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Neighborhood Tree Domatic Number and Connectivity of Graphs

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***Abstract*: Let G = (V, E) be a connected graph. A subset D of V is called a dominating set if every vertex in V−D is adjacent to some vertex in D. A dominating set D of G is called a neighborhood tree dominating set (ntr-set), if the induced subgraph 〈 N(D) 〉 is a tree. The minimum cardinality of a ntr-set of G is called the neighborhood tree domination number of G and is denoted by $\gamma_{ntr}(G)$. The connectivity κ(G) of G is the minimum number of vertices whose removal results in a disconnected or trivial graph. A partition $\{V_1, V_2, V_3, \ldots ,V_n\}$ of V(G), in which each V_i is a ntr - set in G is called a neighborhood tree domatic partition of simply ntr partition of G. The maximum order of a ntr - partition of G is called the neighborhood tree domatic number of G and is denoted by $d_{ntr}(G)$.**

In this paper, we find an upper bound for the sum of the neighborhood tree domination number and connectivity of a graph and to find bounds for $d_{ntr}(G)$ and its exact value for some particular classes of graphs are studied.

***Keywords*: Domination number, tree domination , neighborhood tree domination number, connectivity, domatic number.**
Subject classification: 05C69.

INTRODUCTION

The graph G = (V, E), we mean a finite, undirected, connected simple graph. The order and size of G are denoted by n and m respectively. The open neighborhood and the closed neighborhood of v∈V are denoted by N(v) and N[v] = N(v) ∪ {v} respectively. If D ⊆ V, then $N(D) = \bigcup_{v\in D} N(v)$ and N[D] = N(D) ∪ D.

The study of domination in graphs has found rapid growth in the recent years. It is a highly flourishing area of research in graph theory. So far, hundreds of research articles have appeared on this topic of research in view of its growing real life application.

A subset D of V is called a dominating set of G if N[D] = V. The minimum cardinality of a minimal dominating set of G is called the domination number of G and is denoted by γ(G). An excellent treatment of the fundamentals of domination is given in the book by Haynes et al. [4]. A survey of several advanced topics in domination is given in the book edited by Haynes et al. [5].

Xuegang Chen, Liang Sun and Alice McRac [8] introduced the concept of tree domination in graphs. A dominating set D of G is called a tree dominating set, if the induced subgraph 〈 D 〉 is tree. The minimum cardinality of a tree dominating set of G is called the tree domination number of G and is denoted by $\gamma_{tr}(G)$. S. Arumugam and C. Sivagnanam introduced the concepts of neighborhood connected and neighborhood total domination in graphs[1,2]. A dominating set D of G is called a neighborhood connected dominating set (ncd-set), if the induced subgraph 〈 N(D) 〉 is connected. The minimum cardinality of a ncd-set of G is called the neighborhood connected domination number of G and is denoted by $\gamma_{nc}(G)$.

A dominating set D of G is called a neighborhood total dominating set, if the induced subgraph 〈 N(D) 〉 has no isolated vertices. The minimum cardinality of a ntd-set of G is called the neighborhood total domination number of G and is denoted by $\gamma_{ntd}(G)$.

We introduced the concept of neighborhood tree dominating set in [6]. Zelinka[10] studied the connected domatic number of a graph. Chen et al. [9] studied the tree domatic number of a graph.

A dominating set D of a connected graph G is called a neighborhood tree dominating set(ntr-set), if the induced subgraph 〈 N(D) 〉 is a tree. The minimum

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cardinality of a ntr-set of G is called the neighborhood tree domination number of G and is denoted by $\gamma_{ntr}(G)$. The tree domatic number of G is the maximum number of pairwise disjoint tree dominating sets in V(G) and is denoted by $d_{tr}(G)$.

