2) H^i(a) \text{ is a partial intuitionistic fuzzy sub graph of } H^j(a) \text{ for all } a \in A.$$

where $H^i(a)$ and $H^j(a)$ are the corresponding intuitionistic fuzzy graph of G^i and G^j for each $a \in A$

Example II.1

Consider a simple graph $G=(V,E)$, where

$$V=\{v_1, v_2, v_3\}, E=\{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$$

Let $A=\{a_1, a_2, a_3\}$ be the parameter set. Then intuitionistic

fuzzy soft graph $G_{A, V, E}$.

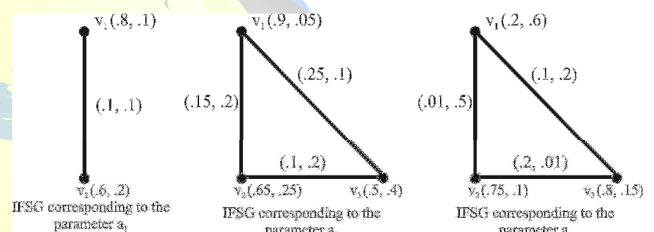


Fig.1

Proposition II.1

Union of two intuitionistic fuzzy soft graph is an IFSG.

Proposition II.2

Intersection of two intuitionistic fuzzy soft graph is an IFSG.

Definition II.2

Any operator δ acting on a lattice L is called dilation if it is extensive, increasing and distributes over the supremum. Any operator ϵ acting on a lattice L is called erosion if it is anti extensive, increasing and distributes over the infimum. The pair (ϵ, δ) form an adjunction if $\delta(A) \subseteq B \Leftrightarrow A \subseteq \epsilon(B)$.

ϵ and δ are dual with respect to complementation if $\overline{\epsilon(A)} = \delta(\overline{A})$.

Theorem II.1

If (ϵ, δ) is an adjunction, then ϵ is an erosion and δ is a dilation.

Now we introduce n -path adjacency vertices (P_n -adjacency vertices), n -path adjacency edges and (P_n -adjacency edges). We also propose path adjacency dilated IFSG which includes both P_n -adjacency vertex dilation and P_n -adjacency edge dilation and path adjacency eroded IFSG which includes both P_n -adjacency vertex erosion and P_n -adjacency edge erosion.



Throughout this paper, let P is the parameter set, $A, B \subseteq P$, $a \in A$. Let $G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i)$ be an Intuitionistic Fuzzy Soft Graph (IFSG), where G^* is the underlying vertex set, G^\times is the set of all elements in $G^* \times G^*$, μ_{1a}^i and γ_{1a}^i are the membership function and non-membership function of vertex set of IFSG and μ_{2a}^i and γ_{2a}^i are the membership function and non-membership function of edge set of IFSG. Let \mathcal{G} be the set of all Intuitionistic Fuzzy Softgraphs $G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i)$ defined on $G = (G^*, G^\times)$, where each pair in \mathcal{G} satisfies property of subgraph. We define a relation \subseteq on \mathcal{G} as follows: Let 0 be the IFSG in \mathcal{G} with all vertices and edges of membership function 0 and non-membership function 1 and let 1 be the IFSG in \mathcal{G} with all vertices and edges of membership function 1 and non-membership function 0. Let ϕ be the empty set. We represent the vertices of the underlying graph by u_i and the edge between u_i and u_j by $e_{u_i-u_j}$. We represent that u_i and u_j are P_n -adjacency vertices in G as $u_i P_n\text{-adj } u_j$.

Definition II.3

Let u_i and u_j be two vertices in the IFSG

$G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in \mathcal{G}$. Then, u_j is said to be 1-path adjacency vertex (P_1 -adjacency vertex) to u_i if they are connected by at most 1 edge. Then, u_j is 1-path adjacency vertex of u_i and vice versa. Illustrated example is given in Example II.2.

Definition II.4

Let u_i and u_j be two vertices in the IFSG $G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in \mathcal{G}$. Then, u_j is said to be 2-path adjacency vertex (P_2 -adjacency vertex) to u_i if they are connected by at most 2 edges. Then u_j is 2 path adjacency vertex of u_i and vice versa. Illustrated example is given in Example II.2.

Observation II.1

In general, Let u_i and u_j be two vertices in the IFSG

$G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in \mathcal{G}$. Then, u_j is said to be n -path adjacency vertex (P_n -adjacency vertex) to u_i if they are connected by at most n edges. Then u_j is n -path adjacency vertex of u_i and vice versa.

Definition II.5

Let $e_{u_i-u_j}$ and $e_{u_k-u_l}$ be two edges in the

IFSG $G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in \mathcal{G}$.

Then, $e_{u_k-u_l}$ is said to be 1-path adjacency edge (P_1 -

adjacency vertex) to $e_{u_i-u_j}$ if either u_i or u_j is connected to u_k or u_l by at most 1 edge. Illustrated example is given in Example II.2.

Definition II.6

Let $e_{u_i-u_j}$ and $e_{u_k-u_l}$ be two edges in the IFSG

$G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in \mathcal{G}$. Then, $e_{u_k-u_l}$ is said to be 2-path adjacency edge (P_2 -adjacency vertex) to $e_{u_i-u_j}$ if either u_i or u_j is connected to u_k or u_l by at most 2 edges. Illustrated example is given in Example 2.2.

In general, Let $e_{u_i-u_j}$ and $e_{u_k-u_l}$ be two edges in the

IFSG $G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in \mathcal{G}$. Then, $e_{u_k-u_l}$ is said to be n -path adjacency edge (P_n -adjacency vertex) to $e_{u_i-u_j}$ if either u_i or u_j is connected to u_k or u_l by at most n edge.

