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ON $\psi\alpha g$-NEIGHBOURHOODS IN TOPOLOGICAL SPACES

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Abstract: In this paper we introduce $\psi\alpha g$-derived set, $\psi\alpha g$-border, $\psi\alpha g$-frontier, $\psi\alpha g$-exterior and further the relationship between them are derived.

Keywords: *$\psi\alpha g$-derived set, $\psi\alpha g$-border, $\psi\alpha g$-frontier and $\psi\alpha g$-exterior.*

I. INTRODUCTION

The importance of general topological spaces rapidly increases in many fields of applications such as data mining [3]. Information systems are basic tools for production knowledge from data in any real life field. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative ones. Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology and real analysis concerns the variously modified forms of continuity, separation axioms etc. By utilizing generalized open sets. One of the most well known notions and also an inspiration source is the notion of $\psi\alpha g$-open [5] sets introduced by V. Kokilavani and P.R. Kavitha. In this paper, we will continue the study of related functions with $\psi\alpha g$-open and $\psi\alpha g$-closed sets. We introduce and characterize the concept of $\psi\alpha g$-derived set, $\psi\alpha g$-border, $\psi\alpha g$-frontier, $\psi\alpha g$-exterior and $\psi\alpha g$-saturated and further the relationship between them are derived.

II. PRELIMIERIES

Throughout the present paper, spaces X and Y always mean topological spaces. Let X be a topological space and A, a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$ respectively.

Definition: 2.1 A subset A of a space (X,τ) is called

(i) semi open set [2] if $A\subseteq cl(int(A))$.

(ii) semi pre open set [1] if $A\subseteq cl(int(cl(A)))$.

(iii) α-open set [4] if $A\subseteq int(cl(int(A)))$.

III. $\psi\alpha g$-Derived and $\psi\alpha g$-Border

Definition: 3.1 Let A be a subset of a space X. A point $x\in X$ is said to be $\psi\alpha g$-limit point of A if for each $\psi\alpha g$-open set U containing x, $U\cap(A-\{x\})\neq\phi$. The set of all $\psi\alpha g$-limit point of A is called $\psi\alpha g$-derived (briefly. $D_{[\psi\alpha g]}$) set of A and is denoted by $D_{[\psi\alpha g]}(A)$.

Theorem 3.2 For subsets A,B of a space X, the following statements hold:

(i) If $A\subset B$, then $D_{[\psi\alpha g]}(A)\subset D_{[\psi\alpha g]}(B)$.

(ii) $D_{[\psi\alpha g]}(A)\cup D_{[\psi\alpha g]}(B)\subset D_{[\psi\alpha g]}(A\cup B)$

(iii) $D_{[\psi\alpha g]}(D_{[\psi\alpha g]}(A))-A\subset D_{[\psi\alpha g]}(A)$

(iv) $D_{[\psi\alpha g]}(A\cup D_{[\psi\alpha g]}(A))\subset A\cup D_{[\psi\alpha g]}(A)$

Proof: (i) Let $x\in D_{[\psi\alpha g]}(A)$ iff $(U\backslash\{x\})\cap A\neq\phi$ for every open set U containing x. But $B\supset A$; hence so $x\in D_{[\psi\alpha g]}(A)$ implies $x\in D_{[\psi\alpha g]}(B)$. That is $D_{[\psi\alpha g]}(A)\subset D_{[\psi\alpha g]}(B)$.

(ii) From (i), $A\subset A\cup B$ implies $D_{[\psi\alpha g]}(A)\subset D_{[\psi\alpha g]}(A\cup B)$and $B\subset A\cup B$ implies $D_{[\psi\alpha g]}(B)\subset D_{[\psi\alpha g]}(A\cup B)$. So $D_{[\psi\alpha g]}(A)\cup D_{[\psi\alpha g]}(B)\subset D_{[\psi\alpha g]}(A\cup B)$.