A partition $\{V_1, V_2, V_3, \dots, V_n\}$ of V(G), in which each V_i is a ntr - set in G is called a neighborhood tree domatic partition of simply ntr partition of G. the maximum order of a ntr partition of G is called the neighborhood tree domatic number of G and is denoted by $d_{ntr}(G)$. The connectivity $\kappa(G)$ of G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

In this paper, we find an upper bound for the sum of the neighborhood tree domination number and connectivity of a graph and to find bounds for $d_{ntr}(G)$ and its exact value for some particular classes of graphs are studied.

2. PRIOR RESULTS

THEOREM: 2.1[6] Let G be a connected graph on $n \geq 3$ vertices. Then $0 \leq \gamma_{ntr}(G) \leq n - 1$ and $\gamma_{ntr}(G) = n - 1$ iff $G \cong P_3$.

THEOREM: 2.2[1] For any graph G, $\kappa(G) \leq \delta(G)$.

3. MAIN RESULTS

THEOREM: 3.1 For any connected graph G with n vertices, $\gamma_{ntr}(G) + \kappa(G) \leq 2n - 3$, $n \geq 3$.

PROOF:

By theorem 2.1 and 2.2,

$\gamma_{ntr}(G) + \kappa(G) \leq n - 1 + \delta(G) \leq n - 1 + n - 1 \leq 2n - 2$.

If $\gamma_{ntr}(G) + \kappa(G) = 2n - 2$, then the following cases are to be considered.

(i) $\gamma_{ntr}(G) = n$ and $\kappa(G) = n - 2$.
(ii) $\gamma_{ntr}(G) = n - 1$ and $\kappa(G) = n - 1$

Since $\gamma_{ntr}(G) \leq n - 1$ the case (ii) alone be considered. But $\gamma_{ntr}(G) = n - 1$ iff $G \cong P_3$ and $\kappa(P_3) = 1 \neq n - 1$.Therefore, there is no connected graph G with $\gamma_{ntr}(G) + \kappa(G) = 2n - 2$. Hence, $\gamma_{ntr}(G) + \kappa(G) \leq 2n - 3$, $n \geq 3$.

THEOREM: 3.2 Let G be a connected graph. Then $\gamma_{ntr}(G) + \kappa(G) = 2n - 3$ $(n \geq 3)$ if and only if G is isomorphic to one of the graphs C_3 and P_3.

PROOF:

If $G \cong P_3$ then $\gamma_{ntr}(G) = 2$ and $\kappa(G) = 1$ and hence $\gamma_{ntr}(G) + \kappa(G) = 3 = 2n - 3$.

If $G \cong C_3$ then $\gamma_{ntr}(G) = 1$ and $\kappa(G) = 2$ and $\gamma_{ntr}(G) + \kappa(G) = 3 = 2n - 3$.

Conversely, assume $\gamma_{ntr}(G) + \kappa(G) = 2n - 3$, for $n \geq 3$. Then the following cases are to considered.

(i) $\gamma_{ntr}(G) = n$ and $\kappa(G) = n - 3$
(ii) $\gamma_{ntr}(G) = n - 1$ and $\kappa(G) = n - 2$
(iii) $\gamma_{ntr}(G) = n - 2$ and $\kappa(G) = n - 1$.

Case(i): $\gamma_{ntr}(G) = n$ and $\kappa(G) = n - 3$.

Since for any connected graph G, $\gamma_{ntr}(G) \leq n-1$, this case is not possible.

Case(ii): $\gamma_{ntr}(G) = n - 1$ and $\kappa(G) = n - 2$

$\gamma_{ntr}(G) = n-1$ if and only if $G \cong P_3$ and $\kappa(P_3) = 1 = n - 2$. Therefore $G \cong P_3$.

Case(iii): $\gamma_{ntr}(G) = n - 2$ and $\kappa(G) = n - 1$.

If $\kappa(G) = n - 1$,then $G \cong K_n$, $n \geq 3$.But $\gamma_{ntr}(G) = 0$ for $G \cong K_n$, $n \geq 4$. Therefore, $G \cong K_3$ (or) C_3. Also $\gamma_{ntr}(C_3) = 1 = n - 2$.

Therefore, from case(ii) and case(iii), $G \cong P_3$ (or) C_3.