Example II.2

Consider a simple graph $G = (V, E)$, where

$V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, $E = \{e_{u_1-u_2}, e_{u_2-u_3}, e_{u_3-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}\}$, Let $A = \{a_1, a_2\}$ be the

parameter set. Then intuitionistic fuzzy soft graph $G_{A, V, E}$ is given in Table-3 and its graph is given in figure 2.

Table 3

μ_{1a}	u_1	u_2	u_3	u_4	u_5	u_6
a_1	.7	.4	.5	.4	.8	1
a_2	.5	.2	.2	.3	0	0

γ_{1a}	u_1	u_2	u_3	u_4	u_5	u_6
a_1	.3	.6	.4	.5	.1	0
a_2	.4	.4	.1	.4	0	0



μ_{2a}	$e_{u_1-u_2}$	$e_{u_2-u_3}$	$e_{u_3-u_4}$	$e_{u_4-u_5}$	$e_{u_5-u_6}$	$e_{u_1-u_6}$
a_1	.4	.3	.2	.3	.2	.1
a_2	.2	.1	.2	0	0	.1

γ_{2a}	$e_{u_1-u_2}$	$e_{u_2-u_3}$	$e_{u_3-u_4}$	$e_{u_4-u_5}$	$e_{u_5-u_6}$	$e_{u_1-u_6}$
a_1	.4	.6	.4	.5	.2	.5
a_2	.4	.4	.4	0	0	.3

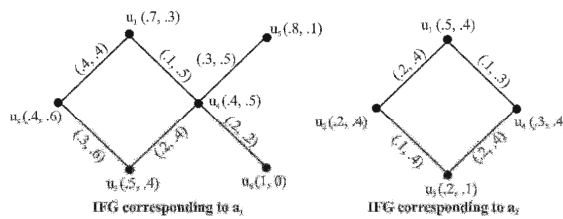


Fig.2

P₁- adjacency vertices and P₂- adjacency vertices of each vertices of IFG corresponding to a₁ are given below Table-4:

Table-4

Vertex	P ₁ - adjacency vertices	P ₂ - adjacency vertices
u_1	u_1, u_2, u_4	$u_1, u_2, u_3, u_4, u_5, u_6$
u_2	u_1, u_2, u_3	u_1, u_2, u_3, u_4
u_3	u_2, u_3, u_4	$u_1, u_2, u_3, u_4, u_5, u_6$
u_4	u_3, u_1, u_4, u_5, u_6	$u_1, u_2, u_3, u_4, u_5, u_6$
u_5	u_4, u_5	u_2, u_3, u_4, u_5, u_6
u_6	u_4, u_6	u_1, u_3, u_4, u_5, u_6

P₁-adjacency edges and P₂- adjacency edges of each edges of IFG corresponding to a₁ are given below Table-5:

Till now, we defined P_n-adjacency of vertices and P_n-adjacency of edges in IFSG in detail. So we reached half way to the journey of our destination. Now we are going to define the corresponding basic morphological operators in IFSG in the following section.

III. P_n-adjacency Dilated IFSG&P_n-adjacency Eroded IFSG

At first, we define P₁- adjacency dilation of each vertices and edges and then it leads to the P_n-adjacency dilated IFSG corresponding to each $a \in A$ in Definition III.1.

Definition III.1

Let P is the parameter set, $A \subseteq P$, $a \in A$. Let $G^i = (G^*, G^\times, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in G$ be an Intuitionistic Fuzzy Soft Graph (IFSG), where $G^* = \{u_1, u_2, \dots, u_n\}$ is the underlying vertex set, G^\times is the set of all elements in $G^* \times G^*$, μ_{1a}^i and γ_{1a}^i are the membership function and non-membership function of vertex set of IFSG and μ_{2a}^i and γ_{2a}^i are the membership function and non-membership function of edge set of IFSG. We define the following:

1. For all elements in G^* , $\delta_{1a}^i: G^* \rightarrow [0, 1]$ by

$$\delta_{1a}^i(u_k) = V_{u_j}(\mu_{1a}^i, \gamma_{1a}^i) = (Sup_{u_j}(\mu_{1a}^i), Inf_{u_j}(\gamma_{1a}^i)) = (\delta_{\mu_{1a}^i}^i, \delta_{\gamma_{1a}^i}^i)$$

where $u_j = u_k$ or u_j P₁-adj u_k

Then $\delta_{1a}^i(u_k)$ is the P₁- adjacency dilation of each

vertex u_k corresponding to the parameter each $a \in A$ in the given IFSG.

2. For all elements in G^\times ,

$$\delta_{2a}^i: G^\times \rightarrow [0, 1] \text{ by } \delta_{2a}^i(e_{u_i-u_j}) = V_{e_{u_i-u_k}}(\mu_{2a}^i, \gamma_{2a}^i) = (Sup_{e_{u_i-u_k}}[\mu_{2a}^i(e_{u_i-u_k})], Inf_{e_{u_i-u_k}}[\gamma_{2a}^i(e_{u_i-u_k})]) = (\delta_{\mu_{2a}^i}^i, \delta_{\gamma_{2a}^i}^i)$$

where $e_{u_i-u_k}$ is $e_{u_i-u_j}$ itself or $e_{u_i-u_k}$ P_n-

adjacency edge to $e_{u_i-u_j}$.

Then $\delta_{2a}^i(e_{u_i-u_j})$ is the P₁-adjacency dilation of each

edge $e_{u_i-u_j}$ corresponding to the parameter each $a \in A$ in the given IFSG.

