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On some Multiplicative indices of special types of Nanotubes and Nanotori

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Abstract: Topological index is the numerical quantity that characterizes the property of the molecule. In this paper we calculate, Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F-index of some special types of Nanotubes and Nanotori..

Keywords: Multiplicative topological indices, Harmonic index, ISI index, F-index, Nanotubes and Nanotori.

I. Introduction

Let G be a finite, simple and connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v.

A molecular graph is pictorial representation of a chemical compound, in which each vertex denotes an atom of the molecule and its edges denotes the bonds between atoms. The topological indices help to find the relation between structure of the compound and its physio-chemical properties.

There are many topological indices so far introduced.

In 2014 Jianxi Li and Chee Shiu introduced another variant of the Randi´c index named the Harmonic index which first appeared in [3], and was defined as

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_u + d_v}$$

Discrete Adriatic indices have been defined by Vukičević and Gašperov in 2010 [4]. One among such indices named as the inverse sum indeg index, and is defined as

$$ISI\ (G) = \sum_{uv \in E(G)} \frac{1}{\frac{1}{d_u} + \frac{1}{d_v}} = \sum_{uv \in E(G)} \frac{d_u d_v}{d_u + d_v}$$

Followed by the first and second Zagreb indices, in 2015 Furtula and Gutman [5] introduced forgotten topological index (also called *F*-index) which was defined as

$$F(G) = \sum_{v \in V(G)} [d_G(v)]^3$$

It is easy to see that

$$F(G) = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]$$

In [6], we introduced the multiplicative Harmonicindex as

$$H\pi(G) = \prod_{uv \in E(G)} \frac{2}{d_u + d_v}$$

In [7], we introduced the multiplicative ISI index and the multiplicative F-index as

$$ISI\pi\ (G) = \prod_{uv \in E(G)} \frac{d_u d_v}{d_u + d_v} \quad \text{and}$$
$$F\pi(G) = \prod_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]$$

II. NANO-STRUCTURES

Carbon nanotubes (CNTs) are types of nanostructure which are allotropes of carbon and having a cylindrical shape. Carbon nanotubes, a type of fullerene, have potential in fields such as nanotechnology, electronics, optics, materials science, and architecture. Carbon nanotubes provide a certain potential for metal-free catalysis of inorganic and organic reactions. Carbon nanotubes have applications in fields of electronic and electrochemical and mechanical reinforcements in high performance composites. Nanotube-based field emitters have applications as nanoprobes in metrology and biological and chemical investigations and as templates for the creation of other nanostructures.

III. MAIN RESULTS

In this section, we compute the Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F index for H-Naphtalenic nanotubes, $TUC_4[m, n]$ nanotubes and V-Phenylenic Nanotubes and Nanotorus.

(a)H-Naphtalenic Nanotubes

In this section, we compute the certain topological indices for H-Naphtalenic nanotubes. This nanotube is a trivalent decoration having sequence of $C_6, C_6, C_4, C_6, C_6, C_4$ ... in first row and a sequence of C_6, C_8, C_6, C_8 ... in other row. In other words, the whole lattice is a plane tiling of C_4, C_6 and C_8 and this type of

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tiling can either cover a cylinder or a torus. These nanotubes usually symbolized $NPHX[m,n]$ in which m is the number of pairs of hexagons in first row and n is the number of alternative hexagons in a column as depicted in Figure 3.1.

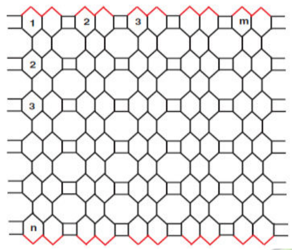


Fig. 3.1. A 2D-lattice of H-Naphtalenic nanotube $NPHX[m,n]$ showing the edge partition based on the degrees of end vertices of each edge.