THEOREM: 3.3 There is no connected graph G with $\gamma_{ntr}(G) + \kappa(G) = 2n - 4$, where $n \geq 3$.

PROOF:

Assume $\gamma_{ntr}(G) + \kappa(G) = 2n - 4$, $n \geq 3$. Then the following cases are to be considered.

(i) $\gamma_{ntr}(G) = n$ and $\kappa(G) = n - 4$
(ii) $\gamma_{ntr}(G) = n - 1$ and $\kappa(G) = n - 3$
(iii) $\gamma_{ntr}(G) = n - 2$ and $\kappa(G) = n - 2$
(iv) $\gamma_{ntr}(G) = n - 3$ and $\kappa(G) = n - 1$

There is no connected graph G with $\gamma_{ntr}(G) = n$, $\kappa(G) = n - 4$ and $\gamma_{ntr}(G) = n - 1$, $\kappa(G) = n - 3$.

Case(iii): $\gamma_{ntr}(G) = n - 2 = \kappa(G)$

Since $\kappa(G) \leq \delta(G)$, $\delta(G) \geq n - 2$.

(a) If $\delta(G) > n - 2$, then $G \cong K_n$, $n \geq 3$.But $\gamma_{ntr}(G) = 0$ for $G \cong K_n$, $n \geq 4$. Therefore, $G \cong K_3$, and $\kappa(K_3) = 2 \neq n - 2$.

(b) Assume $\delta(G) = n - 2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in K_n, $n \geq 3$ and $\gamma_{ntr}(G) \leq 2$.If $\gamma_{ntr}(G) = 2 = n - 2$ then $n = 4$.Therefore, $G \cong K_4 - e$, C_4. If $G \cong C_4$, then $\gamma_{ntr}(G) = 0$. If $G \cong K_4 - e$, then $\gamma_{ntr}(G) = 1 \neq n - 2$. If $\gamma_{ntr}(G) < 2$, then $n - 2 < 2$.That is, $n \leq 3$. Therefore $n = 3$ and $G \cong P_3$ (or) C_3. If $G \cong P_3$, $\gamma_{ntr}(P_3) = n - 1$.If $G \cong C_3$, $\kappa(C_3) = 2 \neq n - 2$.

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Case(iv): $\gamma_{ntr}(G) = n - 3$ and $\kappa(G) = n - 1$

If $\delta(G) = n - 1$, then $G \cong K_n$, $n \geq 3$. But $\gamma_{ntr}(K_n) = 0$ for $n \geq 4$. Therefore, $G \cong K_3$ and $\gamma_{ntr}(G) = \gamma_{ntr}(K_3) = 1 \neq n - 3$. Therefore, there is no connected graph G with $\gamma_{ntr}(G) + \kappa(G) = 2n - 4$.

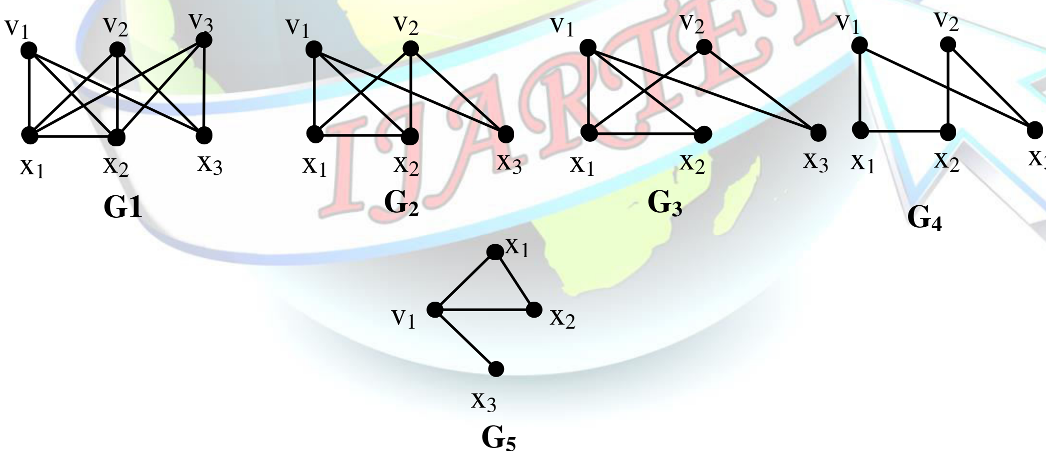
THEOREM: 3.4 For any connected graph G, $\gamma_{ntr}(G) + \kappa(G) = 2n - 5$ $(n \geq 4)$ if and only if $G \cong K_4 - e$, $K_5 - \{e_1, e_2\}$.