(iii)If $x \in D_{[\psi\alpha g]}\left(D_{[\psi\alpha g]}(A)\right) - A$ and U is an $\psi\alpha g$-open set containing x, then $U \cap (D_{[\psi\alpha g]}(A) - \{x\}) \neq \phi$. Let $y \in U \cap (D_{[\psi\alpha g]}(A) - \{x\})$. Then since $y \in D_{[\psi\alpha g]}(A)$ and $y \in U$, $U \cap (A - \{y\}) \neq \phi$. Let $z \in U \cap (A - \{y\})$. Then $z \neq x$ for $z \in A$ and $x \notin A$. Hence $U \cap (A - \{x\}) \neq \phi$. Therefore $x \in D_{[\psi\alpha g]}(A)$.

(iv)Let $x \in D_{[\psi\alpha g]}(A \cup D_{[\psi\alpha g]}(A))$. If $x \in A$, the result is obvious. So let $x \in D_{[\psi\alpha g]}\left(A \cup D_{[\psi\alpha g]}(A)\right) - A$, then for $\psi\alpha g$-open set containing x, $U \cap (A \cup D_{[\psi\alpha g]}(A) - \{x\}) \neq \phi$. Thus $U \cap (A - \{x\}) \neq \phi$ or $U \cap (D_{[\psi\alpha g]}(A) - \{x\}) \neq \phi$. Now it follows (iii) that $U \cap (A - \{x\}) \neq \phi$. Hence $x \in D_{[\psi\alpha g]}(A)$. Therefore, in any case $D_{[\psi\alpha g]}(A \cup D_{[\psi\alpha g]}(A)) \subset A \cup D_{[\psi\alpha g]}(A)$.

Theorem 3.3 For any subset A of a space X, $\psi\alpha g_{cl}(A) = A \cup D_{[\psi\alpha g]}(A)$.

Proof: Since $D_{[\psi\alpha g]}(A) \subset \psi\alpha g_{cl}(A)$, $A \cup D_{[\psi\alpha g]}(A) \subset \psi\alpha g_{cl}(A)$. On the other hand, let $x \in \psi\alpha g_{cl}(A)$. If $x \in A$, then the proof is complete. If $x \notin A$, then each $\psi\alpha g$-open set U containing x intersects A at a point distinct from x. Therefore $x \in D_{[\psi\alpha g]}(A)$. Thus $\psi\alpha g_{cl}(A) \subset A \cup D_{[\psi\alpha g]}(A)$ which implies that $\psi\alpha g_{cl}(A) = A \cup D_{[\psi\alpha g]}(A)$. This completes the proof.

Theorem: 3.4 For subsets A, B of a space X, the following statements hold:

(i) $\psi\alpha g_{Int}(A)$ is the largest $\psi\alpha g$-open set contained in A.

(ii) A is an $\psi\alpha g$-open if and only if $A = \psi\alpha g_{Int}(A)$.

(iii) $\psi\alpha g_{Int}(\psi\alpha g_{Int}(A)) = \psi\alpha g_{Int}(A)$

(iv) $\psi\alpha g_{Int}(A) = A - D_{[\psi\alpha g]}(X - A)$

(v) $X - \psi\alpha g_{Int}(A) = \psi\alpha g_{cl}(X - A)$

(vi) $X - \psi\alpha g_{cl}(A) = \psi\alpha g_{Int}(X - A)$

(vii)$A \subset B$, then $\psi\alpha g_{Int}(A) \subset \psi\alpha g_{Int}(B)$

(viii) $\psi\alpha g_{Int}(A) \cup \psi\alpha g_{Int}(B) \subset \psi\alpha g_{Int}(A \cup B)$

Proof: (i)For each point $x \in A$, $x \in U_x \subset A$. Hence $\cup\{U_x : x \in A\} = A$. And so A is a union of all open sets contained in A. The interior of a set A is the union of all open subsets of A. That is if U is an open subsets of A then $U \subset \psi\alpha g_{Int}(A) \subset A$.

(ii)If A is $\psi\alpha g$-open then $U \subset \psi\alpha g_{Int}(A) \subset A$ or $A = \psi\alpha g_{Int}(A)$. If $A = \psi\alpha g_{Int}(A)$ then A is $\psi\alpha g$-open. Since $\psi\alpha g_{Int}(A)$ is open.

