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Isolate Eccentric Domination in Graphs

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Abstract: **A subset D of the vertex set V(G) of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is said to be an eccentric dominating set if for every $v \in V-D$, there exists at least one eccentric vertex of v in D. An eccentric dominating set S of G is an isolate eccentric dominating set if the induced subgraph $\langle S \rangle$ has atleast one isolated vertex. The minimum of the cardinality of the isolate eccentric dominating set of G is called the isolate eccentric domination number $\gamma_{oed}(G)$. In this paper, we obtain some bounds for $\gamma_{oed}(G)$. Exact values of $\gamma_{oed}(G)$ for some particular classes of graphs are obtained.**

Keywords: **Domination, Eccentric domination, Isolate Domination, Isolate Eccentric Domination.**
Mathematics Subject Classification: **05C12, 05C69.**

I. Introduction

Let G be a finite, simple, undirected (p, q) graph with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary [5], Buckly and Harary [3].

Janakiraman, Bhanumathi and Muthammai [6] introduced Eccentric domination in Graphs. Sahul Hamid and Balamurugan studied the concept of Isolate domination in graphs [7, 8, 9].

DEFINITION 1.1: Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) = d(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. The vertex v is a central vertex if e(v) = r(G). The center C(G) is the set of all central vertices. For a vertex v, each vertex at a distance e(v) from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G) / d(u, v) = e(v)\}$.

DEFINITION 1.2 [4, 10]: A set $D \subseteq V$ is said to be a dominating set in G if every vertex in V–D is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$.

DEFINITION 1.3 [1]: A subset $S \subseteq V(G)$ is called an isolate set if the subgraph induced by S has an isolated vertex. This set S is an isolate dominating set if it is both isolate and dominating. The minimum cardinality of an isolate dominating set is called the isolate domination number and is denoted by $\gamma_o(G)$.

DEFINITION 1.4 [6]: A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V-D$, there exists at least one eccentric vertex of v in D. The minimum cardinality of an eccentric dominating set is called the eccentric domination number and is denoted by $\gamma_{ed}(G)$. If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set.

THEOREM: 1.1 [4] For any graph G, $\lceil p/(1+\Delta(G)) \rceil \leq \gamma(G) \leq p-\Delta(G)$.

THEOREM: 1.2 [6] $\gamma_{ed}(K_p) = 1$.

THEOREM: 1.3 [6]

$$\gamma_{ed}(P_p) = \begin{cases} \left\lceil \frac{p}{3} \right\rceil & \text{if} \quad p = 3k+1 \\ \left\lceil \frac{p}{3} \right\rceil + 1 & \text{if} \quad p = 3k \quad \text{or} \quad p = 3k+2 \end{cases}$$

THEOREM: 1.4 [6] (i) $\gamma_{ed}(C_p) = p/2$ if p is even.
(ii) $\gamma_{ed}(C_p) = \lceil p/3 \rceil$ or $\lceil p/3 \rceil + 1$ if p is odd.
THEOREM: 1.5 [9] $\gamma_o(P_p) = \lceil p/3 \rceil$.
THEOREM: 1.6 [9] $\gamma_o(C_p) = \lceil p/3 \rceil$.

II. ISOLATE ECCENTRIC DOMINATION IN GRAPHS

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Now, we define isolate eccentric dominating set of a graph as follows.

A set $S \subseteq V(G)$ is a isolate eccentric dominating set if S is an eccentric dominating set and also the induced sub graph $\langle S \rangle$ has atleast one isolated vertex. The minimum of the cardinality of the isolate eccentric dominating set of G is called the isolate eccentric domination number $\gamma_{oed}(G)$.

In this section, we study the isolate eccentric domination number for some graphs G.

Obviously, $\gamma(G) \le \gamma_o(G) \le \gamma_{oed}(G)$ and $\gamma_{ed}(G) \le \gamma_{oed}(G)$.

EXAMPLE: 2.1

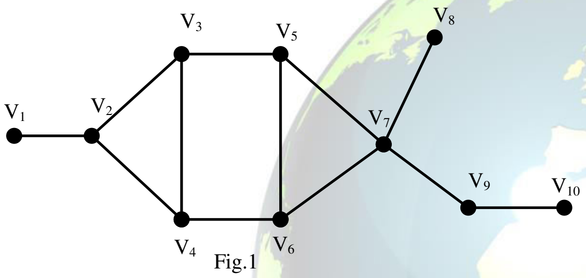

Fig.1

$S_1 = \{v_2, v_7, v_{10}\}$ is a dominating set of G and also an isolate dominating set of G. $\gamma(G) = \gamma_o(G) = 3$.

$S_2 = \{v_1, v_4, v_7, v_{10}\}$ is an eccentric dominating set of G and also an isolate eccentric dominating set of G. $\gamma_{ed}(G) = \gamma_{oed}(G) = 4$.

