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Characterizations of Rarely Continuous Multifunctions in Ideal Topological Spaces

M. Rameshkumar

Department of Mathematics, P. A College of Engineering and Technology, Pollachi - 642002, Coimbatore, Tamil Nadu, India.
E-mail: rameshngm@gmail.com

Abstract**:** This paper aims to introduce a new class of functions called rarely generalized ideal (gI)-continuous multifunctions. This paper is devoted to the study of upper (and lower) rarely gI-continuous multifunctions.

Keywords: Rare set, gI -open, rarely gI-continuous multifunctions.

I. INTRODUCTION

In 1979, Popa [16] introduced the notion of rare continuity as a generalization of weak continuity [11] which has been further investigated by Long and Herrington [13] and Jafari [5] and [6]. Levine [12] introduced the concept of generalized closed sets of a topological space. Caldas and Jafari[2] further generalized rare continuity as rarely g-continuity in topological spaces and investigate some of its basic properties. Authors [9] introduced the concept of rarely gI-continuous function in ideal topological spaces. Authors [3] introduced the concept of rarely g-continuous multifunctions. Rameshkumar[17] study the concept of rarely gp-continuous in topological spaces. Authors [18, 19] studied the concepts of semi local function in ideal topological spaces. In this paper we study some characterization of rarely gI-continuous multifunctions.

II. PRELIMINARIES

Recall that a rare set is a set R such that $Int(R)= \phi$. Let (X, τ) be a topological space with no separation properties are assumed. An ideal I on a topological space (X, τ) is a non-empty collection of subset of X which satisfies the following properties.

1. $A \in I$ and $B \subset A$ implies $B \in I$.
2. $A \in I$ and $B \in I$ implies $A \cup B \in I$.

If (X, τ) is a topological space and I is an ideal on X, then (X, τ, I) is called is called an ideal topological space or ideal space. For $A \subseteq X$, $A^*(I; \tau) = \{ x \in X / U \cap A \notin I$ for every open set U containing $x\}$is called the local function[10] of A with respect to I and τ. We simply write A^* instead of $A^*(I; \tau)$ in case there is no confusion. For $A \subseteq X$, $A_*(I; \tau) = \{ x \in X / U \cap A \notin I$ for every $U \in SO(X)\}$ is called the semi-local function[7] of A with respect to I and τ, where $SO(X) = \{ U \in SO(X) / x \in U\}$. We simply write A_* instead of $A_* (I; \tau)$ in case there is no ambiguity.

Definition 1 A set A in X is called g-closed [12] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X. The complement of a g-closed set is called g-open.

Definition 2 A subset A of an ideal space $(X; \tau; I)$ is said to be gI-closed[13] (resp. Ig-closed [14]) if $A_* \subset U$ (resp. $A^* \subset U$) whenever $A \subset U$ and U is open in X. The complement of a gI-closed (resp. Ig-closed) set is said to be gI-open (resp. Ig-open).

The family of all gI-open sets will be denoted by GIO(X, x) $= \{U / x \in U \in GIO(X)\}$.

Definition 3 A function $f : X \rightarrow Y$ is called g-continuous [1] if the inverse image of every closed set in Y is g-closed in X.

Definition 4 A function $f : X \rightarrow Y$ is called rarely continuous [16] (resp. rarely g-continuous [2], rarely gp-continuous [4]) if for each $x \in X$ and each $G \in O(Y, f(x))$, there exist a rare set R_G with $G \cap Cl(R_G) = \phi$ and $U \in O(X, x)$ (resp. $U \in GO(X, x)$ and $U \in GPO(X, x)$) such that $f(U) \subset G \cup R_G$.

Definition 5 A function $f : (X, \tau_x, I) \rightarrow (Y, \tau_y)$ is called rarely gI-continuous [9] if for each $x \in X$ and each $G \in O(Y, f(x))$, there exist a rare set R_G with $G \cap Cl(R_G) = \phi$ and gI-open set U in X containing x with $f(U) \subset G \cup R_G$.

III. UPPER (LOWER) RARELY

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gp-CONTINUOUS MULTIFUNCTIONS

We provide the following definitions which will be used in the sequel. Let $F: X \rightarrow Y$ be a multifunction. The upper and lower inverses of a set $V \subset Y$are denoted by $F^{+}(V)$ and $F^{-}(V)$ respectively, that is,

$$F^{+}(V) = \{x \in X \,/\, F(x) \subset V\} \text{ and } F^{-}(V) = \{x \in X \,/\, F(x) \cap V = \phi\}.$$

Definition 6: A multifunction $F: (X, \tau_x, I) \rightarrow (Y, \tau_y)$ is said to be

i) upper rarely gI-continuous (briefly u.r.gI.c) at $x \in X$ if for each $V \in O(Y, F(x))$, there exist a rare set R_V with $V \cap Cl(R_V) = \phi$ and $U \in GIO(X, x)$ such that $F(U) \subset V \cup R_V$,

ii) lower rarely gI-continuous (briefly l.r.gI.c) at $x \in X$ if for each $V \in O(Y, F(x))$ with $F(x) \cap V = \phi$ there exist a rare set R_V with $V \cap Cl(R_V) = \phi$ and $U \in GIO(X, x)$ such that $F(u) \cap (V \cup R_V) \neq \phi$ for every $u \in U$,

iii) upper/ lower rarely gI-continuous if it is upper / lower rarely gI-continuous at each point of X.