(iii) It follows from (i) and (ii).

(iv)If $x \in A - D_{[\psi\alpha g]}(X - A)$, then $x \notin D_{[\psi\alpha g]}(X - A)$ and so there exists an $\psi\alpha g$-open set U containing x such that $U \cap X - A = \phi$. Then $x \in U - A$ and hence $x \in \psi\alpha g_{Int}(A)$. That is $A - D_{[\psi\alpha g]}(X - A) \subset \psi\alpha g_{Int}(A)$. On the other hand, if $x \in \psi\alpha g_{Int}(A)$, then $x \notin D_{[\psi\alpha g]}(X - A)$. Since $\psi\alpha g_{Int}(A)$ is an $\psi\alpha g$-open and $\psi\alpha g_{Int}(A) \cap (X - A) = \phi$. Hence $\psi\alpha g_{Int}(A) = A - D_{[\psi\alpha g]}(X - A)$.

(v)

$$X - \psi\alpha g_{Int}(A) = X - \left(A - D_{[\psi\alpha g]}(X - A)\right) = (X - A) \cup D_{[\psi\alpha g]}(X - A) = \psi\alpha g_{cl}(X - A)$$

.

(vi)Using (iv) and Theorem 3.3, we get,

$$\psi\alpha g_{Int}(X - A) = (X - A) - D_{[\psi\alpha g]}(X - A) = X - \left(A \cup D_{[\psi\alpha g]}(A)\right) = X - \psi\alpha g_{cl}(A)$$

.

(vii)Let $x \in \psi\alpha g_{Int}(A)$, the union of all $\psi\alpha g$-open sets contained in A. But $A \subset B$, so the union of all $\psi\alpha g$-open sets contained in B which implies $x \in \psi\alpha g_{Int}(B)$. Hence $\psi\alpha g_{Int}(A) \subset \psi\alpha g_{Int}(B)$.

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(viii) From (i), $A \subset A \cup B$ implies $\psi\alpha g_{Int}(A) \subset \psi\alpha g_{Int}(A \cup B)$and $B \subset A \cup B$ implies $\psi\alpha g_{Int}(B) \subset \psi\alpha g_{Int}(A \cup B)$. So $\psi\alpha g_{Int}(A) \cup \psi\alpha g_{Int}(B) \subset \psi\alpha g_{Int}(A \cup B)$.

Definition:3.5 For any subset A of a space X, $Bd^{<\psi\alpha g>}(A) = A - \psi\alpha g_{Int}(A)$ is said to be $\psi\alpha g$-border of A.

Theorem:3.6 For a subset A of a space X, the following statements hold:

(i) $A = \psi\alpha g_{Int}(A) \cup Bd^{<\psi\alpha g>}(A)$

(ii) $\psi\alpha g_{Int}(A) \cap Bd^{<\psi\alpha g>}(A) = \phi$

(iii) A is an $\psi\alpha g$-open iff $Bd^{<\psi\alpha g>}(A) = \phi$

(iv) $Bd^{<\psi\alpha g>}(\psi\alpha g_{Int}(A)) = \phi$

(v) $\psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A)) = \phi$

(vi) $Bd^{<\psi\alpha g>}\left(Bd^{<\psi\alpha g>}(A)\right) = Bd^{<\psi\alpha g>}(A)$

(vii) $Bd^{<\psi\alpha g>}(A) = A \cap \psi\alpha g_{cl}(X - A)$

(viii) $Bd^{<\psi\alpha g>}(A) = D_{[\psi\alpha g]}(X - A)$

Proof: (i)Obvious.

(ii)If $x \in \psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A))$, then $x \in Bd^{<\psi\alpha g>}(A)$. On the other hand, since $Bd^{<\psi\alpha g>}(A) \subset A$, $x \in \psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A)) \subset \psi\alpha g_{Int}(A)$. Hence $x \in \psi\alpha g_{Int}(A) \cap Bd^{<\psi\alpha g>}(A)$, which contradicts (ii). Thus $\psi\alpha g_{Int}(A) \cap Bd^{<\psi\alpha g>}(A) = \phi$.

