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On Generalized PreSemi Closed Sets in Intuitionistic Topological Spaces

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Abstract: The purpose of this paper is to study a new class of generalized closed set namely intuitionistic generalized presemi closed sets and intuitionistic generalized presemi open sets in intuitionistic topological spaces (classical). We also provide separation axioms on intuitionistic generalized presemi closed sets, by defining $IPST_{1/2}$ space, $IPST^{*}_{1/2}$ space, IGPS – T_0 space and IGPS − T_1 space and study some of their basic properties.

Keywords: Intuitionistic set, intuitionistic topological space, intuitionistic generalized presemi closed sets, IGPS – T_0 space and IGPS − T_1 space.

I. INTRODUCTION

In 1970, Levine [8] introduced the concept of g-closed sets in general topology. Atanassov [1] introduced the concept of "**Intuitionistic fuzzy set**" as a generalization of fuzzy sets. Coker [4] generalized topological structures in fuzzy topological spaces [2] to "**Intuitionistic fuzzy topological spaces**" using intuitionistic fuzzy sets. On the other hand Coker [3] introduced the concept of "**Intuitionistic sets**" in 1996. This is a discrete form of intuitionistic fuzzy set, where all the sets are entirely the crisp sets. In 2000, Coker [5] also introduced the concept of "**Intuitionistic topological spaces**" using intuitionistic sets and investigated basic properties of continuous functions and compactness. Later C. Duraisamy et.al [6] and Gnanambal Ilango et.al [7] studied some weakly open functions and generalized pre regular closed sets in intuitionistic topological spaces respectively. In this paper, we study few properties of intuitionistic generalized presemi closed sets and intuitionistic generalized presemi open sets in intuitionistic topological space. Also provide some separation axioms, by defining IGPS – T_0 space, IGPS − T_1 space, IGPS – S_0 space and IGPS − S_1 space and study further properties of these spaces.

II. PRELIMINARIES

Throughout the present study, (X, τ) or X denotes the intuitionistic topological spaces (briefly ITS). We recall some basic definitions that are used in the sequel.

Definition 2.1: [3] Let X be a non-empty set. An intuitionistic set (IS in short) A in X is an object having the form $A = \langle X, A_1, A_2\rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A. Denote by IS(X), the set of all intuitionistic sets in X.

Definition 2.2: [3] Let X be a non-empty set and let A, B be an intuitionistic sets of the form $A = \langle X, A_1, A_2\rangle$, $B = \langle X, B_1, B_2\rangle$ respectively. Then

(i) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \langle X, A_2, A_1\rangle$,
(iv) $\phi_{\sim} = \langle X, \phi, X\rangle$, $X_{\sim} = \langle X, X, \phi\rangle$,
(v) $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2\rangle$,
(vi) $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2\rangle$
(vii) $A - B = A \cap B^c$,

Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle X, A_i^{(1)}, A_i^{(2)}\rangle$. Then

(i) $\cap A_i = \langle X, \cap A_i^{(1)}, \cup A_i^{(2)}\rangle$.
(ii) $\cup A_i = \langle X, \cup A_i^{(1)}, \cap A_i^{(2)}\rangle$.

Remark 2.3: [3] Any topological spaces (X, τ) is obviously an intuitionistic topological spaces of the form $\tau = \{A' : A \in \tau\}$ where $A' = \langle X, A, A^c\rangle$.

Definition 2.4: [4] An intuitionistic topology (IT in short) on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

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(i) $\phi_\sim, X_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic topological space (ITS in short) and any IS in τ is known as an intuitionistic open set (IOS in short) in X. The complement A^c of an IOS A in an ITS (X, τ) is called an intuitionistic closed set (ICS in short) in X.

Definition 2.5: [4] Let (X, τ) be an ITS and $A = \langle X,A_1, A_2\rangle$ be an IS in X. Then the interior and closure of A are defined by
(i) $Iint(A) = \cup\{ G / G$ is an IOS in X and $G \subseteq A \}$,
(ii) $Icl(A) = \cap\{ K / K$ is an ICS in X and $A \subseteq K \}$.