Theorem 1: The following statements are equivalent for a multifunction $F: (X, \tau_x, I) \rightarrow (Y, \tau_y)$:

i) F is u.r.gI.c at $x \in X$,

ii) For each $V \in O(Y, F(x))$, there exists $U \in GIO(X, x)$ such that $Int[F(U) \cap (Y - V)] = \phi$

iii) For each $V \in O(Y, F(x))$, there exists $U \in GIO(X, x)$ such that $Int[F(U)] \subset Cl(V)$.

Proof: (i) ⇒ (ii): Let $V \in O(Y, F(x))$. By $F(x) \subset V \subset Int(Cl(V))$ and the fact that $Int(Cl(V)) \in O(Y,F(x))$, there exist a rare set R_V with $Int(Cl(V)) \cap Cl(R_V) = \phi$ and a gI-open set $U \subset X$ containing x such that $F(U) \subset Int(Cl(V)) \cup R_V$. We have $Int\,[F(U) \cap (Y - V)] = Int(F(U)) \cap Int(Y - V) \subset Int(Cl(V) \cup R_V) \cap (Y - Cl(V)) \subset (Cl(V) \cup Int(R_V)) \cap (Y - Cl(V)) = \phi$.

(ii) ⇒ (iii) : Obvious.

(iii) ⇒ (i) : Let $V \in O(Y, F(x))$. Then, by (iii) there exists $U \in GIO(X, x)$ such that $Int[F(U)] \subset Cl(V)$. Thus $F(U) = [F(U) - Int(F(U))] \cup Int[F(U)] \subset [F(U) - Int(F(U))] \cup Cl(V) = [F(U) - Int(F(U))] \cup V \cup (Cl(V) - V) = [(F(U) - Int(F(U)) \cap (Y - V)] \cup V \cup (Cl(V) - V)$. Put $P = (F(U) - Int(F(U))) \cap (Y - V)$ and $G = Cl(V) - V$, then P and G are rare sets. Moreover, $R_V = P \cup G$ is a rare set such that $Cl(R_V) \cap V = \phi$ and $F(U) \subset V \cup R_V$. Hence F is u.r.gI.c.

Theorem 2: The following are equivalent for a multifunction $F: (X, \tau_x, I) \rightarrow (Y, \tau_y)$:

i) F is l.r.gI.c at $x \in X$,

ii) For each $V \in O(Y)$ such that $F(x) \cap V \neq \phi$ there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gI}(F^{-}(V \cup R_V))$,

iii) For each $V \in O(Y)$ such that $F(x) \cap V \neq \phi$ there exists a rare set R_V with $Cl(V) \cup R_V = \phi$ such that $x \in Int_{gI}(F^{-}(Cl(V) \cup R_V))$,

iv) For each $V \in RO(Y)$ such that $F(x) \cap V \neq \phi$ there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gI}(F^{-}(V \cup R_V))$.

Proof: (i) ⇒(ii): Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. By (i), there exist a rare set R_V with $V \cap Cl(R_V) = \phi$ and $U \in GIO(X, x)$ such that $F(x) \cap (V \cup R_V) \neq \phi$for each $u \in U$. Therefore, $u \in F^{-}(V \cup R_V)$ for each $u \in U$ and hence $U \subset F^{-}(V \cup R_V)$. Since $U \in GIO(X, x)$, we obtain $x \in U \subset Int_{gI}(F^{-}(V \cup R_V))$.

(ii) ⇒ (iii): Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. By (ii), there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gI}(F^{-}(V \cup R_V))$. We have $R_V \subset Y - V = (Y - Cl(V)) \cup (Cl(V) - V)$ and hence $R_V \subset [R_V \cap (Y - Cl(V))] \cup (Cl(V) - V)$. Now, put $P = R_V \cap (Y - Cl(V))$. Then P is a rare set and$P \cap Cl(V) = \phi$. Moreover, we have $x \in Int_{gI}(F^{-}(V \cup R_V)) \subset Int_{gI}\,(F^{-}(P \cup Cl(V)))$.

(iii) ⇒ (iv): Let V be any regular open set of Y such that $F(x) \cap V \neq \phi$. By (iii), there exists a rare set R_V with $Cl(V) \cap R_V = \phi$ such that $x \in Int_{gI}(F^{-}(Cl(V) \cup R_V))$. Put $P = R_V \cup (Cl(V) - V)$, then P is a rare set and $V \cap Cl(P) = \phi$. Moreover, we have $x \in Int_{gI}(F^{-}(Cl(V) \cup R_V)) = Int_{gI}(F^{-}(R \cup (Cl(V) - V) \cup V)) = Int_{gI}(F^{-}(P \cup V))$.

(iv) ⇒ (i) : Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. Then $F(x) \cap Int(Cl(V)) \neq \phi$ and $Int(Cl(V))$ is regular open in Y . By (iv), there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gI}(F^{-}(V \cup R_V))$. Therefore, there exists $U \in GIO(X, x)$ such that $U \subset F^{-}(V \cup R_V)$; hence $F(u) \cap (V \cup R_V) \neq \phi$; for each $u \in U$. This shows that F is lower rarely gI-continuous at x.

Corollary 1: The following statements are equivalent for a function $f: (X, \tau_x, I) \rightarrow (Y, \tau_y)$:

i) f is rarely gI-continuous at $x \in X$,

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ii) For V ∈ O(Y, f(x)), there exists U ∈ GIO(X, x) such that $Int[f(U) \cap (Y - V)] = \phi$,

iii) For each V ∈ O(Y, f(x)), there exists U ∈ GIO(X, x) such that $Int[f(U)] \subset Cl(V)$.

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