(iii)Since $\psi\alpha g_{Int}(A) \subseteq A$, it follows from theorem 3.4(ii), that A is $\psi\alpha g$-open

$\Leftrightarrow A = \psi\alpha g_{Int}(A) \Leftrightarrow Bd^{<\psi\alpha g>}(A) = A - \psi\alpha g_{Int}(A) = \phi$

.

(iv)Since A is an $\psi\alpha g$-open, it follows from (3) that $Bd^{<\psi\alpha g>}(\psi\alpha g_{Int}(A)) = \phi$.

(v)If $x \in \psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A))$, then $x \in Bd^{<\psi\alpha g>}(A) \subseteq A$ and $x \in \psi\alpha g_{Int}(A)$. Since $\psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A)) \subseteq \psi\alpha g_{Int}(A)$. Thus $x \in \psi\alpha g_{Int}(A) \cap Bd^{<\psi\alpha g>}(A) = \phi$, which is a contradiction. Hence $\psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A)) = \phi$.

(vi)Using (v), we get $Bd^{<\psi\alpha g>}\left(Bd^{<\psi\alpha g>}(A)\right) = Bd^{<\psi\alpha g>}(A) - \psi\alpha g_{Int}(Bd^{<\psi\alpha g>}(A)) = Bd^{<\psi\alpha g>}(A)$

.

(vii)

$Bd^{<\psi\alpha g>}(A) = A - \psi\alpha g_{Int}(A) = A - (X - \psi\alpha g_{cl}(X - A)) = A \cap \psi\alpha g_{cl}(X - A)$

$Bd^{<\psi\alpha g>}(A) = A - \psi\alpha g_{Int}(A) = A - \left(A - D_{[\psi\alpha g]}(X - A)\right) = D_{[\psi\alpha g]}(X - A)$

IV. $\psi\alpha g$-Frontier and $\psi\alpha g$-Exterior

Definition:4.1 For a subset A of a space X, $F_{\psi\alpha g}(A) = \psi\alpha g_{cl}(A) \cap \psi\alpha g_{cl}(X \backslash A)$ is said to be $\psi\alpha g$-Frontier of A.

Theorem:4.2 For subsets A of a space X, the following statements hold:

(i) $\psi\alpha g_{cl}(A) = \psi\alpha g_{Int}(A) \cup F_{\psi\alpha g}(A)$

(ii) $\psi\alpha g_{Int}(A) \cap F_{\psi\alpha g}(A) = \phi$

(iii) $Bd^{<\psi\alpha g>}(A) \subset F_{\psi\alpha g}(A)$

(iv) $F_{\psi\alpha g}(A) = Bd^{<\psi\alpha g>}(A) \cup D_{[\psi\alpha g]}(A)$

(v) A is $\psi\alpha g$-open set iff $F_{\psi\alpha g}(A) = D_{[\psi\alpha g]}(A)$

(vi) $F_{\psi\alpha g}(A) = \psi\alpha g_{cl}(A) \cap \psi\alpha g_{cl}(X \backslash A)$

(vii) $F_{\psi\alpha g}(A) = F_{\psi\alpha g}(X \backslash A)$

(viii) $F_{\psi\alpha g}(A)$ is $\psi\alpha g$-closed.

(ix) $F_{\psi\alpha g}\left(F_{\psi\alpha g}(A)\right) \subset F_{\psi\alpha g}(A)$

(x) $F_{\psi\alpha g}\left(\psi\alpha g_{Int}(A)\right) \subset F_{\psi\alpha g}(A)$

(xi) $F_{\psi\alpha g}\left(\psi\alpha g_{cl}(A)\right) \subset F_{\psi\alpha g}(A)$

(xii) $\psi\alpha g_{Int}(A) = A - F_{\psi\alpha g}(A)$

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Proof:

(i)

$\psi\alpha g_{Int}(A) \cup F_{\psi\alpha g}(A) = \psi\alpha g_{Int}(A) \cup (\psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(A)) = \psi\alpha g_{cl}(A)$

.