It can be also shown that Icl(A) is an ICS and Iint(A) is an IOS in X. Let A be an ICS in X iff Icl(A) = A and A be an IOS in X iff Iint(A) = A.

Remark 2.6: [4] Let (X, τ) be an ITS and $A = \langle X,A_1, A_2\rangle$ be an IS in X. Then
(i) $Icl(A^c) = (Iint(A))^c$,
(ii) $Iint(A^c) = (Icl(A))^c$.

Definition 2.7: [3] Let X be a non empty set and $p \in X$. Then the IS $p_\sim = \langle X, \{p\}, \{p\}^c\rangle$ is called an intuitionistic point (IP in short) in X. The intuitionistic point $p_\sim$ is said to be contained in $A = \langle X, A_1, A_2\rangle$ (ie, $p_\sim \in A$) if and only if $p \in A_1$.

Definition 2.8: [10] Let (X, τ) be an ITS. An IS $A = \langle X, A_1, A_2\rangle$ of X is said to be an
(i) intuitionistic regular closed set (IRCS in short) if $Icl(Iint(A)) = A$,
(ii) intuitionistic regular open set (IROS in short) if $A = Iint(Icl(A))$,
(iii) intuitionistic semi closed set (ISCS in short) if $Iint(Icl(A)) \subseteq A$,
(iv) intuitionistic semi open set (ISOS in short) if $A \subseteq Icl(Iint(A))$,
(v) intuitionistic pre closed set (IPCS in short) if $Icl(Iint(A)) \subseteq A$,
(vi) intuitionistic pre open set (IPOS in short) if $A \subseteq Iint(Icl(A))$,
(vii) intuitionistic α-closed set (IαCS in short) if $Icl(Iint(Icl(A))) \subseteq A$,
(viii) intuitionistic α-open set (IFαOS in short) if $A \subseteq Iint(Icl(Iint(A)))$.

Definition 2.9: [10] Let (X, τ) be an ITS and $A = \langle X, A_1, A_2\rangle$ be an IS in X. Then
(i) $Ipint(A) = \cup\{G : G$ is an IPOS in X and $G \subseteq A\}$,
$Ipcl(A) = \cap\{K : K$ is an IPCS in X and $A \subseteq K\}$.
(ii) $I\alpha int(A) = \cup\{G : G$ is an IαOS in X and $G \subseteq A\}$,
$I\alpha cl(A) = \cap\{K : K$ is an IαCS in X and $A \subseteq K\}$.

Definition 2.10: Let (X, τ) be a nonempty ITS and $A = \langle X, A_1, A_2\rangle$ be an IS in X. Then A is said to be an
(i) intuitionistic generalized closed set (IGCS in short) [10] if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IOS in X,
(ii) intuitionistic w-closed set (IWCS in short) [9] if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is an ISOS in X,
(iii) intuitionistic generalized pre closed set (IGPCS in short) [10] if $Ipcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an intuitionistic open set in X.

An IS A of an ITS (X, τ) is called an intuitionistic generalized open (resp, intuitionistic w-open, intuitionistic generalized pre open) (IGOS (resp,IWOS and IFGPOS) in short) if its complement A^c is IGCS (resp, IWCS and IGPCS).

Definition 2.11: [7] Let (X, τ) be a nonempty ITS and $A = \langle X, A_1, A_2\rangle$ be an IS in X. Then A is said to be an intuitionistic generalized pre regular closed set (IGPRCS in short) if $Ipcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IROS in X. The complement Ac of an IGPRCS A in (X, τ) is called an intuitionistic generalized pre regular open set (IGPROS in short) in X.