EXAMPLE: 2.2

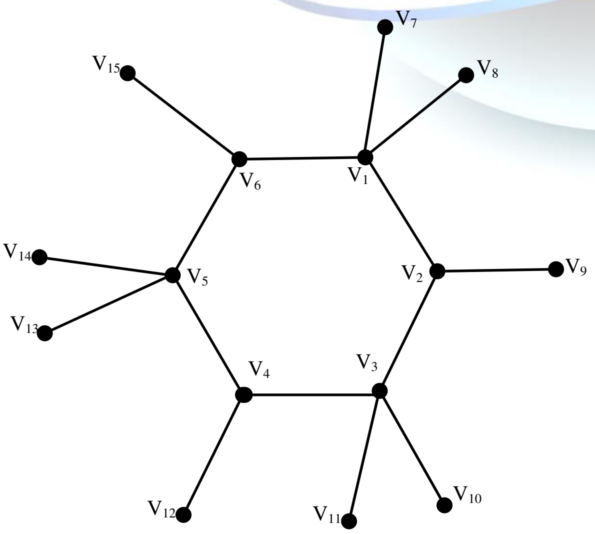

Fig. 2

$S_1 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is a dominating set of G, $\gamma(G) = 6$.

$S_2 = \{v_1, v_2, v_3, v_4, v_5, v_{15}\}$ is an isolate dominating set of G, $\gamma_o(G) = 6$.

$S_3 = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$ is an eccentric dominating set of G and also an isolate eccentric dominating set of G. $\gamma_{ed}(G) = \gamma_{oed}(G) = 9$.

OBSERVATION: 2.1 For any graph G, $1 \le \gamma_{oed}(G) \le p$.

The bounds are sharp, since $\gamma_{oed}(G) = 1$ if and only if $G = K_p$ and $\gamma_{oed}(G) = p$ if and only if $G = \overline{K_p}$.

PROPOSITION: 2.1

(i) $\gamma_{oed}(K_p) = 1$.
(ii) $\gamma_{oed}(K_{1,p}) = p$.
(iii) $\gamma_{oed}(P_p \circ K_1) = p$.
(iv) $\gamma_{oed}(C_p \circ K_1) = p$.

THEOREM: 2.1

$$\gamma_{oed}(P_p) = \begin{cases} \left\lceil \frac{p}{3} \right\rceil & if \quad p = 3k+1 \\ \left\lceil \frac{p}{3} \right\rceil + 1 & if \quad p = 3k \quad or \quad p = 3k+2 \end{cases}$$

PROOF:

Case(i) p = 3m

$S = \{v_1, v_4, v_7, \ldots, v_{3m-2}, v_{3m}\}$ is the minimum eccentric dominating set in P_p. $\langle S \rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(P_p) = \gamma_{ed}(P_p) = \lceil p/3 \rceil + 1$.

Case(ii) p = 3m+1

$S = \{v_1, v_4, v_7, \ldots, v_{3m-2}, v_{3m+1}\}$ is the minimum eccentric dominating set in P_p. $\langle S \rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(P_p) = \gamma_{ed}(P_p) = \lceil p/3 \rceil$.

Case(iii) p = 3m+2

$S = \{v_1, v_2, v_5, v_8, \ldots, v_{3m-1}, v_{3m+2}\}$ is the minimum eccentric dominating set in P_p. $\langle S \rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(P_p) = \gamma_{ed}(P_p) = \lceil p/3 \rceil + 1$.

Example:

P_6

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$S = \{v_1, v_4, v_6\}$ is a dominating set of P_6, an isolate dominating set of P_6, an eccentric dominating set of P_6 and also an isolate eccentric dominating set of P_6. $\gamma(P_6) = \gamma_o(P_6) = \gamma_{ed}(P_6) = \gamma_{oed}(P_6) = 3$.

THEOREM: 2.2 (A) $\gamma_{oed}(C_p) = p/2$ if p is even.
(B) $\gamma_{oed}(C_p) = \lceil p/3 \rceil$ or $\lceil p/3 \rceil + 1$ if p is odd.

PROOF:

Proof of (A):
If p = 4, any two adjacent vertices of C_4 is an eccentric dominating set of C_4 but does not exists an isolate eccentric dominating set in C_4.
Let p = 2m and m > 2.

Let the cycle C_p be $v_1v_2v_3\ldots v_{2m}v_1$. Each vertex of C_p has exactly one eccentric vertex (that is C_p is unique eccentric vertex graph).

Hence, $\gamma_{oed}(C_p) \geq \gamma_{ed}(C_p) \geq p/2$. (1)

Case(i) m-odd.