Thus $G_D^i = (\delta_{1a}^i, \delta_{2a}^i)$ or δ_G^i is called P₁- adjacency dilation in IFSG G^i corresponding to $a \in A$. Illustrated example is given in Example III.1.



Table-4

Vertex	P ₁ - adjacency vertices	P ₂ - adjacency vertices
$e_{u_1-u_2}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_1-u_4}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$
$e_{u_2-u_3}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$
$e_{u_2-u_4}$	$e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$
$e_{u_4-u_5}$	$e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$
$e_{u_4-u_6}$	$e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$
$e_{u_1-u_4}$	$e_{u_1-u_2}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$	$e_{u_1-u_2}, e_{u_2-u_3}, e_{u_2-u_4}, e_{u_4-u_5}, e_{u_4-u_6}, e_{u_1-u_4}$

Observation III.1

It is obvious that we can extend the above formula to P_n-adjacency dilated IFSG corresponding to each $a \in A$.

Now we define P₁- adjacency erosion of each vertices and edges and then it leads to the P_n-adjacency eroded IFSG corresponding to each $a \in A$ in following Definition III.2.

Definition III.2

Let P is the parameter set, $A \subseteq P$, $a \in A$. Let $G^i = (G^*, G^x, \mu_{1a}^i, \gamma_{1a}^i, \mu_{2a}^i, \gamma_{2a}^i) \in G$ be an Intuitionistic Fuzzy Soft Graph (IFSG), where $G^* = \{u_1, u_2, \dots, u_n\}$ is the underlying vertex set, G^x is the set of all elements in $G^* \times G^*$. μ_{1a}^i and γ_{1a}^i are the membership function and non-membership function of vertex set of IFSG and μ_{2a}^i and γ_{2a}^i are the membership function and non-membership function of edge set of IFSG. We define the following:

1. For all elements in G^* , $\delta_{1a}^i: G^* \rightarrow [0,1]$ by

$$\epsilon_{1a}^i(u_k) = \bigwedge_{u_j} (\mu_{1a}^i, \gamma_{1a}^i) = (\inf_{u_j} (\mu_{1a}^i), \sup_{u_j} (\gamma_{1a}^i))$$

where $u_j = u_k$ or u_j P₁-adj u_k

Then $\epsilon_{1a}^i(u_k)$ is the P₁- adjacency erosion of each vertex u_k corresponding to the parameter each $a \in A$ in the given IFSG.

2. For all elements in G^x ,

$$\epsilon_{2a}^i: G^x \rightarrow [0,1] \text{ by } \epsilon_{2a}^i(e_{u_i-u_j}) = \epsilon_{2a}^i(e_{u_i-u_j} \wedge_{e_{u_i-u_k}} (\mu_{2a}^i, \gamma_{2a}^i)) \\ = (\inf_{e_{u_i-u_k}} [\mu_{2a}^i(e_{u_i-u_k})], \sup_{e_{u_i-u_k}} [\gamma_{2a}^i(e_{u_i-u_k})])$$

where $e_{u_i-u_k}$ is itself or $e_{u_i-u_k}$ P_n-adjacency edge to $e_{u_i-u_j}$. Then $\epsilon_{2a}^i(e_{u_i-u_j})$ is the

P₁-adjacency erosion of each edge $e_{u_i-u_j}$ corresponding to the parameter each $a \in A$ in the given IFSG.

Thus $G_E^i = (\epsilon_{1a}^i, \epsilon_{2a}^i)$ is called P₁-adjacency eroded IFSG corresponding to $a \in A$. Illustrated example is given in Example III.1.

Observation III.2

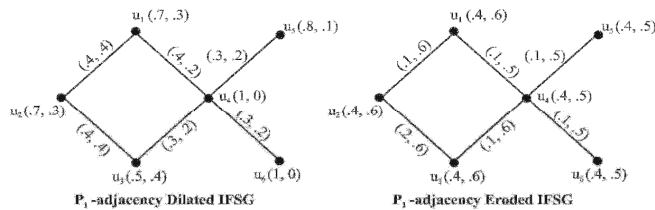
It is obvious that we can extend the above formula to P_n-adjacency eroded IFSG corresponding to each $a \in A$.

Example III.1

Given below are the P₁-adjacency dilated IFSG corresponding to each $a_1 \in A$ and P₁-adjacency eroded IFSG corresponding to $a_1 \in A$ using corresponding formulae in definition 3.1 and definition III.2 for P₁-adjacency dilation for each vertices and P₁-adjacency erosion for each edges in Intuitionistic Fuzzy Soft Graph corresponding to the parameter $a_1 \in A$ in figure-2 in the above example II.2.



Fig.3



Observation III.3

In the similar manner, we can find out the P_1 -adjacency dilated IFSG corresponding to each $a_2 \in A$ and P_1 -adjacency eroded IFSG corresponding to $a_2 \in A$ of the IFSG corresponding to $a_2 \in A$ in the given above example II.2.

Observation III.4

Further, it is easy to find out the P_2 -adjacency dilated IFSG and P_2 -adjacency eroded IFSG corresponding to each parameters a_1 and a_2 in the given IFSG in example II.2.

Now we have to prove that the above proposed P_n -adjacency dilation and P_n -adjacency erosion in definition III.1 and definition III.2 is morphological operators. So we define a lattice on the set of all intuitionistic fuzzy soft sub graphs and adjunction between the above defined operators in the following section IV.