3.1. Results for H-Naphtalenic Nanotubes

At first, we compute the Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F index for H-Naphtalenic nanotubes

Lemma 3.1.1[1]

Let $NPHX[m,n]$ be the graph of H-Naphtalenic nanotubes with $(m,n) > 1$, then the number of vertices is $|V(NPHX[m,n])| = 10mn$

Lemma 3.1.2[1]

Consider the graph of H-Naphtalenic nanotubes $NPHX[m,n]$ with $(m,n) > 1$, then the number of edges is $|E(NPHX[m,n])| = 15mn - 2m$

Theorem 3.1 Consider the graph of

$G = NPHX[m,n]$ nanotubes, then

(i) $H\pi(G) = (38.698)^m \times (6.969e^{-8})^{mn}$

(ii) $ISI\pi(G) = (0.0746)^m \times (437.894)^{mn}$

(iii) $F\pi(G) = (2.2847e^{-4})^m \times (18)^{15mn}$

Proof. Consider the graph of $NPHX[m,n]$. There are two type of edges in 2D-lattice of this nanotube as shown by different colours in Figure 1, in which red colour shows the edges ab with $d_u = 2$ and $d_v = 3$ and black colour shows the edges ab with $d_u = d_v = 3$ Table I shows cardinalities of these two partite sets of edge set of $NPHX[m,n]$ nanotube.

II. (d_u, d_v)	III. $(2,3)$	IV. $(3,3)$
V. Number of edges	VI. 8m	VII. 15mn-10m

Table 3.1. Edge partition of 2D-lattice of H-Naphtalenic nanotubes based on degrees of end vertices of each edge.

Hence

(i) $H\pi(G) = \prod_{uv\in E(NPHX[m,n])} \frac{2}{d_u+d_v}$

$= \left(\frac{2}{2+3}\right)^{8m} \times \left(\frac{2}{3+3}\right)^{15mn-10m} = \left(\frac{2}{5}\right)^{8m} \times \left(\frac{1}{3}\right)^{15mn-10m}$

$= (38.698)^m \times (6.969e^{-8})^{mn}$

(ii) $ISI\pi(G) = \prod_{uv\in E(NPHX[m,n])} \frac{d_u d_v}{d_u+d_v}$

$= \left(\frac{2\times3}{2+3}\right)^{8m} \times \left(\frac{3\times3}{3+3}\right)^{15mn-10m} = \left(\frac{6}{5}\right)^{8m} \times \left(\frac{3}{2}\right)^{15mn-10m}$

$= (0.0746)^m \times (437.894)^{mn}$

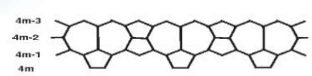
(iii) $F\pi(G) = \prod_{uv\in E(NPHX[m,n])} [d_G(u)^2 + d_G(v)^2]$

$= [2^2 + 3^2]^{8m} \times [3^2 + 3^2]^{15mn-10m}$

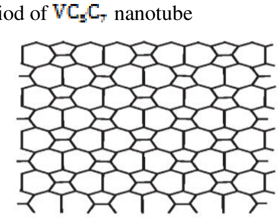
$= [13]^{8m} \times [18]^{15mn-10m} = [2.2847e^{-4}]^m \times [18]^{15mn}$

(b) $VC_5C_7[p,q]$ $(p,q > 1)$, Nanotubes

This nanotube is a C_5C_7 net and constructed by alternating C_5 and C_7 following the trivalent decoration as shown in Figure 2(b). This type of tiling can either cover a cylinder or a torus also. The graph of $VC_5C_7[p,q]$, in which p is the number of pentagons in one row and q is the number of periods in whole lattice. A period consists of four rows as in Figure 2(a) in which m-th period is shown.



(a) m-th period of VC_5C_7 nanotube



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(b) The graph of $VC_5C_7[p,q]$ nanotube with p = 3 and q = 4.

Fig. 2. $VC_5C_7[p,q]$ Nanotubes and m-th period.

3.2. Results for $VC_5C_7[p,q](p,q > 1)$ Nanotubes

In this section, we compute the Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F index for $VC_5C_7[p,q]$ nanotubes.