$S = \{v_1, v_3, \ldots, v_m, v_{m+2}, \ldots, v_{2m-1}\}$ is the minimum eccentric dominating set in C_p. $\langle S\rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(C_p) \leq p/2$. (2)

From (1) and (2), $\gamma_{oed}(C_p) = p/2$.

Case(ii) m-even.

$S = \{v_1, v_3, \ldots, v_{m-1}, v_{m+2}, \ldots, v_{2m}\}$ is the minimum eccentric dominating set in C_p. $\langle S\rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(C_p) \leq p/2$. (3)

From (1) and (3), $\gamma_{oed}(C_p) = p/2$.

Proof of (B):

When p is odd, each vertex of C_p has exactly two eccentric vertices. If p = 2m+1, $v_i \in V(G)$ has v_{i+m}, v_{i+m+1} as eccentric vertices.

Case(i) p = 3n, p-odd $\Rightarrow$ n-odd.

$S = \{v_1, v_4, v_7, \ldots, v_m, v_{m+3}, \ldots, v_{2m-1}\}$ is the minimum eccentric dominating set in C_p. $\langle S\rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(C_p) = p/3$.

Case(ii) p = 3n+1, p-odd $\Rightarrow$ n-even.

$S = \{v_1, v_4, \ldots, v_{m+1}, v_{m+3}, v_{m+6}, \ldots, v_{2m-1}\}$ is the minimum eccentric dominating set in C_p. $\langle S\rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

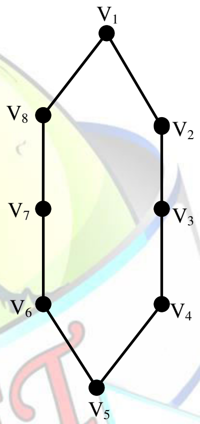
Therefore, $\gamma_{oed}(C_p) = \lceil p/3 \rceil$.

Case(iii) p = 3n+2 $\Rightarrow$ 3n-odd $\Rightarrow$ n-odd.

$S = \{v_1, v_4, \ldots, v_{m-1}, v_m, v_{m+3}, \ldots, v_{2m+1}\}$ is the minimum eccentric dominating set in C_p. $\langle S\rangle$ contains isolated vertices. Hence S is also an isolate eccentric dominating set.

Therefore, $\gamma_{oed}(C_p) = \lceil p/3 \rceil + 1$.

Example:



C_8

$S_1 = \{v_1, v_4, v_7\}$ is a dominating set of C_8 and also an isolate dominating set of C_8. $\gamma(C_8) = \gamma_o(C_8) = 3$.

$S_2 = \{v_1, v_3, v_6, v_8\}$ is an eccentric dominating set of C_8 and also an isolate eccentric dominating set of C_8. $\gamma_{ed}(C_8) = \gamma_{oed}(C_8) = 4$.

THEOREM: 2.3 If G is self-centered of diameter two, then $\gamma_{oed}(G) \leq (p - \deg_G w + 1)/2$, if there exists $w \in V(G)$ with $S \subseteq N_2(w)$ such that vertices in $N_1(w)$ and $N_2(w) - S$ has eccentric vertices in S.

PROOF: Let $w \in V(G)$ such that $\deg_G w = \delta(G)$. w and $N_2(w)$ dominates all other vertices of G. Let S be a subset of $N_2(w)$ with minimum cardinality such that vertices in $N_1(w)$ and $N_2(w) - S$ has their eccentric vertices in S. Then $|S| \leq |N_2(w)|/2 \leq (p - \deg_G w - 1)/2$. Now $\{w\} \cup S$ is an isolate eccentric dominating set of G. Since w is an isolated vertex of $\langle \{w\} \cup S\rangle$.

$$\begin{aligned} \text{Hence, } \gamma_{oed}(G) &\leq |w| + |S| \\ &\leq 1 + (p - \deg_G w - 1)/2 \\ &\leq (p - \deg_G w + 1)/2. \end{aligned}$$

THEOREM: 2.4 If G is of radius two and diameter three and if G has a pendant vertex h of eccentricity three then $\gamma_{oed}(G) \leq \Delta(G)$.

PROOF: If G has a pendant vertex h of eccentricity three then its support f is of eccentricity two. In this case, N(f) is an isolate eccentric dominating set, since h is an isolated vertex in $\langle N(f) \rangle$.

Thus, $\gamma_{oed}(G) \leq deg_G(f) \leq \Delta(G)$.

THEOREM: 2.5 If G is of radius two with a unique central vertex w then $\gamma_{oed}(G) \leq p - deg(w)$.

PROOF: If G is of radius two with a unique central vertex w then w dominates N[w] and the vertices in V – N[w] dominate themselves and each vertex in N[w] has eccentric vertices in V – N[w] only. Therefore, D = V – N(w) is an eccentric dominating set and w is an isolated vertex of $\langle D \rangle$.

