ISSN2394-3777 (Print)
ISSN2394-3785 (Online)
Available online atwww.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

# Characterization of Nano P*G-Homeomorphisms in Nano topological spaces

K.Geetha[1], M.Vigneshwaran[2]

Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, India [1&2]

**Abstract**:The objective of this paper is to introduce and investigate the Nano p*g-homeomorphism in Nano topological spaces.

**Keywords**: Nano Topology, Nano p*g-closed set, Nano p*g-closed map, Nano p*g-homeomorphism.

## I. Introduction

The word topology is used in the mathematical discipline for a family of sets with certain properties that are used to define a topological space. In topology the notion of homeomorphism is playing a very important role. Many researchers in topological spaces have generalised the notion of homeomorphism, g-homeomorphism, gc-homeomorphism were introduced by Maki.et.al.[5]. Nano topology was introduced by LellisThivagar [4] and more, he investigated Nano homeomorphism in Nano topological spaces. Bhuvaneswari.et.al.[1] introduced and distinguished some properties of Nano generalised homeomorphism in Nano topological spaces. K . MythiliGnanapriya[2] introduced some properties in Nano generalised pre-homeomorphism. In this paper, a new class of homeomorphism called Nano p*g-homeomorphism is introduced and some properties are discussed.

## II. Preliminaries

**Definition 2.1**[**3**]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X⊆U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup$ where R(x) denotes the equivalence class determined by X∊U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x\in U}\{R(x):R(X)\cap X \neq \emptyset\}$

3. The boundary of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X)=U_R(X)-L_R(X)$.

**Property: [3]:** If (U, R) is an approximation space and X, Y⊆U, then

(i) $L_R(X)\subseteq X\subseteq U_R(X)$

(ii) $L_R(\Phi)=U_R(\Phi)=\Phi$ and $L_R(U)=U_R(U)=U$

(iii) $U_R(X\cup Y)= U_R(X) \cup U_R(Y)$

(iv) $U_R(X\cap Y)\subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X\cup Y)\supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X\cap Y)=L_R(X) \cap L_R(Y)$

ISSN2394-3777 (Print)
ISSN2394-3785 (Online)
Available online atwww.ijartet.com



(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R$ (XC) = $[L_R(X)]$ C and $L_R$ (XC) = $[U_R(X)]$ C
(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**Research Article :**

**Definition 2.2[3]:** Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X ⊆ U. Let $\tau_R(X)=\{U,\Phi, L_R(X), U_R(X), B_R(X)\}$ Then $\tau_R(X)$ is a topology on U, called as the Nano topology with respect to X. Elements of the Nano topology are known as the Nano-open sets in U and U, $\tau_R(X)$) (is called the Nano topological space. $[\tau_R(X)]^c$ is called as the dual Nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as Nano closed sets.

**Definition 2.3[3]:** If $\tau_R(X)$ is the Nanotopology on U with respect to X, then the set $B=\{U,L_R(X),U_R(X),B_R(X)\}$ is the basis for $\tau_R(X)$.

**Definition 2.4[3]:** If ( U, $_R(X)$) is a Nano topological space with respect to X where X⊆U and if A⊆U , then the Nano interior of A is defined as the union of all Nano-open subsets of A and it is denoted by Nint(A). That is Nint (A), is the largest Nano open subset of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing Aand is denoted by Ncl (A). That is Ncl (A), is the smallest Nano closed set containing A.

**Definition 2.5[3]:** Let (U, $\tau_R(X)$) be a Nano topological space and A⊆U. Then A is said to be
Nano semi open if A ⊆Ncl (Nint(A))
Nano pre-open if A⊆Nint (Ncl(A))
Nano $\alpha$ -open if A⊆Nint (Ncl (Nint(A)))
Nano b-open if A⊆Ncl(Nint(A))∪Nint(Ncl(A))
Nano regular-open if A=Nint(Ncl(A)).

Throughout this paper (U, $\tau_R(X)$) is a Nano topological space with respect to X where X⊆U, R is equivalence relation on U, U/R denotes the family of equivalence classes of U by R In this section we define Nano *gα- closed sets and Nano p*g – closed sets and explain about Nano p*g-homeomorphism.

**Definition 2.6[5]:** Abijective f: (x, τ) → (y, σ) is called a generalized homeomorphism (g-homeomorphism) (5) if f is both g- continuous and g - open.

**Definition 2.7[5]:**A bijective f: (x, τ) → (y, σ) is called a gc- homeomorphism if both f and $f^{-1}$ are gc- irresolute functions.

**Definition 2.8 [4]:** A bijective f: (U, $\tau_R$ (X )) → ( V, $\tau_R'$ (Y )) is said to be Nano homeomorphism if, f is 1-1 and onto, f is Nano continuous and f is Nano open.