Lemma 3.2.1[1]

Let $VC_5C_7[p,q]$ be the graph of nanotubes with $(p,q) > 1$.There are 16p vertices in one period and 3p vertices which are joined at the end of the graph of these nanotubes, then the number of vertices is $|V(VC_5C_7[p,q])| = 16pq + 3p$

Lemma 3.2.2 [1]

Consider the graph of $VC_5C_7[p,q]$ nanotubes with$(p,q > 1)$. There are 24p edges in one period and 3p extra edges which are joined to the end of the graph in these nanotubes then the number of edges is $|E(VC_5C_7[p,q])| = 24pq - 3p$

Theorem 3.2 Consider the graph of

$G = VC_5C_7[p,q]$ Nanotubes, then

(i) $H\pi(G) = (250.765)^p \times (3.5407 \times e^{-12})^{pq}$

(ii) $ISI\pi(G) = (0.0212)^p \times (207.829)^{pq}$

(iii) $F\pi(G) = [2.9424 \times e^{-6}]^p \times [18]^{24pq}$

Proof. Let the graph G be $VC_5C_7[p,q]$. Table 3 shows edge partition of graph of $VC_5C_7[p,q]$ nanotubes.

VIII. (d_u, d_v)	IX. (2,2)	X. (2,3)	XI. (3,3)
XII. Number of edges	XIII. p	XIV. 10p	XV. 24pq −14p

Table 2. Edge partition of $VC_5C_7[p,q]$ nanotubes based on degrees of end vertices of each edge

Hence

$$(i) H\pi(G) = \prod_{uv \in E(VC_5C_7[p,q])} \frac{2}{d_u + d_v}$$

$$= \left(\frac{2}{2+2}\right)^p \times \left(\frac{2}{2+3}\right)^{10p} \times \left(\frac{2}{3+3}\right)^{24pq-14p}$$

$$= (250.765)^p \times (3.5407 \times e^{-12})^{pq}$$

$$(ii) ISI\pi(G) = \prod_{uv \in E(VC_5C_7[p,q])} \frac{d_u d_v}{d_u + d_v}$$

$$= \left(\frac{2 \times 2}{2+2}\right)^p \times \left(\frac{2 \times 3}{2+3}\right)^{10p} \times \left(\frac{3 \times 3}{3+3}\right)^{24pq-14p}$$

$$= (0.0212)^p \times (207.829)^{pq}$$

$$(iii) F\pi(G) = \prod_{uv \in E(VC_5C_7[p,q])} [d_G(u)^2 + d_G(v)^2]$$

$$= [2^2 + 2^2]^p \times [2^2 + 3^2]^{10p} \times [3^2 + 3^2]^{24pq-14p}$$

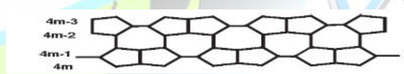
$$= [2.9424 \times e^{-6}]^p \times [18]^{24pq}$$

(c) $HC_5C_7[p,q](p,q > 1)$, Nanotubes

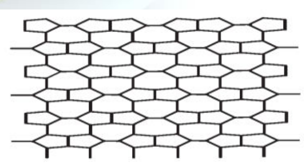
This nanotube is a C_5C_7 net and its two dimensional lattice is constructed by alternating

C_5 and C_7 following the trivalent decoration as shown in Figure 3(b). This tessellation of C_5 and C_7 can either cover a cylinder or a torus also. The 2-dimensional lattice of $HC_5C_7[p,q]$, in which p is the number of heptagons in first row and q is the number of periods in whole lattice.

A period consists of four rows as in Figure 3(a), in which m-th period is shown.



(a) m-th period of HC_5C_7 nanotube



(b) The graph of $HC_5C_7[p,q]$ nanotube with p = 3 and q = 3.

Fig. 3. $HC_5C_7[p,q]$ Nanotubes and m-th period.