PROOF:

Assume $\gamma_{ntr}(G) + \kappa(G) = 2n - 5$, then the following cases are to be considered.

(i) $\gamma_{ntr}(G) = n$ and $\kappa(G) = n - 5$
(ii) $\gamma_{ntr}(G) = n - 1$ and $\kappa(G) = n - 4$
(iii) $\gamma_{ntr}(G) = n - 2$ and $\kappa(G) = n - 3$
(iv) $\gamma_{ntr}(G) = n - 3$ and $\kappa(G) = n - 2$
(v) $\gamma_{ntr}(G) = n - 4$ and $\kappa(G) = n - 1$

There is no connected graph G satisfying (i), (ii), and (v).



Case (iii): $\gamma_{ntr}(G) = n - 2$ and $\kappa(G) = n - 3$

Then $\delta(G) \geq n - 3$, since $\kappa(G) \leq \delta(G)$. If $\delta(G) = n - 1$, then $G \cong K_n$, $n \geq 4$. But $\gamma_{ntr}(K_n) = 0$ for $n \geq 4$. If $\delta(G) = n - 2$, then G is isomorphic to $K_n - Y$, where Y is matching in K_n. Then $\gamma_{ntr}(G) \leq 2$. If $\gamma_{ntr}(G) = 2$, then $n = 4$. Therefore $K_4 - e$, C_4. If $G \cong C_4$, $\gamma_{ntr}(C_4) = 0$.If $G \cong K_4 - e$, $\kappa(K_4 - e) = 1 \neq n - 2$. Therefore, $G \cong P_4$. If $\gamma_{ntr}(G) < 2$, then $n \leq 3$. But $n \geq 4$. Therefore, $\delta(G) = n - 3$.

Let $X = \{v_1, v_2, v_3, \ldots, v_{n-3}\}$ be a vertex cut of G and $V - X = \{x_1, x_2, x_3\}$.

Subcase: 3.1$\langle V - X \rangle \cong \overline{K_3}$

Since $\delta(G) = n - 3$, each vertex in $V - X$ is adjacent to all the vertices of X.

(1)Assume $E(\langle X \rangle) = \phi$. If $|X| \geq 4$, then $\langle X \rangle$ contains atleast one edge. Therefore, $|X| \leq 3$. Hence, $G \cong K_{3,3}$, $K_{2,3}$ or $K_{1,3}$. But $\gamma_{ntr}(K_{3,3}) = \gamma_{ntr}(K_{2,3}) = 0$. Therefore, $G \cong K_{1,3}$.

(2) Let $E(\langle X \rangle) \neq \phi$. Let $\langle X \rangle$ contains exactly one edge say $(v_1, v_2) \in E(\langle X \rangle)$. Then $\{v_1, v_2, \ldots, v_{n-3}\}$ is a ntr - set of G and hence $\gamma_{ntr}(G) \leq n - 4$. If $\langle X \rangle$ is a tree, then $V - X$ is a ntr - set of G and hence $\gamma_{ntr}(G) \leq |V - X| = 3$.

That is., $n - 2 \leq 3 \Rightarrow n \leq 5$ and hence $|X| \leq 2$. If $|X| = 2$, then $G \cong K_2 + 3K_1$.If $|X| = 2$, then $G \cong K_{1,3}$. If $|X|$ contains atleast two edge and $\langle X \rangle$ is not a tree, then $\gamma_{ntr}(G) = 0$.