**Definition 2.9[1]:** A bijectivef: (U, $\tau_R$ (X )) → (V, $\tau_R'$ (Y )) is called Nano generalized homeomorphism (Ng-homeomorphism) if f is both Ng continuous and Ng-open.

**Definition 2.10**[2]: A bijective f:(U, $\tau_R$ (X )) → ( V, $\tau_R'$ (Y )) is called Nano generalized pre homeomorphism (Ngp-homeomorphism) if f is both Ngp- continuous and Ngp-open.

**Definition 2.11:** A subset A of (U,$\tau_R(X)$) is called Nano *gα closed set (Briefly N*gα-closed) if Ncl(A) ⊆U and V is Nano gα- open in (U,$\tau_R(X)$).

**Definition 2.12 :**A subset A of (U,$\tau_R(X)$) is called Nano pre star generalised closed sets (Briefly Np*gα-closed) if Npcl(A) ⊆ U and V is Nano *gα- open in (U,$\tau_R(X)$).

**Definition 2.13:** A function f: (U, $\tau_R$ (X)) → (V, $\tau_R'$(Y)) is Nano p*g- continuous if and only if the inverse image of every Nano p*g- closed set in V is Nano p*g-closed in U.

ISSN2394-3777 (Print)
ISSN2394-3785 (Online)
Available online atwww.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

## III. NP*G- HOMEOMORPHISM

**Definition 3.1:** A function f: (U, $\tau_R$ (X )) → (V, $\tau_R{}'$ (Y )) is said to beNp*g – homeomorphism if

(i) f is one to one and onto
(ii) f is a Np*g – Continuous
(iii) f is Np*g – open.

**Definition 3.2:** A Mapf: (U, $\tau_R$ (X )) → ( V, $\tau_R{}'$ (Y )) is said to beNano generalized p*g-closed map if the image of every Nano closed set in (U, $\tau_R$ (X )) is Np*g-closed in ( V, $\tau_R{}'$ (Y )).

**Theorem3.3:**Letf:(U,$\tau_R$ (X )) → ( V, $\tau_R{}'$ (Y )) be an one to one and onto mapping. Then f is Np*g – homeomorphism if and only if f is Np*g – closed and Np*g- continuous.

**Proof:**

Let f: (U, $\tau_R$ (X )) → ( V, $\tau_R{}'$(Y )) be a Np*g – homeomorphism. Then f is Np*g – Continuous. Let A be an arbitrary Nanoclosed set in (U, $\tau_R$ (X )). Then U-A is Nano open. Since f is Np*g – open. f (U-A) is Np*g- open in (V, $\tau_R{}'$ (Y )) . That is V-f(A) is Np*g- open in (V, $\tau_R{}'$ (Y )). Hence f(A) is Np*g – closed in (V, $\tau_R{}'$ (Y )) for every Nano closed set A is (U,$\tau_R$ (X )).Hencef: (U,$\tau_R$ (X )) → (V, $\tau_R{}'$ (Y ))is Np*g – closed.Conversely, let f be Np*g – closed and Np*g – Continuous function. Let G bea Nano open set in (U, $\tau_R$ (X )). Then U-G is Nano closed in (U, $\tau_R$ (X )). Since f is Np*g-closed, f(U-G) is Np*g closed in ( V, $\tau_R{}'$ (Y )) . That is f ( U-G) = V-F(G) isNp*g- closed in (V, $\tau_R{}'$ (Y )) for everyNano open set G in (U,$\tau_R$ (X )). Thus , f is Np*g – open and hence f:(U,$\tau_R$ (X )) → (V,$\tau_R{}^1$ (Y )) isNp*g – homeomorphism.

**Example 3.4:** Let U = {a,b,c} with U/R = {{a},{b,c}} andX={a,c},Then $\tau_R$ (X )={U,ϕ,{a},{b,c}}. Then $\tau_R{}'$ (Y ) = { V, ϕ,{b},{b,c}} withV/R'= { V ,ϕ,{b}, {b,c}} and Y={a,b}.Define f : ((U, $\tau_R$ (X )) → (V, $\tau_R{}'$ (Y )) as f(a) =b, f(b) =a, f(c)=c. Np*g -closed sets are {U,ϕ,{a},{b},{c},{a,b},{b,c},{a,c}}.Then f is bijective, Np*g –continuous and Np*g -open. Therefore f is an Np*g homomorphism.