III. Intuitionistic Generalized PreSemi Closed Sets

Definition 3.1: Let (X, τ) be a nonempty ITS and $A = \langle X, A_1, A_2\rangle$ be an IS in X. Then A is said to be an intuitionistic generalized pre semi closed set (IGPSCS in short) if $Ipcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an ISOS in (X, τ). The family of all IGPSCSs of an ITS (X, τ) is denoted by IGPSC(X).

The complement of IGPSCS's are intuitionistic generalized pre semi open sets (IGPSOS in short) and the family of all IGPS open subsets of (X, τ) is denoted by IGPSO(X).

Example 3.2: Let X= {a, b, c, d, e} and the family $\tau = \{ \phi_\sim, X_\sim, A_1, A_2, A_3, A_4\}$ where $A_1= \langle X, \{a, b, c\}, \{d\}\rangle$, $A_2 = \langle X, \{c, d\}, \{e\}\rangle$, $A_3 = \langle X, \{c\}, \{d, e\}\rangle$, $A_4 = \langle X, \{a, b, c, d\}, \phi\rangle$. Then (X, τ) is an ITS on X and the IS $A = \langle X, \{a, b\}, \{d\} \rangle$ is an IGPSCS in X.

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Proposition 3.3: Every ICS in (X, τ) is an IGPSCS in (X, τ) but not conversely.

Proof: Let A be an ICS and A ⊆ U where U is an ISOS in (X, τ). Then Ipcl(A) ⊆ Icl(A) = A⊆U. Hence A is an IGPSCS in (X, τ).

Example 3.4: In Example 3.2., the IS A= ⟨ X, {a, b}, {d} ⟩ is an IGPSCS but not an ICS in (X, τ).

Proposition 3.5: Every IRCS in (X, τ) is an IGPSCS in (X, τ) but not conversely.

Proof: Let A be an IRCS in (X, τ). Since every IRCS is an ICS, by Proposition 3.3., A is an IGPSCS in (X, τ).

Example 3.6: In Example 3.2., the IS A= ⟨ X, {a, b}, {d} ⟩ is an IGPSCS but not an IRCS in (X, τ).

Proposition 3.7: Every IαCS in (X, τ) is an IGPSCS in (X, τ) but not conversely.

Proof: Let A be an IαCS (X, τ) and A ⊆ U where U is an ISOS in (X, τ).Then Ipcl(A)⊆Iαcl(A) = A⊆U. Hence A is an IGPSCS in (X, τ).

Example 3.8: In Example 3.2., the IS A= ⟨ X, {a, b}, {d} ⟩ is an IGPSCS but not an IαCS in (X, τ).

Proposition 3.9: Every IPCS in (X, τ) is an IGPSCS in (X, τ) but not conversely.

Proof: Let A be an IPCS in (X, τ) and A ⊆U where U is an ISOS in (X, τ). Then Ipcl(A) = A⊆ U. Hence A is an IGPSCS in (X, τ).

Example 3.10: Let X= {a, b, c, d} and the family τ = { $\phi_\sim$, $X_\sim$, A_1, A_2, A_3 } where A_1= ⟨X, {a, b}, {c, d}⟩, A_2 = ⟨X, {c}, {a, b, d}⟩, A_3 = ⟨X, {a, b, c}, {d}⟩. Then (X, τ) is an ITS on X and the IS A= ⟨ X, {a, c}, {b} ⟩ is an IGPSCS in (X, τ) but not IPCS in (X, τ).

Proposition 3.11: Every IWCS in (X, τ) is an IGPSCS in (X, τ) but not conversely.

Proof: Let A be an IWCS and A ⊆ U where U is an ISOS in (X, τ). Then Ipcl(A) ⊆ Icl(A) ⊆ U. Hence A is an IGPSCS in (X, τ).

Example 3.12: In Example 3.2., the IS A= ⟨X, {a, b}, {d}⟩ is an IGPSCS but not an IWCS in (X, τ).

Proposition 3.13: Every IGCS and IGPSCS are independent to each other in an ITS (X, τ) which are seen from the following examples.