Sub case: 3.2 $\langle V - X \rangle \cong K_2 \cup K_1$.

Let $(x_1, x_2) \in E(G)$. Since $\delta(G) = n - 3$, x_3 is adjacent to all the vertices in X. That is, $d(x_3) = n - 3$.

(1) $E(\langle X \rangle) = \phi$. Therefore, $d(v_i) \leq 3$, $i = 1, 2, \ldots, n - 3$.

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$\Rightarrow \delta(G) \le d(v_i) \le 3 \Rightarrow n - 3 \le 3 \Rightarrow n \le 6$

$\Rightarrow |X| \le 3.$

(a) $|X| = 3$. Then $|V(G)| = 6$ and $\delta(G) = n - 3 \le 3$. Each vertex in X is adjacent to all the vertices of $V - X$. If $G \cong G_1$, then $\gamma_{ntr}(G_1) = 2 \ne n - 2$.

(b) $|X| = 2$. Then $|V(G)| = 5$ and $\delta(G) = n - 3 \le 2$. If $G \cong G_2$ and $d(x_1) = d(x_2) = 3$ and $d(x_3) = 2$ and then $\gamma_{ntr}(G_2) = 2 \ne n - 2$. If $G \cong G_3$ and $d(x_1) = 3$ and $d(x_2) = d(x_3) = 2$ and then $\gamma_{ntr}(G_3) = 2 \ne n - 2$. If $G \cong G_4$ and $d(x_1) = d(x_2) = d(x_3) = 2$ and then $\gamma_{ntr}(G_4) = 0$.

(c) $|X| = 1$. Then $|V(G)| = 4$ and $\delta(G) = n - 3 \le 1$. If $G \cong G_5$ and $d(x_1) = d(x_2) = 2$ and $d(x_3) = 1$ and then $\gamma_{ntr}(G_5) = 2 = n - 2$.

(2). $E(\langle X \rangle) \ne \phi$. Since $\delta(G) = n - 3$, $d(x_1), d(x_2) \ge n - 3$. Therefore, x_1, x_2 are adjacent to atleast $(n - 4)$ vertices of $v_1, v_2, v_3, \ldots, v_{n-3}$. Let $d(x_1) = d(x_2) = n - 2$. If $\langle X \rangle$ contains an edge, then any dominating set D of G containing $(n - 2)$ vertices, $\langle N(D) \rangle$ contains a cycle and hence $\gamma_{ntr}(G) = 0$. Let $d(x_1) = d(x_2) = n - 3$ and $(v_{n-4}, v_{n-3}) \in E(G)$. Let x_1 be adjacent to $v_1, v_2, v_3, \ldots, v_{n-4}$. Then $\{v_1, v_2, v_3, \ldots, v_{n-4}\}$ is a ntr-set of G and $\gamma_{ntr}(G) \le n - 4$. Let $d(x_1) = n - 2$, $d(x_2) = n - 3$ and let x_2 be non adjacent to v_{n-3}. Then $\{v_1, v_2, v_3, \ldots, v_{n-4}\}$ is a ntr-set of G and $\gamma_{ntr}(G) \le n - 4$. If $\langle X \rangle$ contains atleast two edges then $\gamma_{ntr}(G) = 0$.

Sub case: 3.3 $\langle V - X \rangle \cong P_3$

(1) If $E(\langle X \rangle) = \phi$, then X is an ntr-set of G and hence $\gamma_{ntr}(G) \le |X| = n - 3$.

(2) If $E(\langle X \rangle) \ne \phi$, and if $\langle X \rangle$ contains exactly one edge, then $\gamma_{ntr}(G) \le n - 4$. If $E(\langle X \rangle) \ne \phi$, and if $\langle X \rangle$ contains exactly two edge, then $\gamma_{ntr}(G) = 0$.