IV. P_n -ADJACENCY DILATION AND P_n -ADJACENCY EROSION AS MORPHOLOGICAL OPERATORS

Definition IV.1

We define the partial order \sqsubseteq on \mathcal{G} as $G^1 \sqsubseteq G^2 \Leftrightarrow G^1$ is an intuitionistic fuzzy soft sub graph of G^2 . It is obvious that this relation is reflexive, anti-symmetric and transitive. This leads us to the following:

Proposition IV.1

$(\mathcal{G}, \sqsubseteq)$ is a poset.

Supremum and infimum of two intuitionistic fuzzy soft sub graphs in \mathcal{G} are defined as follows:-

Definition IV.2

For all $G^1, G^2 \in \mathcal{G}$, we define

$$G^1 \vee G^2 = (G^*, G^x, \mu_{1a}^1 \vee \mu_{1a}^2, \gamma_{1a}^1 \wedge \gamma_{1a}^2, \mu_{2a}^1 \vee \mu_{2a}^2, \gamma_{2a}^1 \wedge \gamma_{2a}^2)$$

$$G^1 \wedge G^2 =$$

$$G^1 \cap G^2 = (G^*, G^x, \mu_{1a}^1 \wedge \mu_{1a}^2, \gamma_{1a}^1 \vee \gamma_{1a}^2, \mu_{2a}^1 \wedge \mu_{2a}^2, \gamma_{2a}^1 \vee \gamma_{2a}^2)$$

The following proposition will prove that union (intersection) of finite collection of intuitionistic fuzzy soft

sub graphs in \mathcal{G} is the supremum (infimum) of this collection.

Proposition IV.2

Let $\mathcal{E} = (G^1, G^2, \dots, G^n)$ be a family of elements of \mathcal{G} .

Then $\text{Sup } \mathcal{E} = G^1 \vee G^2 \vee \dots \vee G^n$ and $\text{Inf } \mathcal{E}$

$$= G^1 \wedge G^2 \wedge \dots \wedge G^n$$

Proof

$$\text{Let } X = G^1 \vee G^2 \vee \dots \vee G^n$$

$$= G^1 \cup G^2 \cup \dots \cup G^n$$

$$(G^*, G^x, \bigvee_{i=1}^n \mu_{1a}^i, \bigvee_{i=1}^n \gamma_{1a}^i, \bigvee_{i=1}^n \mu_{2a}^i, \bigvee_{i=1}^n \gamma_{2a}^i)$$

For corresponding vertices and edges of

$$G^1, G^2, \dots, G^n,$$

$$\mu_{1a}^i \leq \bigvee_{i=1}^n \mu_{1a}^i, \gamma_{1a}^i \leq \bigvee_{i=1}^n \gamma_{1a}^i,$$

$$\mu_{2a}^i \leq \bigvee_{i=1}^n \mu_{2a}^i, \gamma_{2a}^i \leq \bigvee_{i=1}^n \gamma_{2a}^i$$

$$\text{By definition, } G^i \subseteq X \quad \forall i=1,2,\dots,n$$

Assume that $G^i \subseteq Y \quad \forall i=1,2,\dots,n$ where $Y =$

$$(G^*, G^x, \mu_{1a}^y, \gamma_{1a}^y, \mu_{2a}^y, \gamma_{2a}^y)$$

$$\Rightarrow \mu_{1a}^i \leq \mu_{1a}^y, \gamma_{1a}^i \leq \gamma_{1a}^y, \mu_{2a}^i \leq \mu_{2a}^y \text{ and } \gamma_{2a}^i \leq \gamma_{2a}^y \quad \forall i=1,2,\dots,n$$

$$\Rightarrow \bigvee_{i=1}^n \mu_{1a}^i \leq \mu_{1a}^y, \bigvee_{i=1}^n \gamma_{1a}^i \leq \gamma_{1a}^y,$$

$$\bigvee_{i=1}^n \mu_{2a}^i \leq \mu_{2a}^y, \bigvee_{i=1}^n \gamma_{2a}^i \leq \gamma_{2a}^y$$

for corresponding vertices and edges of

$$G^1, G^2, \dots, G^n$$

$$\Rightarrow \bigcup_{i=1}^n G^i \subseteq Y$$

$$\Rightarrow X \subseteq Y$$

Since it is true for all IFSG Y , we can conclude that $\text{Sup } \mathcal{E} =$

$$G^1 \vee G^2 \vee \dots \vee G^n$$

Similarly we can prove that $\text{Inf } \mathcal{E}$

$$= G^1 \wedge G^2 \wedge \dots \wedge G^n \blacksquare$$

Theorem IV.1

$(\mathcal{G}, \wedge, \vee, 0, 1)$ is a complete lattice.

Proof

Proof follows from the Proposition II.1, II.2, III.1 and III.2 ■



Now we prove P_n -adjacency dilated IFSG and P_n -adjacency eroded IFSG are again IFSG in the following theorem IV.2 and IV.3.

Theorem IV.2

$$(G^*, G^\times, \delta_{\mu_{1a}}^i, \delta_{\gamma_{1a}}^i, \delta_{\mu_{2a}}^i, \delta_{\gamma_{2a}}^i) \in \mathcal{G}.$$

Proof

It is enough to prove that

$$0 \leq \delta_{\mu_{1a}}^i(u_i) + \delta_{\gamma_{1a}}^i(u_i) \leq 1 \dots\dots\dots(a)$$

$$\delta_{\mu_{2a}}^i(e_{u_i-u_j}) \leq \delta_{\mu_{1a}}^i(u_i) \wedge \delta_{\mu_{1a}}^i(u_j) \dots\dots\dots(b)$$

$$\gamma_{\mu_{2a}}^i(e_{u_i-u_j}) \leq \delta_{\gamma_{1a}}^i(u_i) \vee \delta_{\gamma_{1a}}^i(u_j) \dots\dots\dots(c)$$

$$0 \leq \delta_{\mu_{2a}}^i(e_{u_i-u_j}) + \delta_{\mu_{2a}}^i(e_{u_i-u_j}) \leq 1 \dots\dots\dots(d)$$