Example 3.14: In Example 3.2., the IS A= ⟨X, {a, b}, {d}⟩ is an IGPSCS but not an IGCS in (X, τ).

Example 3.15: Let X= {a, b, c} and the family τ = {$\phi_\sim$, $X_\sim$, A_1, A_2, A_3, A_4, A_5, A_6, A_7} where A_1= ⟨X, {a, b}, {c}⟩, A_2 = ⟨X, {c}, {a, b}⟩, A_3 = ⟨X, {a}, {ϕ}⟩, A_4 = ⟨X, {a, b}, {ϕ}⟩, A_5 = ⟨X, {a}, {c}⟩, A_6 = ⟨X, {a, c}, {ϕ}⟩, A_7 = ⟨X, {ϕ}, {a, b}⟩. Then (X, τ) is an ITS on X and the IS A= ⟨ X, {b}, {c} ⟩ is an IGCS but not IGPSCS in (X, τ).

Proposition 3.16: Every IGPSCS in (X, τ) is an IGPCS in (X, τ) but not conversely.

Proof: Let A be an IGPSCS and A ⊆ U where U is an ISOS in (X, τ). Since every IOS in (X, τ) is an ISOS in (X, τ). Then by hypothesis, Ipcl(A)⊆U. Hence A is an IGPCS in (X, τ).

Example 3.17: Let X= {a, b, c} and τ = {$\phi_\sim$, $X_\sim$, A_1, A_2} where A_1= ⟨X, {a}, {b}⟩, A_2 = ⟨X, {a, b}, {ϕ}⟩. Then (X, τ) is an ITS on X and the IS A= ⟨ X, {a, c}, {ϕ} ⟩ is an IGPCS but not IGPSCS in (X, τ).

Proposition 3.18: Every IGPSCS in (X, τ) is an IGPRCS in (X, τ) but not conversely.

Proof: Let A be an IGPSCS and A ⊆U where U is an IROS in (X, τ). Since every IROS in (X, τ) is an ISOS in (X, τ). Then Ipcl(A)⊆U. Hence A is an IGPRCS in (X, τ).

Example 3.19: Let X= {a, b} and τ = {$\phi_\sim$, $X_\sim$, A_1, A_2, A_3} where A_1= ⟨X, {a}, {ϕ}⟩, A_2 = ⟨X, {a}, {b}⟩, A_3 = ⟨X, {ϕ}, {b}⟩. Then (X, τ) is an ITS on X and the IS A= ⟨ X, {a}, {ϕ}⟩ is an IGPRCS in (X, τ) but not IGPSCS in (X, τ).

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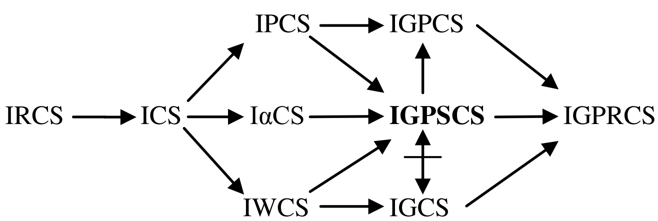
Remark 3.20: Summing up the above propositions, we have the following diagram.



In this diagram by "A ⟶ B" we mean A implies B but not conversely and "A ⟷ B" means A and B are independent of each other.

Remark 3.21: The union of any two IGPSCSs need not be an IGPSCS in (X, τ) which is seen from the following example.

Example 3.22: Let X= {a, b, c} and the family τ = { $\phi_{\sim}$, $X_{\sim}$, ⟨X, {a}, {b}⟩, ⟨X, {a, b}, {ϕ}⟩} then (X, τ) is an ITS on X and the intuitionistic subsets ⟨X, {a}, {b, c}⟩ and ⟨X, {c}, {ϕ}⟩ are IGPSCS but their union ⟨X, {a, c}, {ϕ}⟩ is not IGPSCS in (X, τ).