Sub case: 3.4 $\langle V - X \rangle \cong C_3$

Then any dominating set D of G containing $(n - 2)$ vertices contain a cycle and hence $\gamma_{ntr}(G) = 0$.

case (iv): $\gamma_{ntr}(G) = n - 3$ and $\kappa(G) = n - 2$

Therefore, $\delta(G) \ge n - 2$. If $\delta(G) = n - 1$, then $G \cong K_n$, $n \ge 4$. But $\gamma_{ntr}(K_n) = 0$, $n \ge 4$. Let $\delta(G) = n - 2$. Then G is isomorphic to K_n -Y, where Y is a matching in G, $n \ge 4$ and $\gamma_{ntr}(G) \le 2$. If $\gamma_{ntr}(G) = 2$, then $n = 5$. Therefore, $G \cong K_5 - e$ (or) $K_5 - (e_1, e_2)$, where (e_1, e_2) is a matching. If $G \cong K_5 - e$, then $\gamma_{ntr}(G) = 0$. If $G \cong K_5 - (e_1, e_2)$, then $\gamma_{ntr}(G) = 2 = n - 3$ and $\kappa(G) = 3 = n - 2$. If $\gamma_{ntr}(G) = 1$, then $n = 4$. Therefore, $G \cong K_4 - e$, C_4. But $\gamma_{ntr}(C_4) = 0$. If $G \cong K_4 - e$, then $\gamma_{ntr}(G) = 1 = n - 3$ and $\kappa(G) = 2 = n - 2$.

4. NEIGHBORHOOD TREE DOMATIC NUMBER

In this section we define a new parameter known as neighborhood tree domatic partition of a given graph and study that parameter.

DEFINITION: 4.1

A ***domatic partition*** of G is a partition $\{V_1, V_2, V_3, \ldots, V_n\}$ of V(G), in which each V_i is a dominating set of G. The maximum order of a domatic partition of G is called the domatic number of G and is denoted by d(G).

DEFINITION: 4.2

A partition $\{V_1, V_2, V_3, \ldots, V_n\}$ of V(G), in which each V_i is a ncd - set in G is called a ***neighborhood connected domatic partition*** of simply ncd partition of G. the maximum order of a ncd partition of G is called the neighborhood connected domatic number of G and is denoted by $d_{ncd}(G)$.

DEFINITION: 4.3

A partition $\{V_1, V_2, V_3, \ldots, V_n\}$ of V(G), in which each V_i is a ntr -set in G is called a ***neighborhood tree domatic partition*** of simply ntr partition of G. the maximum order of a ntr partition of G is called the neighborhood tree domatic number of G and is denoted by $d_{ntr}(G)$.

EXAMPLE: 4.1

$D_1 = \{ v_1, v_4 \}$, $V - D_1 = \{ v_2, v_3 \}$

$D_2 = \{ v_2, v_3 \}$, $d_{ntr}(G) = 2$

REMARKS:

a) If $G \cong C_3$, then $d_t(G) = d_{ntr}(G) = \kappa(G)$ where $\kappa(G)$ is the connectivity of G.

b) Since any tree domatic partition of G is a ntr – domatic partition, we have $d_{tr}(G) \le d_{ntr}(G) \le d(G)$.

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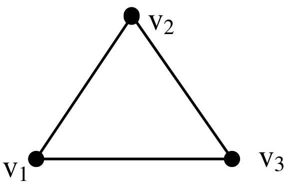
c) Let $v \in V(G)$ and $d(v) = \delta$. Since any ntr – set of G must contains either v (or) neighbor of v, it follows that $d_{ntr}(G) \leq \delta(G) + 1$.

EXAMPLE 4.2:



$d_{ntr}(G) \leq \delta(G) + 1 = 3$

$d_t(G) = d_{ntr}(G) = \kappa(G) = 3$

Now we give some observations, theorems relating neighborhood tree domatic numbers of some classes of graphs.