Case 1

$$\text{If possible let, } \delta_{\mu_{1a}}^i(u_i) + \delta_{\gamma_{1a}}^i(u_i) > 1 \dots\dots\dots(e)$$

By definition of dilation, $\delta_{\mu_{1a}}^i(u_i)$ is the supremum and $\delta_{\gamma_{1a}}^i(u_i)$ is the infimum. Then one of the P_n -adjacency vertices to u_i must have membership function as $\delta_{\mu_{1a}}^i(u_i)$, where the non-membership function is greater than or equal to $\delta_{\gamma_{1a}}^i(u_i)$.

Hence by our assumption (e), the sum of membership function and non-membership function of vertex u_i must be greater than 1 in the given IFSG.

As sum of membership function and non-membership function should be in between 0 and 1, this contradicts the fact that the given graph is IFSG.

Since membership function and non-membership function of each vertices are greater than zero, $0 \leq \delta_{\mu_{1a}}^i(u_i) +$

$$\delta_{\gamma_{1a}}^i(u_i)$$

This proves (a).

The same argument used in the proof of (a) proves (d).

Case 2

If possible let,

$$\delta_{\mu_{2a}}^i(e_{u_i-u_j}) > \delta_{\mu_{1a}}^i(u_i) \wedge \delta_{\mu_{1a}}^i(u_j) \dots\dots\dots(f)$$

(1)

(2)

(3)

By definition of dilation, $\delta_{\mu_{2a}}^i(e_{u_i-u_j})$ is the supremum.

The membership functions $\mu_{1a}^i(u_i)$ and $\mu_{1a}^i(u_j)$ of vertices u_i and u_j in the given IFSG should be greater than or equal to the membership of the edge in the IFSG whose membership function is $\delta_{\mu_{2a}}^i(e_{u_i-u_j})$.

The membership function of the vertices u_i and u_j and the edge $e_{u_i-u_j}$ in the P_n -adjacency dilated graph are the supremums of membership functions of the corresponding P_n -adjacency vertices and P_n -adjacency edges.

Hence the assumption (f) will occur only when the membership functions

$\mu_{1a}^i(u_i)$ and $\mu_{1a}^i(u_j)$ of vertices u_i and u_j in the given IFSG are less than the membership function of the edge in the IFSG whose membership function is

$$\delta_{\mu_{2a}}^i(e_{u_i-u_j}).$$

This leads to a contradiction to the fact that given graph is IFSG.

This proves (b).

The same argument used in the proof of (b) for non-membership functions proves (d). ■

Theorem IV.3

$$(G^*, G^\times, \epsilon_{\mu_{1a}}^i, \epsilon_{\gamma_{1a}}^i, \epsilon_{\mu_{2a}}^i, \epsilon_{\gamma_{2a}}^i) \in \mathcal{G}.$$

Proof

Proof is obvious from the proof of the above Theorem. ■

Remarks

From the definitions, it is clear that $G^i \subseteq G_D^i$ or

$$G^i \subseteq \delta_{G^i}. \text{ Hence } \delta_{G^i} \text{ is extensive.}$$

Similarly, $G_E^i \subseteq G^i$ or $\epsilon_{G^i} \subseteq G^i$. Hence ϵ_{G^i} is anti-extensive.

The following theorem proves the operations δ_{G^i} and ϵ_{G^i} are increasing.

Theorem IV.4

Let $G^i = (G^*, G^\times, \epsilon_{\mu_{1a}}^i, \epsilon_{\gamma_{1a}}^i, \epsilon_{\mu_{2a}}^i, \epsilon_{\gamma_{2a}}^i) \in \mathcal{G}$, $i=1,2$. Then

$$G^1 \sqsubseteq G^2 \Rightarrow G_D^1 \sqsubseteq G_D^2$$

$$G^1 \sqsubseteq G^2 \Rightarrow G_E^1 \sqsubseteq G_E^2$$



Proof:

(1) Let u_i be any vertex. Let A be the parameter set.

Suppose $G^1 \sqsubseteq G^2$, then for any $a \in A$,

$$\mu_{1a}^1(u_i) \leq \mu_{1a}^2(u_i) \text{ and } \gamma_{1a}^1(u_i) \geq \gamma_{1a}^2(u_i)$$

If possible let $G_B^1 \not\subseteq G_B^2$, then either $\delta_{\mu_{1a}^1}^1(u_i) > \delta_{\mu_{1a}^2}^2(u_i)$

$$\text{or } \gamma_{\mu_{1a}^1}^1(u_i) \geq \gamma_{\mu_{1a}^2}^2(u_i)$$

At first assume $\delta_{\mu_{1a}^1}^1(u_i) > \delta_{\mu_{1a}^2}^2(u_i)$.

Then \exists a P_n -adjacency vertex u_k of u_i in G^1 whose membership function is $\delta_{\mu_{1a}^1}^1(u_k)$.

Since $\delta_{\mu_{1a}^2}^2(u_i)$ is the supremum of membership function of P_n -adjacency vertices of u_i ,

$$\mu_{1a}^1(u_k) = \delta_{\mu_{1a}^1}^1(u_i) > \delta_{\mu_{1a}^2}^2(u_i) > \mu_{1a}^2(u_k)$$

$\Rightarrow \mu_{1a}^1(u_k) > \mu_{1a}^2(u_k)$, which is a contradiction to our assumption $G^1 \sqsubseteq G^2$.

$$\therefore G_B^1 \subseteq G_B^2$$

(2) Similarly we can prove that the operator ϵ_{G^i} is increasing. ■

Now we are going to prove the operator δ_{G^i} is distributive with respect to union and ϵ_{G^i} is distributive with respect to intersection.