Remark 3.23: The intersection of any two IGPSCSs need not be an IGPSCS in (X, τ) which is seen from the following example.

Example 3.24: Let X= {a, b, c} and τ = { $\phi_{\sim}$, $X_{\sim}$, ⟨X, {c}, {a, b}⟩, ⟨X, {b}, {c}⟩, ⟨X, {b, c}, {ϕ}⟩} then (X, τ) is an ITS on X and the intuitionistic subsets ⟨X, {c}, {a}⟩ and ⟨X, {c}, {b}⟩ are IGPSCS but their intersection ⟨X, {c}, {a, b}⟩ is not IGPSCS in (X, τ).

Theorem 3.25: Let (X, τ) be an ITS. Then for every A ∈ IGPSC(X) and for every B ∈ IS(X), A⊆B ⊆ Ipcl(A) implies B ∈ IGPSC(X).

Proof: Let B ⊆ U and U is an ISOS in (X, τ). Then since A ⊆ B, A ⊆ U. Since A is an IGPSCS, it follows that Ipcl(A) ⊆ U. Now B ⊆Ipcl(A) implies Ipcl(B) ⊆ Ipcl(Ipcl(A)) = Ipcl(A) ⊆ U. Thus Ipcl(B) ⊆U. Hence B ∈ IGPSC(X).

Theorem 3.26: If A is an ISOS and an IGPSCS in (X, τ) then A is an IPCS in (X, τ).

Proof: Since A ⊆A and A is an ISOS in (X, τ). By hypothesis, Ipcl(A)⊆A. But since A ⊆ Ipcl(A). Therefore Ipcl(A) = A. Hence A is an IPCS in (X, τ).

Theorem 3.27: For every $\{p_{\sim}\}$∈ X, X – $\{p_{\sim}\}$ is an IGPSCS or ISOS in (X, τ).

Proof: Suppose X – $\{p_{\sim}\}$ is not an ISOS. Then X is the only ISOS containing X – $\{p_{\sim}\}$. This implies Ipcl(X – $\{p_{\sim}\}$) ⊆ X. Hence X – $\{p_{\sim}\}$ is an IGPSCS in X.

Definition 3.28: Let A be an IS in an ITS (X, τ). Then intuitionistic generalized pre semi interior of A (Igpsint(A) in short) and intuitionistic generalized pre semi closure of A (Igpscl(A) in short) are defined by

(i) Igpsint(A) = ∪{G : G is an IGPSOS in X and G ⊆ A},
(ii) Igpscl(A) = ∩{K : K is an IGPSCS in X and A ⊆ K}.

Note that for any IS A in (X, τ), we have Igpscl(A^c) = $(\text{Igpsint}(A))^c$ and Igpsint(A^c) = $(\text{Igpscl}(A))^c$.

Theorem 3.29: An IS A of an ITS (X, τ) is an IGPSOS in (X, τ) if and only if F ⊆Ipint(A) whenever F is an ISCS in (X, τ) and F ⊆ A.

Proof: Necessity: Suppose A is an IGPSOS in (X, τ). Let F be an ISCS in (X, τ) such that F ⊆A. Then F^c is an ISOS and A^c⊆F^c. By hypothesis, A^c is an IGPSCS in (X, τ), we have Ipcl(A^c) ⊆F^c. But Ipcl(Ac) = $(\text{Ipint}(A))^c$ ⊆ F^c, which implies F ⊆ Ipint (A).
Sufficiency: Let U be an ISOS in (X, τ) such that A^c ⊆ U. By hypothesis, U^c ⊆ Ipint(A). Therefore Ipcl(A^c) ⊆ U and A^c is an IGPSCS in (X, τ). Hence A is an IGPSOS in (X, τ).

Lemma 3.30: Let A and B are intuitionistic subsets of (X, τ). Then the following results obvious.