(ii)

$\psi\alpha g_{Int}(A) \cap F_{\psi\alpha g}(A) = \psi\alpha g_{Int}(A) \cap (\psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(A)) = \phi$

.

(iii)Since $A \subseteq \psi\alpha g_{cl}(A)$, we have $Bd^{<\psi\alpha g>}(A) = A - \psi\alpha g_{Int}(A) \subseteq \psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(A) = F_{\psi\alpha g}(A)$

.

(iv)Since

$\psi\alpha g_{Int}(A) \cup F_{\psi\alpha g}(A) = \psi\alpha g_{Int}(A) \cup Bd^{<\psi\alpha g>}(A) \cup D_{[\psi\alpha g]}(A)$

, $F_{\psi\alpha g}(A) = Bd^{<\psi\alpha g>}(A) \cup D_{[\psi\alpha g]}(A)$.

(v)It follows from (iv), Theorem(3.6)(iii) and Theorem (3.4)(ii).

(vi)

$\psi\alpha g_{cl}(A) \cap \psi\alpha g_{cl}(X \backslash A) = \psi\alpha g_{cl}(A) \cap (X \backslash \psi\alpha g_{Int}(A)) = \psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(A) = F_{\psi\alpha g}(A)$

.

(vii)It follows from (vi).

(viii)

$\psi\alpha g_{cl}\left(F_{\psi\alpha g}(A)\right) = \psi\alpha g_{cl}(\psi\alpha g_{cl}(A) \cap \psi\alpha g_{cl}(X \backslash A)) \subset \psi\alpha g_{cl}(\psi\alpha g_{cl}(A)) \cap \psi\alpha g_{cl}(\psi\alpha g_{cl}(X \backslash A)) = F_{\psi\alpha g}(A)$

. Hence $F_{\psi\alpha g}(A)$ is $\psi\alpha g$-closed.

(ix)

$F_{\psi\alpha g}\left(F_{\psi\alpha g}(A)\right) = \psi\alpha g_{cl}\left(F_{\psi\alpha g}(A)\right) \cap \psi\alpha g_{cl}\left(X - F_{\psi\alpha g}(A)\right) \subset \psi\alpha g_{cl}\left(F_{\psi\alpha g}(A)\right) = F_{\psi\alpha g}(A)$

.

(x)Using Theorem 3.4(iii), we get,

$F_{\psi\alpha g}(\psi\alpha g_{Int}(A)) = \psi\alpha g_{cl}\left(\psi\alpha g_{Int}(A) \backslash \psi\alpha g_{Int}(\psi\alpha g_{Int}(A))\right) \subseteq \psi\alpha g_{cl}(A) \backslash \psi\alpha g_{Int}(A) = F_{\psi\alpha g}(A)$

.

(xi)

$F_{\psi\alpha g}(\psi\alpha g_{cl}(A)) = \psi\alpha g_{cl}(\psi\alpha g_{cl}(A)) - \psi\alpha g_{Int}(\psi\alpha g_{cl}(A)) = \psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(\psi\alpha g_{cl}(A)) = \psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(A) = F_{\psi\alpha g}(A)$

.

(xii)

$A - F_{\psi\alpha g}(A) = A - (\psi\alpha g_{cl}(A) - \psi\alpha g_{Int}(A)) = \psi\alpha g_{Int}(A)$

.

Definition:4.3 For a subset A of a space X, $\mathcal{E}xt^{<\psi\alpha g>}(A) = \psi\alpha g_{Int}(X - A)$ is said to be $\psi\alpha g$-exterior of A.

Theorem:4.4 For subsets A of a space X, the following statements hold:

(i) $\mathcal{E}xt^{<\psi\alpha g>}(A)$ is $\psi\alpha g$-open.

