ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5,Special Issue 12, April 2018

Contra pre b-I-open functions

S.P.Jothiprakash

Assistant professor, Department of Mathematics, Nehru Institute of Engineering and Technology, Coimbatore, India.
spjpmath@gmail.com

***Abstract*: In this paper we introduce Contra pre b-I-open, Contra pre b-I-closed functions and investiage some of their properties.**

Keywords: Ideal topological spaces, b-I-open sets, Contra pre b-I-open functions

INTRODUCTION

One of the important and basic topic in the theory of classical point set topology and several branches of mathematics, which have been researched by many authors, is continuity of functions. In this direction, Dontchev [4] introduced the stronger form of LC-continuity called contra-continuity. The concept of ideals in topological spaces has been studied by Kuratowski [6] and Vaidyanathaswamy [7]. In 1996, Andrijevic [2] introduced and studied a weak form of open set called b-open set. In 2005, A. Caksu Guler and G. Aslim [3] introduced and studied b-I-open sets, b-I-continuity and gave a decomposition of continuity.

An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X satisfying the following properties:(1) $A \in I$ and $B \subseteq A$ imply $B \in I$ (heredity); (2) $A \in I$ and $B \in I$ imply $A \cup B \in I$ (finite additivity). A topological space (X, τ) with an ideal I on X is called an ideal topological space and is denoted by (X, τ, I). For a subset $A \subseteq X$, $A^*(I) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$, is called the local function [6] of A with respect to I and τ. We simply write A^* in case there is no chance for confusion.

A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I)$ called the *- topology finer than τ is defined by $Cl^*(A) = A \cup A^*$ [7].

Let (X,τ) denote a topological space on which no separation axioms are assumed unless explicitly stated. In a topological space (X, τ), the closure and the interior of any subset A of X will be denoted by Cl(A) and Int(A), respectively.

PRELIMINARIES

Definition 2.1. A subset A of and ideal topological space (X,τ,I) is called b-I-open[3] if $A \subset cl^*(int(A)) \cup int(cl^*(A))$.

The complement of b-I-open set is called b-I-closed set.

The largerst (resp. smallest) b-I-open (resp. b-I-closed) set contained in (resp. containing) a subset A of the space (X, τ, I) is called b-I-interior (resp. b-I-closure) of A and is denoted by bIint(A) (resp. bIcl(A)).

The class of all b-I-open sets in (X, τ) will be denoted by $BIO(X, \tau)$.

Definition 2.2. [8] An ideal topological space (X, τ, I) is said to be b-I-T_1 if each pair of distinct points x and y of X, there exist b-I-open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma, I)$ is called

(1). b-I-open [1] if $f(U) \in BIO(Y, \sigma)$ for each $U \in \tau$

(2). contra b-I-open [5] if $f(U) \in BIC(Y, \sigma)$ for each $U \in \tau$

CONTRA PRE B-I-OPEN FUNCTIONS

Definition 3.1. A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be

1) pre b-I-open[5] if $f(U) \in BIO(Y, \sigma)$ for each $U \in BIO(X, \tau)$
2) pre b-I-closed[5] if $f(U) \in BIC(Y,\sigma)$ for each $U \in BIC(X,\tau)$
3) contra pre b-I-open if $f(U) \in BJC(Y,\sigma)$ for each $U \in BIO(X,\tau)$
4) contra pre b-I-closed if $f(U) \in BJO(Y,\sigma)$ for each $U \in BIC(X, \tau)$

Remark 3.2. The following example shows that contra pre b-I-openness and contra pre b-I-closedness are independent.

Example 3.3.. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{c\}, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then the function $f : (X, \tau, I) \to (X,\sigma, I)$ defined by $f(a) = f(c) = c$, $f(b) = b$ is contra pre b-I-open but not contra pre b-I-closed.

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5,Special Issue 12, April 2018

Also the function $g : (X, \sigma, I) \to (X, \tau, I)$ by $g(a) = g(c) = c$, $g(b) = b$ is contra pre b-I-closed but not contra pre b-I-open.

Remark 3.4. The following example shows that contra pre b-I-openness (resp. contra pre b-I-closedness) and pre b-I-opennes (resp. pre b-I-closedness) are independent.

Example 3.5. Let (X, τ, I) and (X, σ, I) be the space as defined in Example 3.3.. Then the function $f : (X, \tau, I) \to (X, \sigma, I)$ defined by $f(a) = c$ $f(b) = b$, $f(c) = a$ is pre b-I-open (resp. pre b-I-closed) but not contra pre b-I-open (resp.contra pre b-I-closed).

Example 3.6. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{c\}, \{a, b\}, X\}$, $\eta = \{\emptyset, \{a, b\}, X\}$ and $I = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then the identity function $f : (X, \eta, I) \to (X, \tau, I)$ is contra pre b-I-open (resp. contra pre b-I-closed) but not pre b-I-open (resp. pre b-I-closed).

Remark 3.7. If the function is bijective, then contra pre b-I-openness and contra pre b-I-closedness are equivalent.