(i) Igpscl($X_{\sim}$) = $X_{\sim}$ and Igpscl($\phi_{\sim}$) = $\phi_{\sim}$.
(ii) If A ⊆B, then Igpscl (A) ⊆ Igpscl (B).
(iii) A ⊆Igpscl (A).
(iv) Igpscl(A) = Igpscl(Igpscl(A)).

Proof: (i) Igpsint($X_{\sim}$) = Largest IGPSOS in X contained in $X_{\sim}$ = $X_{\sim}$ and Igpscl($\phi_{\sim}$) = Smallest IGPSCS in X containing $\phi_{\sim}$ = $\phi_{\sim}$.
(ii) Given A ⊆ B and Icl(A) ⊆ Icl(B) implies that Igpscl(A) ⊆ Igpscl(B).
(iii) Since A ⊆ Icl(A). We have every ICS is an IGPSCS implies that A ⊆ Igpscl (A).

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(iv) Since Icl(A) = Icl(Icl(A)) implies that Igpscl(A) = Igpscl(Igpscl (A)).

IV. SEPARATION AXIOMS OF INTUITIONISTIC GENERALIZED PRESEMI CLOSED SETS

Definition 4.1: If every IGPSCS in (X, τ) is an IPCS in (X, τ) then the space can be called as intuitionistic presemi $T_{1/2}$ space ($IPST_{1/2}$ space in short).

Theorem 4.2: An ITS (X, τ) is an $IPST_{1/2}$ space if and only if IPOS(X) = IGPSO(X).

Proof: Necessity: Let (X, τ) be an $IPST_{1/2}$ space. Let A be an IGPSOS in (X, τ). By hypothesis, A^c is an IGPSCS in (X, τ) and therefore A is an IPOS in (X, τ). Also every IPOS is an IGPSOS. Hence IPO(X) = IGPSO(X).
Sufficiency: Let IPO(X, τ) = IGPSO(X, τ). Let A be an IGPSCS in (X, τ). Then A^c is an IGPSOS in (X, τ). By hypothesis, A^c is an IPOS in (X, τ). Therefore A is an IPCS in (X, τ). Hence (X, τ) is an $IPST_{1/2}$ space.

Theorem 4.3: Let X be an $IPST_{1/2}$ space. Then A ∈ IGPSO(X) if and only if for every IP $p_{\sim}$∈A, there exists an IGPSOS B in X such that $p_{\sim} \in B \subseteq A$.

Proof: Necessity: If A ∈ IGPSO(X), then we can take B = A so that $p_{\sim} \in B \subseteq A$ for every IP $p_{\sim} \in A$.
Sufficiency: Let A be an IS in (X, τ) and assume that there exists B ∈ IGPSO(X) such that $p_{\sim} \in B \subseteq A$. Since X is an $IPST_{1/2}$ space, B is an IPOS. Then $A = \bigcup_{p\sim\in A}\{p_{\sim}\} \subseteq \bigcup_{p\sim\in A} B \subseteq A$. Therefore $A= \bigcup_{p\sim\in A} B$, which is an IPOS. Hence by Theorem 4.2., A is an IGPSOS in (X, τ).

Definition 4.4: An ITS(X, τ) is said to be an intuitionistic presemi $T^*_{1/2}$ space ($IPST^*_{1/2}$ space in short) if every IGPSCS is an ICS in (X, τ).

Theorem 4.5: Every $IPST^*_{1/2}$ space is an $IPST_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an $IPST^*_{1/2}$ space. Let A be an IGPSCS in (X, τ). Then A is an ICS in (X, τ). Since every ICS is an IPCS implies that A is an IPCS in (X, τ). Hence (X, τ) is an $IPST_{1/2}$ space.

Example 4.6: Let X= {a, b, c} and the family τ = { $\phi_{\sim}$, $X_{\sim}$, ⟨X, {a}, {b}⟩, ⟨X, {a, b}, {ϕ}⟩} then (X, τ) is an $IPST_{1/2}$ space but not an $IPST^*_{1/2}$ space, since the IS A = ⟨X, {b}, {ϕ}⟩ is an IGPSCS but not an ICS in (X, τ).