Theorem IV.5

Let $G^i = (G^*, G^*, \epsilon_{\mu_{1a}^i}^i, \epsilon_{\gamma_{1a}^i}^i, \delta_{\mu_{1a}^i}^i, \delta_{\gamma_{1a}^i}^i) \in G$, $i=1,2$. Then

$$1. \delta_{G^1 \cup G^2} = \delta_{G^1} \cup \delta_{G^2}$$

$$2. \epsilon_{G^1 \cap G^2} = \epsilon_{G^1} \cap \epsilon_{G^2}$$

Proof: Let $V_1 \subseteq G^*$ and $E_1 \subseteq G^*$ be vertex set and edge set of IFS subgroups of G^1 and $V_2 \subseteq G^*$ and $E_2 \subseteq G^*$ be vertex set and edge set of IFS subgroups of G^2 in G .

Let A and B be parameter set and $A \sqsubseteq B$

Suppose $G_{A,V_1,E_1}^1 \subseteq G_{B,V_2,E_2}^2$

(1) By definition of union of IFSG's, the possible cases are

$$a) a \in B \setminus A \text{ and } v_i \in V_2 \setminus V_1$$

$$b) a \in B \setminus A \text{ and } v_i \in V_1 \cap V_2$$

$$c) a \in B \cap A \text{ and } v_i \in V_2 \setminus V_1$$

$$d) a \in B \cap A \text{ and } v_i \in V_1 \cap V_2$$

Since $V_1 \subseteq V_2$ and $A \sqsubseteq B$, all membership functions and non membership functions of vertices and edges of $G^1 \cup G^2$ in

cases (a), (b) and (c) will be corresponding membership and non membership functions of vertices and edges of G^2 .

In the case of (d), by definition of union of 2 IFSG's G^1 and G^2 and since $V_1 \subseteq V_2$ and $A \sqsubseteq B$, the membership and non membership functions of vertices and edges of $G^1 \cup G^2$ will be corresponding to membership and non membership functions of vertices and edges in G^2 .

\therefore In all cases, $G^1 \cup G^2$ will be G^2 .

$$\therefore \delta_{G^1 \cup G^2} = \delta_{G^2} \quad (1)$$

By definition of IFS sub graph, $G^1 \cup G^2 \Rightarrow \delta_{G^1} \subseteq \delta_{G^2}$

$$\therefore \delta_{G^1 \cup G^2} = \delta_{G^2} \quad (2)$$

From (1) and (2), $\delta_{G^1 \cup G^2} = \delta_{G^1} \cup \delta_{G^2}$

(2) Let $V_2 = V_1 \cap V_2$ and $E_2 = E_1 \cap E_2$ and $C = A \cap B$

By definition of intersection of IFSG's, V_2 is the set of all vertices and E_2 is the set of all edges in $G^1 \cap G^2$.

$$\text{Since } G^1 \subseteq G^2 \Rightarrow G^1 \cap G^2 = G^1, \epsilon_{G^1 \cap G^2} = \epsilon_{G^1} \quad (3)$$

$$\text{Since } G^1 \subseteq G^2 \Rightarrow \epsilon_{G^1} \subseteq \epsilon_{G^2} \Rightarrow \epsilon_{G^1 \cap G^2} = \epsilon_{G^1} \quad (4)$$

From (1) and (2), $\epsilon_{G^1 \cap G^2} = \epsilon_{G^1} \cap \epsilon_{G^2}$ ■

Observation

We can extend the Theorem IV.5 to finite number of IFSG's.

The following theorem proves an adjunction between the proposed operators P_n -adjacency dilation and P_n -adjacency erosion.

Theorem IV.6

Let $G^i = (G^*, G^*, \mu_{1a}^i, \gamma_{1a}^i, \mu_{1a}^i, \gamma_{1a}^i) \in G$, $i=1,2$ be an Intuitionistic Fuzzy Soft Graphs (IFSG). Then $\delta_{G^i} \subseteq G^i \Leftrightarrow G^i \subseteq \epsilon_{G^i}$.

Proof:

Case 1

Consider, for u_i in G^* ,

$$\delta_{G^1} \subseteq G^1 \Leftrightarrow \delta_{\mu_{1a}^1}^1(u_i) \leq \mu_{1a}^1(u_i) \text{ and } \delta_{\gamma_{1a}^1}^1(u_i) \geq \gamma_{1a}^1(u_i)$$

$$\Leftrightarrow \bigvee_{u_j} \mu_{1a}^1(u_j) \leq \mu_{1a}^1(u_i) \text{ and } \bigvee_{u_j} \gamma_{1a}^1(u_j) \geq \gamma_{1a}^1(u_i)$$

$$\Leftrightarrow \text{where } u_j = u_i \text{ or } u_j \text{ } P_1\text{-adj } u_i$$

$$\mu_{1a}^1(u_j) \leq \mu_{1a}^1(u_i) \text{ and } \gamma_{1a}^1(u_j) \geq \gamma_{1a}^1(u_i) \forall u_j \text{ and}$$

$$\text{where } u_j = u_i \text{ or } u_j \text{ } P_1\text{-adj } u_i$$

$$\Leftrightarrow$$

$$\mu_{1a}^1(u_i) \leq \bigwedge_{u_j} \mu_{1a}^1(u_j) \text{ and } \gamma_{1a}^1(u_i) \geq \bigwedge_{u_j} \gamma_{1a}^1(u_j)$$