Theorem 3.8. For an injective function $f : (X,\tau,I) \to (Y,\sigma, J)$, the following are equivalent

(1). f is contra pre b-I-open.

(2). for every subset B of Y and for every b-I-closed subset F of X with $f^{-1}(B) \subset F$, there exists a b-I-open subset A of Y with $B \subset A$ and $f^{-1}(A) \subset F$.

(3). for every $y \in Y$ and for every b-I-closed subset F of X with $f^{-1}(y) \subset F$, there exists a b-I-open subset A of Y with $y \in A$ and $f^{-1}(A) \subset F$.

Proof: (1) $\Rightarrow$ (2). Let B be a subset of Y and let F be a b-I-closed subset of X with $f^{-1}(B) \subset F$. Put $A = f(F^c)^c$. Since f is contra pre b-I-open, then A is a b-I-open set of Y and since $f^{-1}(B) \subset F$, we have $f(F^c) \subset B^c$ and hence $B \subset A$. Moreover $f^{-1}(A) = f^{-1}(f(F^c)^c) \subset F$.
(2) $\Rightarrow$ (3). It is sufficient to put $B = \{y\}$.
(3) $\Rightarrow$ (1). Let A be a b-I-open subset of X. Then let $y \in f(A)^c$ and let $F = A^c$. By (3), there exists a b-I-open subset By of Y with $y \in B_y$ and $f^{-1}(B_y) \subset F$. Then we see that $y \in B_y \subset f(A)^c$. Hence $f(A)^c = \cup\{B_y : y \in f(A)^c\}$ is b-I-open. Therefore, f(A) is a b-I-closed subset of Y.

Theorem 3.9. For an injective function $f : (X, \tau, I) \to (Y, \sigma, J)$, the following are equivalent

(1). f is contra pre b-I-closed.

(2). for every subset B of Y and for every b-I-open subset A of X with $f{-}1(B) \subset A$, there exists a b-I-closed subset F of Y with $B \subset F$ and $f{-}1(F) \subset A$.

Proof: (1) $\Rightarrow$(2). Let B be a subset of Y and let A be a b-I-open subset of X with $f^{-1}(B) \subset A$. Put $F = f(A^c)^c$. Since f is contra pre b-I-closed, then F is a b-I-closed set of Y and since $f^{-1}(B) \subset A$, we have $f(A^c) \subset B^c$ and hence $B \subset F$. Moreover $f^{-1}(F) \subset A$.

(2) $\Rightarrow$ (1). Let E be a closed subset of X. Put $B = f(E)c$ and let $A = E^c$. Hence $f^{-1}(B) = f^{-1}(f(E)^c) = (f^{-1}(f(E)))^c \subseteq A$. By assumption there exists a b-I-closed $F \subset Y$ for which $B \subset F$ and $f^{-1}(F) \subset A$. It follows that $B = F$. But if $y \in F$ and $y \notin B$, then $y \in f(E)$. Therefore $y = f(x)$ for some $x \in E$ and we have $x \in f^{-1}(F) \subset A = E^c$, which is a contradiction. Since $B = F$, f(E) is b-I-open and hence f is contra pre b-I-closed.

Corollary 3.10. If $f : (X, \tau, I) \to (Y, \sigma, J)$ is contra pre b-I-closed, then for every $y \in Y$ and for every b-I-open subset F of X with $f^{-1}(y) \subset F$, there exists a b-I-closed subset A of Y with $y \in A$ and $f^{-1}(A) \subset F$.

Remark 3.11. The converse of Corollary 3.10. does not hold since for any function with b-I-T1 codomain we may take F to be {y}.

Theorem 3.12. Let $f : (X,\tau,I) \to (Y,\sigma,J)$ be a function. Then,

(1) If f is contra pre b-I-open, then $bIcl(f(A)) \subset f(bIcl(A))$ for every b-I-open subset A of X.

(2) If f is contra pre b-I-closed, then $(f(A) \subset bIint(f(bIcl(A)))$ for every subset A of X.

Proof: (1) Since f is contra pre b-I-open, then $bIcl(f(A)) = f(A) \subset f(bIcl(A))$ for every $A \in BIO(X, \tau)$.

(2) Since f is contra pre b-I-closed and since bIcl(A) is b-I-closed, then $(f(A) \subset f(bIcl(A)) = bIint(f(bIcl(A)))$ for every subset A of X.

Definition 3.13. A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is called

(1) b-I-preclosed if $bIcl(bIint(f(A))) \subset f(A)$ for every b-I-closed set A of X.

(2) b-I-preopen if $f(B) \subset bIint(bIcl(f(B)))$ for every b-I-open set B of X.

Theorem 3.14. For a function $f : (X, \tau, I) \to (Y, \sigma, J)$, the following properties holds:

(1) f is pre b-I-closed, whenever f is contra pre b-I-closed and b-I-preclosed.

(2) f is pre b-I-open, whenever f is contra pre b-I-open and b-I-preopen.

Proof: (1). Let F be a b-I-closed subset of X. Since f is b-I preclosed, $bIcl(bIint(f(F))) \subset f(F)$. Since f(F) is b-I-open, $bIcl(bIint(f(F))) = bIcl(f(F)) \subset f(F)$. Hence f(F) is b-I-closed.