Observation: 4.1

If $\{V_1, V_2, \ldots, Vd_{ntr}\}$ is a neighborhood tree domatic partition of G. Since $|V_k| \geq \gamma_{ntr}$ for each k, it follows that $\gamma_{ntr}(G).\, d_{ntr}(G) \leq n$.

EXAMPLE 4.3:

If $G \cong G_1$ o K_1, where G_1 is any tree then $d_{ntr}(G) = 2$ and $\gamma_{ntr}(G) = n/2$an hence $\gamma_{ntr}(G).\, d_{ntr}(G) = n$.

THEOREM : 4.1

For any connected graph G, $\lfloor d(G)/2 \rfloor \leq d_{ntr}(G) \leq d(G)$ and the bounds are sharp.

PROOF:

Since every neighborhood tree dominating set, we have $d_{ntr}(G) \leq d(G)$. Further, since the union of two disjoint dominating sets is a neighborhood tree dominating set, we have , $\lfloor d(G)/2 \rfloor \leq d_{ntr}(G)$.

Also for the graphs $G \cong P_3, K_{1,\,n-1}, J_{m,\,n}, T_n$. , $\lfloor d(G)/2 \rfloor = d_{ntr}(G)$. For the graph $G = K_3$, $d_{ntr}(G) = d(G) = 3$.

THEOREM: 4.2

If $\gamma_{ntr}(G) > 0$, then $d_{ntr}(G) \leq \dfrac{n}{\gamma_{ntr}(G)}$ and the bound is sharp.

PROOF:

Let$\{D_1, D_2, \ldots, D_k\}$ is a partition of V(G) into k neighborhood tree dominating sets, such thatb , $d_{ntr}(G) = k$. Since each $\langle N(D_i) \rangle$ is a neighborhood tree dominating set, it follows that , $\gamma_{ntr}(G) \leq |D_i|$ for $1 \leq i \leq k$.

Thus, $n = \sum_{1 \leq i \leq k} |D_i| \geq \gamma_{ntr}(G).k$

THEOREM: 4.3

For the path P_n ($n \geq 4$), we have

$$d_{ntr}(P_n) = \begin{cases} 1 & \text{if} \quad n \text{ is odd} \\ 2 & \text{if} \quad n \text{ is even} \end{cases}$$

PROOF:

Let $V(P_n) = \{v_1, v_2, v_3, \ldots, v_n\}$.

If n is odd, $V(P_n)$ is the only ntr – set. Suppose n is even, it follows from remarks that $d_{ntr}(P_n) \leq 2$.

Now, let

$$S = \bigcup_{i=1}^{\lfloor n/4 \rfloor} \{v_{4i-2}, v_{4i-1}\}$$ and

$$\text{let } V_1 = \begin{cases} S & n \equiv 0 \pmod 4 \\ S \cup v_n & n \equiv 1, 2 \pmod 4 \\ S \cup \{v_{n-1}, v_n\} & n \equiv 3 \pmod 4 \end{cases}$$

Then $\{V_1, V\text{-}\ V_1\}$ is a ntr- domatic partition of P_n and hence $d_{ntr}(P_n) = 2$.

Observation 4.2:

For the cycle C_n ($n \geq 3$), we have

$$d_{ntr}(C_n) = \begin{cases} 3 & \text{if} \quad n = 3 \\ 2 & \text{if} \quad n = 4k+2, k \geq 1 \end{cases}$$

Observation 4.3: $d_{ntr}(K_{1,\,n-1}) = 1, n \geq 3$.

Observation 4.4: $d_{ntr}(S_{m,\,n}) = 2$, $m, n \geq 1$.

Observation 4.5: $d_{ntr}(P_n \bullet K_1) = 2, n \geq 2$.

Observation 4.6: $d_{ntr}(P_n + K_1) = 3, n \geq 2$.

Observation 4.7: $d_{ntr}(\overline{P_n}) = \begin{cases} 2 & \text{if} \quad n = 4 \\ 0 & \text{if} \quad n \geq 4 \end{cases}$

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Observation 4.8: $d_{ntr}(\overline{C_6}) = 3$

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