3.3. Results for $HC_5C_7[p,q](p,q > 1)$ Nanotubes

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In this section, we compute the Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F index for $HC_5C_7[p,q]$ nanotubes.

Lemma 3.3.1 [1]

Let $HC_5C_7[p,q]$ be the graph of nanotubes with $(p,q) > 1$. There are 16p vertices in one period of the lattice and 2p vertices are joined to the end of the graph, then its the number of vertices is $|V(HC_5C_7[p,q])| = 16pq + 2p$.

Lemma 3.3.2[1]

Consider the graph of $HC_5C_7[p,q]$ nanotubes with $(p,q > 1)$. There are 24p edges in one period and 2p extra edges which are joined to the end of the graph in these nanotubes, then its the number of vertices is $|E(HC_5C_7[p,q])| = 24pq - 2p$.

Theorem 3.3 Consider the graph of $G = HC_5C_7[p,q]$ Nanotubes, then

(i) $H\pi(G) = (0.0746)^p \times (16834.1122)^{pq}$

(ii) $ISI\pi(G) = (1255.2423)^p \times (16834.1122)^q$

(iii) $F\pi(G) = [2.2847 \times e^{-4}]^p \times [18]^{24pq}$

Proof. Let the graph G be $HC_5C_7[p,q]$. Table 4 shows edge partition of graph of $HC_5C_7[p,q]$ nanotubes.

XVI. (d_u, d_v)	XVII. (2,3)	XVIII. (3,3)
XIX. NUMBER OF EDGES	XX. 8P	XXI. 24PQ −10P

Table 3. Edge partition of $HC_5C_7[p,q]$ nanotubes

Hence

$$(i) H\pi(G) = \prod_{uv\in E(HC_5C_7[p,q])} \frac{2}{d_u+d_v}$$

$$= \left(\frac{2}{2+3}\right)^{8p} \times \left(\frac{2}{3+3}\right)^{24pq-10p}$$

$$= (38.698)^p \times (3.5407 \times e^{-12})^{pq}$$

$$(ii) ISI\pi(G) = \prod_{uv\in E(HC_5C_7[p,q])} \frac{d_u d_v}{d_u+d_v}$$

$$= \left(\frac{2\times3}{2+3}\right)^{8p} \times \left(\frac{3\times3}{3+3}\right)^{24pq-10p}$$

$$= (0.0746)^p \times (16834.1122)^{pq}$$

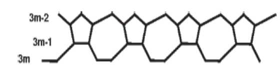
$$(iii) F\pi(G) = \prod_{uv\in E(HC_7[p,q])} [d_G(u)^2 + d_G(v)^2]$$

$$= [2^2 + 3^2]^{8p} \times [3^2 + 3^2]^{24pq-10p}$$

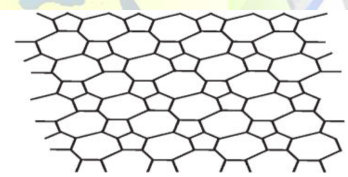
$$= [2.2847 \times e^{-4}]^p \times [18]^{24pq}$$

(d)$SC_5C_7[p,q](p,q > 1)$, Nanotubes

This nanotube is a C_5C_7 net is constructed by alternating C_5 and C_7 following the trivalent decoration as shown in Figure 4. This typical tiling of C_5 and C_7 can either cover a cylinder or a torus too. The $SC_5C_7[p,q]$, in which p is the number of heptagons in each row and q is the number of periods in whole lattice. A period consists of three rows as in Figure 4 in which m-th period is shown.



(a) m-th period of SC_5C_7 nanotube



(b)The graph of $SC_5C_7[p,q]$ nanotube with p = 4 and q = 4.

Fig. 4. $SC_5C_7[p,q]$ Nanotubes and m-th period.

3.4. Results for $SC_5C_7[p,q](p,q > 1)$ Nanotubes

In this section, we compute the Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F index for $SC_5C_7[p,q]$ nanotubes.