$$\text{where } u_j = u_i \text{ or } u_j \text{ } P_1\text{-adj } u_i$$



$$\Rightarrow \mu_{1a}^1(u_i) \leq \epsilon_{\mu_{1a}^2}^2(u_i) \text{ and}$$

$$\gamma_{1a}^1(u_i) \geq \epsilon_{\gamma_{1a}^2}^2(u_i)$$

$$\Rightarrow G^1 \subseteq \epsilon_{G^2}$$

Case 2

For edge $e_{u_i-u_j}$

$$\delta_{G^1} \subseteq G^2 \Rightarrow \delta_{\mu_{2a}^1}^1(e_{u_i-u_j}) \leq \mu_{2a}^2(e_{u_i-u_j}) \text{ and}$$

$$\delta_{\gamma_{2a}^1}^1(u_i) \geq \gamma_{2a}^2(e_{u_i-u_j}) \Rightarrow$$

$$\forall_{e_{u_i-u_k}} \mu_{2a}^1(e_{u_i-u_k}) \leq \mu_{2a}^2(e_{u_i-u_j}) \text{ and}$$

$$\forall_{e_{u_i-u_k}} \gamma_{2a}^1(e_{u_i-u_k}) \geq \gamma_{2a}^2(e_{u_i-u_j})$$

where $e_{u_i-u_k}$ is $e_{u_i-u_j}$ itself or $e_{u_i-u_k}$ P_n -adjacency edge to $e_{u_i-u_j}$

$$\Rightarrow \mu_{2a}^1(e_{u_i-u_k}) \leq \mu_{2a}^2(e_{u_i-u_j}) \text{ and}$$

$$\gamma_{2a}^1(e_{u_i-u_k}) \geq \gamma_{2a}^2(e_{u_i-u_j}) \forall e_{u_i-u_j} \text{ and}$$

where $e_{u_i-u_k}$ is $e_{u_i-u_j}$ itself or

$e_{u_i-u_k}$ P_n -adjacency edge to $e_{u_i-u_j}$

$$\Rightarrow \mu_{2a}^1(e_{u_i-u_j}) \leq \wedge_{e_{u_i-u_k}} \mu_{2a}^2(e_{u_i-u_k}) \text{ and}$$

$$\gamma_{2a}^1(e_{u_i-u_j}) \geq \wedge_{e_{u_i-u_k}} \gamma_{2a}^2(e_{u_i-u_k})$$

where $e_{u_i-u_k}$ is $e_{u_i-u_j}$ itself or $e_{u_i-u_k}$ P_n -adjacency edge to $e_{u_i-u_j}$

$$\Rightarrow \mu_{2a}^1(e_{u_i-u_j}) \leq \epsilon_{\mu_{2a}^2}^2(e_{u_i-u_j}) \text{ and}$$

$$\gamma_{2a}^1(u_i) \geq \epsilon_{\gamma_{2a}^2}^2(e_{u_i-u_j})$$

$$\Rightarrow G^1 \subseteq \epsilon_{G^2}$$

Hence the result is proved.

Observation

The pair of operators $(\epsilon_{G^1}, \delta_{G^1})$ form an intuitionistic fuzzy soft adjunction.

Proposition IV.3

If $i \in I$ and $n > i$, then P_n -adjacency dilation IFSG and $P_{n \pm i}$ -adjacency dilation IFSG are isomorphic if and only if the corresponding vertices and edges have same membership and non-membership functions.

Proposition IV.4

If $i \in I$ and $n > i$, then P_n -adjacency erosion IFSG and $P_{n \pm i}$ -adjacency erosion IFSG are isomorphic if and only if the corresponding vertices and edges have same membership and non-membership functions.

V. P_n -ADJACENCY ALGORITHM

The path adjacency concept in Morphology can apply in the field of Medical Imaging. This helps to identify the region of interest. We propose an algorithm for P_n -adjacency dilation in the following.

- Step 1. Convert the image into graph structure with vertex and edge.
- Step 2. Perform Intuitionistic soft fuzzification of graph structure in step 1.
- Step 3. Use the two formulae given in definition 3.1 to get P_n -adjacency Dilation on the IFSG in step 2.
- Step 4. Compare the obtained membership and non-membership degrees of P_n -adjacency Dilation in IFSG in step 3 with IFSG having threshold membership and non-membership degrees.
- Step 5. This graph generates a particular area of interest in the graph.

VI. CONCLUSION

In this paper, we have introduced the path adjacency vertex, path adjacency edge, P_n -adjacency Dilation, P_n -adjacency Erosion, P_n -adjacency Dilated IFSG, and P_n -adjacency Eroded IFSG. The algorithm presented in this paper is applied to the other areas of knowledge.

REFERENCES

- [1] K. Atanassov, *Intuitionistic fuzzy sets: Theory and Applications*, Heidelberg; Springer-Verlag, 1999.
- [2] De Beats, Kerre, E and Gadan M, "The fundamentals of fuzzy mathematical morphology Part 1: Basic concepts" INT. J. General System, Vol. 23, pp. 155-177, 1995.