Definition 4.7: A space X is an $IGPST_o$ spaces for every pair of points '$a_{\sim}$' and '$b_{\sim}$' there exists an IGPSOS U such that atleast one of the following statement is true.

(i) if $a_{\sim}$ lies in U then $b_{\sim}$ does not lie in U.

(ii) if $b_{\sim}$ lies in U then $a_{\sim}$ does not lie in U.

Definition 4.8: A space X is an $IGPST_1$ space for every pair of distinct points '$a_{\sim}$', '$b_{\sim}$'of X there exists a pair of an IGPSOS's containing '$a_{\sim}$' but not '$b_{\sim}$' and the other containing '$b_{\sim}$' but not'$a_{\sim}$'.

Theorem 4.9: Every $IGPST_1$ space is an $IGPST_o$ space but not conversely.

Proof: Let (X, τ) be an $IGPST_1$ space. Let '$a_{\sim}$' and '$b_{\sim}$' be any pair of disjoint points of X. By $IGPST_1$ space, if there exists a pair of an IGPSOS's U and V such that U containing '$a_{\sim}$' but not '$b_{\sim}$' and V containing '$b_{\sim}$' but not '$a_{\sim}$'. Hence '$a_{\sim}$' lies in U but '$b_{\sim}$' does not lies in U. Therefore (X, τ) is an $IGPST_o$ space.

Example 4.10: Let X={a, b} and the family τ = { $\phi_{\sim}$, $X_{\sim}$, A_1, A_2} where A_1= ⟨X, {b}, {a}⟩, A_2 = ⟨X, {a}, {b}⟩. Then (X, τ) is an $IGPST_o$ space but not an $IGPST_1$ space.

Definition 4.11: A space X is an IGPS-S_o space iff for each IGPSOS G and x ∈ G, Igpscl{x} ⊆ G.

Definition 4.12: A space X is an IGS_o space iff for each IOS G and x ∈ G, Igpscl{x} ⊆ G.

Theorem 4.13: Every IGPS-S_o space is an IGS_o space but not conversely.

Proof: Let (X, τ) be an IGPS-S_o space. Let G be an IOS and $x_{\sim}$ ∈ G. Since every IOS is an IGPSOS. Hence G be an IGPSOS and $x_{\sim}$ ∈ G. By IGPS-S_o space, Igpscl{$x_{\sim}$} ⊆ G. Hence (X, τ) is an IGS_o space.

Example 4.14: Let X = {a, b, c} and the family τ = { $\phi_{\sim}$, $X_{\sim}$, A_1, A_2, A_3} where A_1= ⟨X, {c}, {a,b}⟩, A_2 = ⟨X, {b}, {c}⟩ and A_3 = ⟨X, {b,c}, {ϕ}⟩. Then (X, τ) is an IGS_o space but not IGPS-S_o space.

Theorem 4.15: An ITS (X, τ) is an $IGPST_1$ space iff the singleton sets are IGPSCS.

Proof: Let (X, τ) be an $IGPST_1$ space and $x_{\sim}$ and $y_{\sim}$ be any pair of distinct points in X. Suppose that $y_{\sim}$ ∈ X-{$x_{\sim}$}, there exists two IGPSOS(X, τ), U= ⟨X, U_1,U_2⟩ and V= ⟨X, V_1,V_2⟩ such that $y_{\sim}$ ∈ V but $x_{\sim} \notin V$. Since $y_{\sim} \in V \subseteq$ X-{$x_{\sim}$}. i.e,

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X-$\{x_{\sim}\}$= $\bigcup\{V: y_{\sim} \in X-\{x_{\sim}\}\}$ is a union of all IGPSOS's and so X-$\{x_{\sim}\}$ is an IGPSOS. That is $\{x_{\sim}\}$ is an IGPSCS in X.