Thus, $\gamma_{oed}(G) \leq p - deg(w)$.

COROLLARY: 2.1 If T is a unicentral tree of radius two, then $\gamma_{oed}(T) \leq p - deg(w)$, where w is the central vertex.

THEOREM: 2.6 If H is any self-centered unique eccentric point graph with t vertices and $G = H \circ 2K_1$, then $\gamma_{oed}(G) = 2p/3$ where p = 3t.

PROOF: If H is any self-centered unique eccentric vertex graph, then every vertex of H is an eccentric vertex. Hence, t is even and G has 3t vertices. Let $v_1, v_2, v_3, \ldots, v_t$ represent the vertices of H and $\{v_r', v_r''\}$ for r = 1, 2, 3, …, t be the vertices of t copies of $2K_1$. Then in G, v_r', v_r'' are adjacent to v_r and if v_s is the eccentric vertex of v_r in H, then v_r', v_r'' are eccentric vertices of v_s in G and v_s', v_s'' are eccentric vertices of v_r. It is clear that $\{v_1', v_2', v_3', \ldots, v_t'\} \cup \{v_1'', v_2'', v_3'', \ldots, v_t''\}$ is a minimum isolate eccentric dominating set of G.

Hence, $\gamma_{oed}(G) = 2p/3$.

THEOREM: 2.7 If G is of radius greater than two, then $\gamma_{oed}(G) \leq p - \Delta(G)$.

PROOF: Let w be a vertex of maximum degree $\Delta(G)$. Then w dominates N[w] and the vertices in V – N[w] dominate themselves. Also, since diam(G) > 2, each vertex in N(w) has eccentric vertices in V – N[w] only. Therefore, V – N(w) is an eccentric dominating set of cardinality p – $\Delta(G)$ and u is an isolated vertex in $\langle V - N(w) \rangle$, so that $\gamma_{oed}(G) \leq p - \Delta(G)$.

THEOREM: 2.8 Let G be a connected graph with $|V(G)| = p$. Then $\gamma_{oed}(G \circ K_1) = p$.

PROOF: Let $V(G) = \{v_1, v_2, v_3, \ldots, v_p\}$. Let v_r' be the pendant vertex adjacent to v_r in $G \circ K_1$ for r = 1, 2, 3, …, p. Then $\{v_1', v_2', v_3', \ldots, v_p'\}$ is an isolate eccentric dominating set for $G \circ K_1$ and is also a minimum dominating set for $G \circ K_1$. Hence, $\gamma_{oed}(G \circ K_1) = p$.

OBSERVATION: 2.2 (i) If G = $\overline{K_2} + K_1 + K_1 + \overline{K_2}$, then $\gamma(G) = 2$, $\gamma_{ed}(G) = 4$, $\gamma_o(G) = 3$, $\gamma_{oed}(G) = 4$.
(ii) If $G = K_p + K_1 + K_1 + K_p$, p > 2 then $\gamma(G) = \gamma_{ed}(G) = \gamma_o(G) = \gamma_{oed}(G) = 2$.
(iii) If G = $\overline{K_q} + K_1 + K_1 + \overline{K_p}$ with q ≤ p, then $\gamma(G) = 2$, $\gamma_{ed}(G) = 4$, $\gamma_o(G) = \min(q, p) + 1$, $\gamma_{oed}(G) = \min(q, p) + 2$.

III. CONCLUSION

Here we have initiated the study of isolate eccentric domination in graphs. We have studied isolate eccentric domination in some families of graphs and also studied some bounds for isolate eccentric domination number of a graph.

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BIOGRAPHY

M. Bhanumathi was born in Nagercoil, Tamilnadu, India in 1960. She received her B.Sc., M.Sc. and M.Phil. degrees in Mathematics from Madurai Kamaraj University, India in 1981, 1983 and 1985 respectively. In 1987, she joined as Assistant Professor of Mathematics in M.V.M. Govt. Arts College for Women, Dindigul affiliated to Madurai Kamaraj University, India. Since 1990, she has been with the PG department of Mathematics in Government Arts College for Women, Pudukkottai. She did her research under Dr.T.N.Janakiraman at National Institute of Technology, Trichy for her doctoral degree, and received her Ph.D degree from Bharathidasan University in 2005. She became Reader in 2005 and Associate Professor in 2009. Her current research interests in Graph Theory include Domination in Graphs, Graph Operations, Distance in Graphs, Decomposition of Graphs, Metric dimension and Topological indices of graphs. She has published more than 70 research papers in national/international journals. She is currently the Principal in Government Arts and Science College, Kadaladi, Ramanathapuram district, Tamilnadu, India.

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