**Theorem 3.5:** Let f: ((U, $\tau_R$ (X )) → (V, $\tau_R{}'$ (Y )) be a bijective.Np*g - continuous map. Then the following are equivalent:

(i)f is an Np*g - open map.
(ii) f is an Np*g - homeomorphism.
(iii)f is an Np*g - closed map.

**Proof:**

**(i) →(ii)** By the given hypothesis, the map f: (U, $\tau_R$ (X )) → (V, $\tau_R{}'$ (Y )) is bijectiveNp*g - continuous andNp*g – open . Hence f:(U,$\tau_R$ (X )) → (V,$\tau_R{}'$ (Y )) is Np*g – homeomorphism.

**(ii) →(iii)** Let A be the Nano closed set is (U,$\tau_R$ (X )). Then $A^c$ is Nano openis (U, $\tau_R$ (X )) .By assumption, f ($A^c$) is Np*g – open in(V, $\tau_R{}'$ (Y )). That is f($A^c$) = $(f(A))^c$ is Np*g – open in (V, $\tau_R{}'$ (Y ))and hence f(A) is Np*g – closed in (V, $\tau_R{}'$ (Y )) for every Nano closed set A in (U, $\tau_R$ (X )). Hence the function f: ((U, $\tau_R$ (X )) →(V, $\tau_R{}'$ (Y )) isNp*g – closed map.

**(iii) →(i)** Let B be a Nano open set in (U, $\tau_R$ (X )). Then $B^c$ isNano closed set in (U, $\tau_R$ (X )). By the given hypothesis, f ($B^c$) is Np*g – closed in (V, $\tau_R{}'$ (Y )). But f ($B^c$) = $(f(B))^c$ isNp*g closed . That f (B) isNp*g – open in (V, $\tau_R{}'$ (Y )) for every Nano open set B in ((U, $\tau_R$ (X )). Hence f: ((U, $\tau_R$ (X )) → (V, $\tau_R{}'$ (Y )) is Np*g – open map.

**Theorem 3.6:** Every Nanohomeomorphism is Np*g- homeomorphism but not conversely.

**Proof:**

ISSN2394-3777 (Print)
ISSN2394-3785 (Online)
Available online atwww.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

If f: $((U, \tau_R(X)) \rightarrow (V, \tau_R{}'(Y))$ is Nano homeomorphism, by definition 3.1, f is bijective, Nanocontinuous and Nano open. Thenf: $((U, \tau_R(X)) \rightarrow (V,\tau_R{}'(Y))$ isNp*g - continuous and Np*g- open respectively. Hence the function f:$((U,\tau_R(X))\rightarrow(V,\tau_R{}'(Y))$ is Np*g - homeomorphism. Every Nano continuous function is Np*g-continuous and every Nano open map is Np*g- open.Then f is bijective, Np*g-continuousand Np*g- open.Therefore f isNp*g-homeomorphism.

In the converse part through example we have to prove that f is Np*g- homemorphism but not Nanohomeomophism.

**Example 3.7:** Let U= {a,b,c,d} with U/R = {{a}, {c},{b,d}}. Let X = {a,b}⊆U *and* $\tau_R(X)$ = { U ,ϕ,{ a },{a,b,d}, {b,d}}.Let V = {w,x,y,z} with V/R'= {{ x}, {w}, {y,z}}. Let Y = {x,z}⊆ V and $\tau_R{}'(Y)$ = { V,ϕ, {x},{x,y,z},{y,z}}.Define f: $(U, \tau_R(X)) \rightarrow (V, \tau_R{}'(Y))$ is f(a) = y f(b) =x,f(c) = w,f(d) =z. Then f is one to one, onto and Np*g is open and Np*g is continuous. So that f: $(U, \tau_R(X)) \rightarrow (V, \tau_R{}'(Y))$ is Np*g - homeomorphism.

Now $f^{-1}(V)$ = U, $f^{-1}(\phi)=\phi$,$f^{-1}(x)$={b}, $f^{-1}${y,z}= {a,d}, $f^{-1}${x,y,z}= {a,b,d}. Hence the inverse images of Nano p*g-open in $(V, \tau_R{}'(Y))$ are not Nano p*g-open in $(U, \tau_R(X))$ and hence f is not Nano p*g-continuous. Thus the map f:$(U, \tau_R(X)) \rightarrow (V, \tau_R{}'(Y))$ is notNanohomeomorphism.

**Remark 3.8:** The composition of two Np*g-homeomorphism need not be aNp*g-homeomorphism as seen from the example.

**Example3.9:**Let$(U, \tau_R(X))$, $(V,\tau_R{}'(Y))$ and $(W, \tau_R{}''(Z))$ be three Nano topological spaces and let U = V = W = {a,b,c} then $\tau_R(X)$ = {ϕ,{b}, {a,c}, U } with U/R = {{b}, {a,c}} and x = {b,c}. $\tau_R{}'(Y)$ = {ϕ,{a}, {b,c}, V } with V/R' = {{a}, {b,c}} and Y = {a,c}.
$\tau_R{}''(Z)$ = {ϕ,{a,c}, W} with W/R" = {{a}, {b,c}} and z = {a}.