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5,Special Issue 12, April 2018

(2). Let A be a b-I-open subset of X. Since f is b-I-preopen, $f(A) \subset bInt(bIcl(f(A)))$. Since f(A) is b-I-closed, $bInt(bIcl(f(A))) = bIntf(A)$ and hence $f(A) \subset bInt(f(A))$, that is, $f(A) = bInt(f(A))$. Thus, f(A) is b-I-open.

Regarding the restriction $f|_A$ of a function $f : (X, \tau, I) \to (Y, \sigma, J)$ to a subset A of X, we have the following result.

Theorem 3.15. For a function $f : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are true.

(1) If f is contra pre b-I-closed, and A is b-I-closed set in X, then the function $f|_A : (A, \tau|_A, I|_A) \to (Y, \sigma, J)$ is contra pre b-I-closed.

(2) If f is contra pre b-I-open, and A is b-I-open set in X, then the function $f|_A : (A, \tau|_A, I|_A) \to (Y, \sigma, J)$ is contra pre b-I-open.

Proof: (1). Suppose B be an arbitrary b-I-closed subset of A. Since A is b-I-closed subset of X, then B is b-I-closed in X. Then $f|A(B) = f(B)$ is b-I-open in Y . Thus $f|_A$ is contra pre b-I-closed.

(2). Similar to (1).

Theorem 3.16. Let $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \nu, K)$ be two functions such that $g \circ f : (X, \tau, I) \to (Z, \nu, K)$. Then,

(1) $g \circ f$ is contra pre b-I-open, if f is pre b-I-open and g is contra pre b-I-open.

(2) $g \circ f$ is contra pre b-I-open, if f is contra pre b-I-open and g is pre b-I-closed.

(3) $g \circ f$ is contra pre b-I-closed, if f is pre b-I-closed and g is contra pre b-I-closed.

(4) $g \circ f$ is contra pre b-I-closed, if f is contra pre b-I-closed and g is pre b-I-open.

Proof: Straightforward.

Lemma 3.17. Let (X, τ, I) be an ideal topological space and let $A \subset X$. Then $x \in bIcl(A)$ if and only if for every b-I-open set U of X containing x, $U \cap A \neq \emptyset$.

Definition 3.18.. A subset A of X is said to be b-I-dense [1] in X if and only if $bIcl(A) = X$.

Theorem 3.19.. For a function $f : (X, \tau, I) \to (Y, \sigma, J)$, the following properties holds:

(1) If f is contra pre b-I-open and $B \subset Y$ has the property that B is not contained in proper b-I-open sets then $f^{-1}(B)$ is b-I-dense in X.

(2) If f is contra pre b-I-closed and A is a b-I-dense subset of Y then $f^{-1}(A)$ is not contained in a proper b-I-open set.

Proof: (1). Let $x \in X$ and let $A \in BIO(X, x)$. Then f(A) is b-I-closed and $(f(A))^c$ is a proper b-I-open subset of Y. Thus, B not contained in $(f(A))^c$ and hence there exists $y \in B$ such that $y \in f(A)$. Let $z \in A$ for which $y = f(z)$. Then $z \in A \cap f^{-1}(B)$. Hence $A \cap f^{-1}(B) \neq \emptyset$and thus by Lemma 3.17, $x \in bIcl(f^{-1}(B))$. Then by definition, $f^{-1}(B)$ is b-I-dense in X.

(2). Assume that $f^{-1}(A) \subset B$ where B is a proper b-I-open subset of X. Then we have that $f(B^c)$ is a nonempty b-I-open set if Y such that $f(B^c) \cap A = \emptyset$, which by Lemma 3.17., contradicts the fact that A is b-I-dense.

REFERENCES

[1] M. Akdag, "On b-I-open sets and b-I-continuous functions", *Intern.J. Math. Math. Sci.,* vol 2007, Article ID 75721, 13 pages. doi:10.1155/2007/75721.

[2] D. Andrijevic, "On b-open sets", *Math. Vesnik,* vol.48, pp. 59-64, 1996.

[3] A. Caksu Guler and G.Aslim, "b-I-open sets and decomposition of continuity via idealization", *Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb.* 22(2005), 27-32.

[4] J. Dontchev, "Contra-continuous functions and Strongly S-closed spaces", *Internet. J. Math. Math. Sci.* vol. 19, (2) pp. 303-310, 1996.

[5] S.P. Jothiprakash, "Contra functions via b-I-open sets", *Proc. Indian Math.Society, 79th Annual conference,* Cochin (2013).

[6] K. Kuratowski, *Topology*, Vol. I, New York: Academic press, 1966.

[7] R. Vaidyanathaswamy, "The localization theory in set topology", *Proc. Indian Acad. Sci.* vol. 20, pp. 51-61, 1954.

[8] K. Viswanathan and S.P. Jothiprakash, "Almost contra b-I-continuous functions", *Proc. Indian Science Congress 100th Annual Conference, Coimbatore-Chapter*, Coimbatore (2012).