(ii)

$\mathcal{E}xt^{<\psi\alpha g>}(A) = \psi\alpha g_{Int}(X - A) = X - \psi\alpha g_{cl}(A)$

(iii)

$\mathcal{E}xt^{<\psi\alpha g>}\left(\mathcal{E}xt^{<\psi\alpha g>}(A)\right) = \psi\alpha g_{Int}(\psi\alpha g_{cl}(A))$

(iv)If $A \subset B$, then $\mathcal{E}xt^{<\psi\alpha g>}(A) \supset \mathcal{E}xt^{<\psi\alpha g>}(B)$

(v)$\mathcal{E}xt^{<\psi\alpha g>}(X) = \phi$

(vi)$\mathcal{E}xt^{<\psi\alpha g>}(\phi) = X$

(vii)(viii

$\mathcal{E}xt^{<\psi\alpha g>}(A) = \mathcal{E}xt^{<\psi\alpha g>}\left(X - \mathcal{E}xt^{<\psi\alpha g>}(A)\right)$

(viii)$\psi\alpha g_{Int}(A) \subset \mathcal{E}xt^{<\psi\alpha g>}\left(\mathcal{E}xt^{<\psi\alpha g>}(A)\right)$

(ix)
$X = \psi\alpha g_{Int}(A) \cup \mathcal{E}xt^{<\psi\alpha g>}(A) \cup F_{\psi\alpha g}(A)$

Proof:

(i)Obvious.
(ii)It follows from the Theorem 3.4 (vi).
(iii)
$\mathcal{E}xt^{<\psi\alpha g>}\left(\mathcal{E}xt^{<\psi\alpha g>}(A)\right) = \mathcal{E}xt^{<\psi\alpha g>}\left(\psi\alpha g_{Int}(X\backslash A)\right) = \psi\alpha g_{Int}\left(X\backslash \psi\alpha g_{Int}(X\backslash A)\right) = \psi\alpha g_{Int}\left(\psi\alpha g_{cl}(A)\right) \supset \psi\alpha g_{Int}(A)$
.

(iv)Assume that $A \subset B$. Then
$\mathcal{E}xt^{<\psi\alpha g>}(B) = \psi\alpha g_{Int}(X\backslash B) \subseteq \psi\alpha g_{Int}(X\backslash A) = \mathcal{E}xt^{<\psi\alpha g>}(A)$
by using Theorem (3.4)(vii).
(v)
$\mathcal{E}xt^{<\psi\alpha g>}(A \cup B) = \psi\alpha g_{Int}(X\backslash(A \cap B)) = \psi\alpha g_{Int}((X\backslash A) \cup (X\backslash B)) \supseteq \psi\alpha g_{Int}(X\backslash A) \cup \psi\alpha g_{Int}(X\backslash B) = \mathcal{E}xt^{<\psi\alpha g>}(A) \cup \mathcal{E}xt^{<\psi\alpha g>}(B).$

(vi) and (vii) are obvious.
(viii)
$\mathcal{E}xt^{<\psi\alpha g>}\left(X - \mathcal{E}xt^{<\psi\alpha g>}(A)\right) = \mathcal{E}xt^{<\psi\alpha g>}\left(X - \psi\alpha g_{Int}(X - A)\right) = \psi\alpha g_{Int}\left(X - (X - \psi\alpha g_{Int}(X - A))\right) = \psi\alpha g_{Int}\left(\psi\alpha g_{Int}(X - A)\right) = \psi\alpha g_{Int}(X - A) = \mathcal{E}xt^{<\psi\alpha g>}(A)$
.
(ix)
$\psi\alpha g_{Int}(A) \subset \psi\alpha g_{Int}\left(\psi\alpha g_{cl}(A)\right) = \psi\alpha g_{Int}\left(X - \psi\alpha g_{Int}(X - A)\right) = \psi\alpha g_{Int}\left(X - \mathcal{E}xt^{<\psi\alpha g>}(A)\right) = \mathcal{E}xt^{<\psi\alpha g>}\left(\mathcal{E}xt^{<\psi\alpha g>}(A)\right)$
.
(x)obvious.

V. CONCLUSION

we introduced $\psi\alpha g$-derived set, $\psi\alpha g$-border, $\psi\alpha g$-frontier, $\psi\alpha g$-exterior and further the relationship between them are derived.

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