Define two functions f:$(U, \tau_R(X)) \rightarrow (V,\tau_R{}'(Y))$ andg:$(V,\tau_R{}'(Y)) \rightarrow (W,\tau_R{}''(Z))$ as f (a) = a, f(b) = b, f(c) = c and g(a) = a, g(b) = b, g(c) = c. Here the functionf and g are Np*g- continuous and bijective. Also the image of every Nano open set in $(U, \tau_R(X))$ is Np*g-open in $(V,\tau_R{}'(Y))$.That isNp*g-open = {U,ϕ,{b},{a,b},{b,c},{a,c}. That is f{b} = {b}, f{a,c} = {b,c}. Thus the functionf:$(U,\tau_R(X)) \rightarrow (V,\tau_R{}'(Y))$ is Np*g- open and thus Np*g- homeomorphism. The map g: $(V, \tau_R{}'(Y)) \rightarrow (W, \tau_R{}''(Z))$ is also Np*g-continuous, bijective and open. Hence g is also Np*g- homeomorphism. But their composition gof : $(V, \tau_R{}'(Y)) \rightarrow (W, \tau_R{}''(Z))$ is not a Np*g-homeomorphism. Because for the Nano open set A = {a,b} in $(W, \tau_R{}''(Z))$, $(gof)^{-1}(A)$ = $f^{-1}(g^{-1}(\{c\}))$= $f^{-1}${c} = {c} is not in Np*g open in $(U, \tau_R(X))$. Hence the composition gof : $(U, \tau_R(X)) \rightarrow (W, \tau_R{}''(Z))$ is not Np*g-continuous and thus not a Np*g- homeomorphism. Thus the composition of two Np*g-homeomorphism need not be a Np*g-homeomorphism.

**Theorm 3.10:** Every Np*g – homeomorphism is Ngp -homeomorphism.

**Proof:**

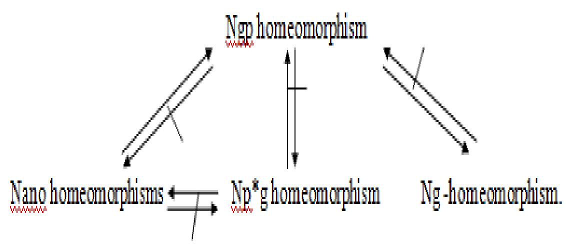
If f: $(U, \tau_R(X)) \rightarrow (V, \tau_R{}'(Y))$ is Nano p*g-homeomorphism, By definition 3.1f is bijective, Nano p*g- continous and Nano p*g open. Thenf:$(U,\tau_R(X)) \rightarrow (V,\tau_R{}'(Y))$ isNgp- continuous and Ngp-open respectively. Hence the function f: $(U, \tau_R(X)) \rightarrow (V, \tau_R{}'(Y))$ is an Ngp-homeomorphism. Therefore every Nano p*g-homeomorphism is Ngp- homeomorphism.

ISSN2394-3777 (Print)
ISSN2394-3785 (Online)
Available online atwww.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

**Remark 3.11:** We obtain the following implication form the above discussion.




## IV. CONCLUSION

In this paper some of the properties of Nano p*g homeomorphismarediscussed. This shall be extended in the future Research with someapplication.

## REFERENCES

[1]. Bhuvaneswari.Kand K.MythiliGanapriya,2015,*OnNano Generalised continuous function in Nano Topological spaces*,International Journal of Mathematical Archive. 6(6); 182-186
[2]. Bhuvaneswari.Kand K.MythiliGanapriya,2016,*Nano Generalized Pre Homeomorphisms in Nano Topological Space,*International Journal of Scientific and Research Publication,6(7);526-530.
[3]. LellisThivagar.M and CarmelRichard,2013,*OnNano forms of weakly open sets,* International Journal of Mathematical and statisticsInvention,1(1);31-37.
[4]. LellisThivagar.M and Carmel Richard,2013,*OnNano Continuity Mathematical Theroy and Modeling*,3(7);32
[5]. Maki.H,Sundaram.P and Balachandran.K,1991,*On generalised homeomorphisms in topological spaces,*Bull.FukuokaUniv.Ed.PartIII., 40; 13-21.
[6]. Pawalk.Z,1982,*Rough sets,* International Journal of Information and Computer Science,11(5); 341-356.