Conversely, Let $x_{\sim}$ and $y_{\sim}$ be any pair of distinct points in X. By assumption $\{x_{\sim}\}$ and $\{y_{\sim}\}$ are IGPSCS's. Then U = X-$\{x_{\sim}\}$ is an IGPSOS containing $y_{\sim}$ but not $x_{\sim}$ and U = X-$\{ y_{\sim}\}$ is an IGPSOS containing $x_{\sim}$ but not $y_{\sim}$.Hence X is an IGPST$_1$ space.

Theorem 4.16: A space X is an IGPS-S$_o$ space iff for each IGPSCS F and $x_{\sim} \notin F$ there exists an IGPSOS U such that F⊆U, $x_{\sim} \notin U$.

Proof: Let X be an IGPS-S$_o$ space and F⊆X be an IGPSCS not containing the points $x_{\sim} \notin X$. Then X-F is IGPSOS and $x_{\sim} \in$ X-F. Sine X is an IGPS-S$_o$ space, Igpscl$\{x_{\sim}\}$⊆x-f. That is F⊆X-Igpscl$\{x_{\sim}\}$. Let U = X- Igpscl$\{x_{\sim}\}$. Then U is an IGPSOS such that F⊆U and $x_{\sim} \notin U$.

Conversely, Let $x_{\sim} \in U$ where U is an IGPSOS in X. Then X-U is an IGPSCS and $x_{\sim} \notin$ X-U. By hypothesis, there is an IGPSOS W such that X-U ⊆U. Therefore, X is an IGPS-S$_o$ space.

Theorem 4.17: If a space X is both IGPST$_o$ space and IGPS-S$_o$ space, then X is an IGPST$_1$ space.

Proof: By hypothesis, the space X is both IGPST$_o$ space and IGPS-S$_o$ space. To show that X is an IGPST$_1$ space. Let $x_{\sim}$, $y_{\sim} \in$ X be any pair of distinct points. Since X is IGPST$_o$ space there exists an IGPSOS G such that $x_{\sim} \in$ G and $y_{\sim} \notin G$ or there exists a IGPSOS H such that $y_{\sim} \in$ H and $x_{\sim} \notin H$. Suppose $x_{\sim} \in X$ and $y_{\sim} \notin X$. Then Igpscl$\{x_{\sim}\} \subseteq G$ and $y_{\sim} \notin$ Igpscl$\{x_{\sim}\}$. Hence $y_{\sim} \in H$ = X-Igpscl$\{X_{\sim}\}$. It is clear that $x_{\sim} \notin$ X-Igpscl$\{x_{\sim}\}$. Therefore there exists IGPSOS's G and H containing $x_{\sim}$ and $y_{\sim}$ respectively such that $y_{\sim} \notin G$ and $x_{\sim} \notin H$. Therefore X is an IGPST$_1$ space.

Theorem 4.18: A space X is an IGS$_o$ space iff for every ICS F and $x_{\sim} \notin F$, there exists an IGPSOS G such that F ⊆ G and $x_{\sim} \notin G$.

Proof: Let X be an IGS$_o$ space and F ⊆ G be an ICS not containing $x_{\sim} \in X$. Then X-F is IOS and $x_{\sim} \in$ X-F. Since X is IGS$_o$ space, Igpscl$\{ x_{\sim} \}$⊆ X-F. Then F ⊆ X-Igpscl$\{ x_{\sim}\}$, then G is an IGPSOS such that F⊆G and $x_{\sim} \notin G$.

Conversely, let $x_{\sim} \in G$, where G is IOS in X. Then X-G is an ICS and $x_{\sim} \in$ X- G. By hypothesis, there exists an IGPSOS H such that X- G⊆ H and $x_{\sim} \notin$ H. Now X- H ⊆ G and $x_{\sim} \in$ X- H. X- H is IGPSCS and so Igpscl$\{ x_{\sim} \}$⊆ X- H ⊆ G. Therefore, X is an IGS$_o$ space**.**

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