Lemma 3.4.1 [1]

Let $SC_5C_7[p,q]$ be the graph of nanotubes with $(p,q) > 1$. There are 8*p* vertices in one period of the lattice, then its the number of vertices is $|V(SC_5C_7[p,q])| = 8pq$.

Lemma 3.4.2 [1]

Consider the graph of $SC_5C_7[p,q]$ nanotubes with $(p,q > 1)$. There are 12*p* edges in one period and 2*p* extra edges which are joined to the end of the graph in these nanotubes, then the number of edges is $|E(SC_5C_7[p,q])| = 12pq - 2p$.

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Theorem 3.4 Consider the graph of

$G = SC_5C_7[p,q]$ nanotubes, then

(i) $H\pi(G) = (40.311)^p \times (1.8817 \times e^{-6})^{pq}$

(ii) $ISI\pi(G) = (0.0777)^p \times (129.7463)^{pq}$

(iii) $F\pi(G) = [1.9467 \times e^{-4}]^p \times [18]^{12pq}$

Proof. Let the graph G be $SC_5C_7[p,q]$. Table 4 shows edge partition of graph of $SC_5C_7[p,q]$ nanotubes.

XXII. (d_u, d_v)	XXIII. (2,2)	XXIV. (2,3)	XXV. (3,3)
XXVI. Number of edges	XXVII. p	XXVIII. 6p	XXIX. 12pq −9p

Table 4. Edge partition of $SC_5C_7[p,q]$ nanotubes based on degrees of end vertices of each edge

Hence

$$(i) H\pi(G) = \prod_{uv\in E(SC_5C_7[p,q])} \frac{2}{d_u+d_v}$$

$$= \left(\frac{2}{2+2}\right)^p \times \left(\frac{2}{2+3}\right)^{6p} \times \left(\frac{2}{3+3}\right)^{12pq-9p} = (40.311)^p \times (1.8817 \times e^{-6})^{pq}$$

$$(ii) ISI\pi(G) = \prod_{uv\in E(SC_5C_7[p,q])} \frac{d_u d_v}{d_u+d_v}$$

$$= \left(\frac{2\times2}{2+2}\right)^p \times \left(\frac{2\times3}{2+3}\right)^{6p} \times \left(\frac{3\times3}{3+3}\right)^{12pq-9p}$$

$(0.0777)^p \times (129.7463)^{pq}$

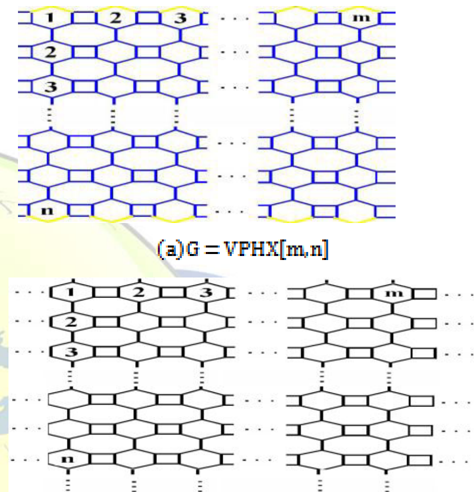
$$(iii) F\pi(G) \prod_{uv\in E(SC_7[p,q])} [d_G(u)^2 + d_G(v)^2]$$

$$= [2^2+2^2]^p \times [2^2+3^2]^{6p} \times [3^2+3^2]^{12pq-9p}$$

$$= [8]^p \times [13]^{6p} \times [18]^{12pq-9p} = [1.9467 \times e^{-4}]^p \times [18]^{12pq}$$

(e) V-Phenylenic Nanotubes and Nanotorus

The structure of these two carbon Nano-structures "V-Phenylenic Nanotubes and Nanotorus" are consist of cycles with length four(C_4), six $(C_6)$and eight(C_8). In other words, in the structure of these nanotubes a $C_4C_6C_8$ net is a trivalent decoration made by alternatingC_4, C_6 andC_8. These can cover either a cylinder or a torus. In Figure 5, a 2-Dimensional lattice of V-Phenylenic Nanotubes $G = VPHX[m,n]$ and V-Phenylenic Nanotorus $H = VPHY[m,n]$ are shown $(\forall m,n > 1)$.