- [3] N.Cagman, S Enginoglu and F.Citak, "Fuzzy soft set and its application", Iranian Journal of Fuzzy systems, Vol.8, No.3, pp.137-147 2001
- [4] P.M Dhanya, A Sreekumar, M Jathavedan, P.B .Ramkumar, *Algebra of Morphological Dilation on Intuitionistic fuzzy hyper graph*, 2018 IJHSRSET/Volume4/Issue1
- [5] P.M Dhanya, A Sreekumar, M Jathavedan, P.B .Ramkumar, *Document Modeling and clustering using Hypergraph*, International Journal of Applied Engineering Research, ISSN(0973-4562) Vol.12 No.10, pp.2127-2135, 2017.
- [6] Eman M , El-Nkeeb, h, Elghawalby, A.A.Sdama and S.A. ElHafeez, *Foundation of Neutrosophic Mathematical Morphology* New Trends in Neutrosophic Theory and Application
- [7] Henk Heijmans & Luc Vincent, *Graph Morphology in Image Analysis, "Mathematical Morphology in Image Processing"*, pp. 171-203, 1992
- [8] John Goutsias and J.A.M.Heijmans, Henk " *Funtamenta Morphologicae Mathematicae, Mathematical Morphology, Fundamenta Informaticae* Vol.41, pp.1-31, 2000.
- [9] K.G.Karunambigai and R.Parvathi, *Intuitionistic fuzzy graphs, Journal of Computational Intelligence: Theory and Applications*, Vol. pp.139-150, 2006
- [10] M.G.Karunambigai, R. Parvathi, O.K.Kalaivani, "A Study on Atanassov's Intuitionistic Fuzzy Graphs" in IEEE International Conference on Fuzzy Systems, 2011
- [11] Kauffman, "Introduction a la theorie des sous ensembles Flous", Massonetcie., vol. 1, 1973
- [12] Laurent Najman and Hauues Talbot, "Mathematical Morphology from theory to Applications", Wiley, 2008
- [13] Laurent Najman, Jean Cousty, "A graph-based mathematical morphology reader", Elsevier, 2014
- [14] M.Mahioub, Q.Shubatah, "Domination in product intuitionistic fuzzy graphs", Advances Computational Mathematics and its Applications ACMA, Vol.1, No.3, 2012, ISSN 2167-6356
- [15] P.K.Maji, R.Biswas and A.R.Roy, "Soft set theory", J. Compt. Math. Appl., Vol. 45, pp. 555-562, 2003.
- [16] Muhammad Akram and Saira Nawaz, "On Fuzzy Soft Graphs", Italian Journal of Pure and Applied Mathematics-N, Vol.34, pp.497-514, 2015
- [17] A.Nagoor Gani, S.Anupriya, "Spilt Domination on Intuitionistic Fuzzy Graph", Advanced in Computational Mathematics and its Applications (ACMA) Vol.2, No.2, 2012, ISSN 2167-6356.
- [18] Renato Keshet (Kresch), "Mathematical Morphology on Complete Semilattices and its Applications to Image Processing, Mathematical Morphology", Fundamenta Informaticae Vol.41, pp.33-56, 2000
- [19] A.Rosenfeld, "Fuzzy Graphs" L.A.Zadeh, K.cS.Fu, K.Tanka and M.Shimura (eds), Fuzzy Sets and their plications to cognitive and decision process, Academic Press, New York, 1975, pp.75-95.
- [20] J.Serra, *Image Analysis and Mathematical Morphology*, Academic Press, New York, 1982
- [21] A.M.Shyla and T.K.Mathew Varkey, "Intuitionistic Fuzzy Soft Graph", International Journal of Fuzzy Mathematical Archive, 2016
- [22] S.Mohinda and T.K.Samanta, "An introduction to fuzzy soft graph", Mathematica Moravica, Vol.19-2, pp.35-48., 2015
- [23] Sundas Shahzadi, Muhammad Akram, "Intuitionistic Fuzzy Soft Graphs with applications", J.Appl.Math.Comput. 2016
- [24] P. Sunitha, "An elementary Introduction to Intuitionistic Fuzzy Soft Graphs", J. Math. Comput. Sci., Vol.6, No. 4, pp. 668-681, 2016
- [25] M.S Sunitha and Sunil Mathew, "Fuzzy Graph Theory: A Survey", Annals of Pure and Applied Mathematics, 2013
- [26] R.K.Thumbakara and B.George, "Soft graph", Gen. Math. Notes, Vol. 21, No.2., pp. 75-86, 2014
- [27] Yunquang Yen, Hongjie Li, Yong Bae Jun, "On algebraic structure of intuitionistic fuzzy soft sets", Computer Mathematics

- with Applications, Vol.64, pp.2896-2911, 2012
- [28] L. A. Zadeh, "Fuzzy sets", Information and Control, Vol.8, pp. 338-353, 1965

BIOGRAPHY



Dr. Ramkumar P.B was born in Cochin, Kerala, India in 1977. He received the PhD from Cochin University of Science & Technology, Cochin, Kerala, India. Now he is working as an Associate Professor in Mathematics at Rajagiri School of Engineering & Technology, Cochin, Kerala, India. His current areas of research are Mathematical Morphology, Graph Morphology, Discrete Mathematics, Fuzzy Sets and Fuzzy Logic, Image Processing, Signal Processing, Stochastic Processes and fractal Geometry. To his credit, he has published many research articles in these areas in the reputed international journals. Moreover, he is guiding many scholars in the above said areas. In addition, he has delivered various invited Lectures in many Academic Institutions.



Mr. Abraham Jacob was born in Ayamkara, Muvattupuzha, Kerala, India in 1974. He received MSc from Mahatma Gandhi University, Kottayam, Kerala, India. He was an Assistant Professor in Mathematics in KMM College of Arts & Science, Thrikkakara, Kerala, India. Now he is a research scholar under Dr. P.B. Ramkumar in APJ Abdul Kalam Technical University, Kerala, India. His area of interest is Fuzzy Mathematical Morphology. He has presented papers in many National and International Conferences.