(a)$G = VPHX[m,n]$

(b)$H = VPHY[m,n]$

Fig. 5. A 2D-lattice of of V-Phenylenic Nanotubes and V-Phenylenic Nanotorus

3.5. Results for V-Phenylenic Nanotubes and Nanotorus

In this section, we compute the Multiplicative Harmonic index, Multiplicative ISI index and multiplicative F -index for two carbon Nano-structures.

Definition 3.5. [8] For a connected graph $G = (V(G), E(G))$ with the minimum and maximum of degrees $\delta = Min\{dv|v \in V(G)\}$ and$\Delta = Max\{dv|v \in V(G)\}$ respectively, there exist vertex and edge partitions as follow
$\forall k: \delta \leq k \leq \Delta, Vk = \{v \in V(G)|\ dv = k\}$ $\forall i: 2\delta \leq i \leq 2\Delta, Ei = \{e = uv \in E(G)| du + dv = i\}$ $\forall j: \delta 2 \leq j \leq \Delta 2, Ej* = \{uv \in E(G)| du \times dv = j\}$.

Lemma 3.5.1[1] let $G = VPHX[m,n]$ be the V-Phenylenic Nanotubes and $H = VPHY[m,n]$ be the V-Phenylenic Nanotorus. Let we denote the number of hexagon in the first row of the 2D-lattice of V-Phenylenic G and H by m and alternatively we denote the number of hexagon in the first column by n. From Figure 6, we see that in each period, there are 6 vertices and we have mn repetition.

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Then its number of vertices are $|V(VPHX[m,n])| = |V(VPHY[m,n])| = 6mn$

Lemma 3.5.2[1] Consider the graph of V-Phenylenic Nanotubes $G = VPHX[m,n]$ and V-Phenylenic Nanotorus $H = VPHY[m,n]$ .Since there are m vertices with degree two in the first row and m vertices with degree two in the last row, too and other vertices have degree three, then the number of edges in V-Phenylenic Nanotubes $G = VPHX[m,n]$ $(\forall m,n > 1)$ is equal to

$$|E(VPHX[m,n])| = \frac{2(2m) + 3(6mn - 2m)}{2} = 9mn - m$$

Since all vertices in V-Phenylenic Nanotorus $H = VPHY[m,n]$ $(\forall m,n > 1)$, have degree three, thus, $|E(VPHX[m,n])| = ½ \times 3(6mn) = 9mn.$

Theorem 3.5.1. Let G be the V-Phenylenic Nanotubes $VPHX[m,n]$ for every $m,n \in \mathbb{N} - \{1\}$, then

(i) $H\pi(VPHX[m,n]) = (6.2208)^m \times (5.0805 \times e^{-5})^{mn}$

(ii) $ISI\pi(VPHX[m,n]) = (0.2731)^m \times (38.4484)^{mn}$

(iii) $F\pi(VPHX[m,n]) = [13]^{4m} \times [18]^{9mn-5m}$

Proof.

Consider the grapgh G, the V-Phenylenic Nanotubes $VPHX[m,n]$ with 6mn vertices and 9mn-m edges. By according to the 2-Dimensional lattice of $G = VPHX[m,n]$ in Figure 6(a), we mark all edges from E_5 or E_6^* by yellow color and all members from E_6or E_9^* by blue color.

According to the Definition 3.6, the edge set of V-Phenylenic Nanotubes $G = VPHX[m,n]$ can be dividing to two partitions, e.g. E_5 and E_6, as follow:

•Forevere $= uv$ belong to $E_6, d_u = 2$ and $d_v = 3$ or $E_5 = E_6^* = \{uv \in E(VPHX[m,n])|$

$d_u + d_v = 5$ & $d_u \times d_v = 6\}$
$\therefore |E_5| = |E_6^*| = 2m + 2m$

• For every $e = uv$ belong to $E_5, du = dv = 3$ or $E_6 = E_9^* = \{uv \in E(VPHX[m,n])|$
$d_u + d_v = 6$ & $d_u \times d_v = 9\}$
$\therefore |E_6| = |E_9^*| = 9mn - 5m$. Hence we have

$$(i) H\pi(G) = \prod_{uv\in E(VPHX[m,n])} \frac{2}{d_u+d_v}$$

$$= \left(\frac{2}{2+3}\right)^{4m} \times \left(\frac{2}{3+3}\right)^{9mn-5m}$$

$$= (6.2208)^m \times (5.0805 \times e^{-5})^{mn}$$

$$(ii)\ ISI\pi(G) = \prod_{uv\in E(VPHX[m,n])} \frac{d_u d_v}{d_u+d_v}$$

$$= \left(\frac{2\times3}{2+3}\right)^{4m} \times \left(\frac{3\times3}{3+3}\right)^{9mn-5m}$$

$$= (0.2731)^m \times (38.4484)^{mn}$$

$$(iii) F\pi(G) = \prod_{uv\in E(VPHX[m,n])} [d_G(u)^2 + d_G(v)^2]$$

$$= [2^2 + 3^2]^{4m} \times [3^2 + 3^2]^{9mn-5m}$$

$$= [13]^{4m} \times [18]^{9mn-5m}$$

Theorem 3.5.2. Let H be the V-Phenylenic Nanotorus $VPHY[m,n]$ for every $m,n \in \mathbb{N} - \{1\}$, then

(i) $H\pi(VPHY[m,n]) = (5.0805 \times e^{-5})^{mn}$

(ii) $ISI\pi(VPHY[m,n]) = (38.4434)^{mn}$

(iii) $F\pi(VPHY[m,n]) = [18]^{9mn}$

Proof.

Consider the graph H, the V-Phenylenic Nanotorus $VPHY[m,n]$ with 6mn vertices and 9mn edges. By according to the 2-Dimensional lattice of $H = VPHY[m,n]$ in Figure 6(b), we see that all member of single edge partition E_6 (orE_9^*) are marked by black color, such that in $H = VPHY[m,n](\forall m,n \in \mathbb{N} - \{1\}), |E_6| = |E_9^*| = 9mn = VPHY[m,n]$
$=| E_9^*|$

=|E(VPHY[m,n])|.According to the definition 3.6, all member in edge set of V-Phenylenic Nanotorus VPHY[m,n] exist in a following partition E_6 or E_9^* since all vertices/atoms have degree three $E_6 = E_9^* = \{uv \in E(VPHY[m,n])| du + dv = 6$

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$\& \; du \times dv = 9\} = E(\mathrm{VPHY}[m,n]) = 9mn$

Hence we have

$$\text{(i)} H\pi(G) = \prod_{uv\in E(\mathrm{VPHY}[m,n])} \frac{2}{d_u + d_v}$$

$$= \left(\frac{2}{3+3}\right)^{9mn} = \left(\frac{1}{3}\right)^{9mn} = (5.0805 \times e^{-5})^{mn}$$

$$\text{(ii)} ISI\pi(G) = \prod_{uv\in E(\mathrm{VPHY}[m,n])} \frac{d_u d_v}{d_u + d_v}$$

$$= \left(\frac{3\times3}{3+3}\right)^{9mn} = \left(\frac{3}{2}\right)^{9mn} = (38.4434)^{mn}$$

$$\text{(iii)} F\pi(G) = \prod_{uv\in E(\mathrm{VPHY}[m,n])} [d_G(u)^2 + d_G(v)^2]$$

$$= [3^2 + 3^2]^{9mn} = [18]^{9mn}$$

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