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Theorems on mΓ group of intuitionistic fuzzy ideals in near rings

S K Mala[1] , Dr. MM Shanmugapriya[2]
Assistant Professor of Mathematics, KG College of Arts and Science, Saravanampatti, Coimbatore, India.[1]

Assistant Professor and Head of Department in Mathematics, Karpagam Academy of Higher Education, Coimbatore, India.[2]

Abstract: In this paper we consider Intuitionistic Fuzzy (IF) ideals of MΓ group and some important theorems with their proofs in MΓ group of IF ideals in near rings.

Keywords: MΓ group, IF ideal, IF ideal of MΓ group in near rings.

I. INTRODUCTION

The concept of a fuzzy subset in non-empty set was first introduced by Zadeh [16] in 1965. Since then, many generalisations of this fundamental concept have been developed. The notion of IF set introduced by Atanassov [1] is one among them.

Subsequently, Jun and Lee [9] studied fuzzy Γ rings. Zhan and Davvaz [17] then explained fuzzy ideals of near rings. Bhavanari and Kuncham [5] explained fuzzy cosets of Γ near rings as an extension of that. Then, the isomorphism theorems for IF submodules of G modules was studied elaborately by Sharma and Kaur [15] and they analysed IF co-sets in Γ near rings.

Now in this paper we study IF ideals of MΓ group G and proof of few theorems of IF ideals of MΓ group in near rings.

II. PRELIMINARIES

A. *Definition 2.1 [6]*

Let M be a Γ-near ring. An additive group G is said to be a Γ-near ring module (or MΓ-module) if there exists a mapping $M \times \Gamma \times G \rightarrow G$ (denote the image of (m, α , g) by m α g for m ∈ M, a ∈ Γ, g ∈ G) satiating the conditions

(i) $(m1 + m2)\alpha 1g = m1\alpha 1g + m2\alpha 1g$ and
(ii) $(m1\alpha 1m2)\alpha 2g = m1\alpha 1\ (m2\alpha 2g)$

For all m1, m2 ∈ M, α 1, α 2 ∈ Γ, and g ∈ G.

All through this section, G stands for an MΓ-module.

B. *Definition 2.3[13]*

Let M be a nonempty set. A fuzzy set A in M is characterized by its membership function $\mu A: M \rightarrow [0, 1]$ and the degree of membership of constituent x in fuzzy set A for each x ∈ M is given as μA (x).

C. *Definition 2.4[13]*

Let μ be a fuzzy set in a G - ring M. For any t ∈ [0, 1], then the level set of μ is given as set $U(\mu, t) = \{x \in M \mid \mu(x) \geqslant t\}$.

D. *Definition 2.5[13]*

A fuzzy set μ in a G - ring M is called a fuzzy left (right) ideal of M, if it conforms to following:

(i) $\mu(x - y) \geqslant \mu(x) \wedge \mu(y)$,
(ii) $\mu(x \alpha y) \geqslant \mu(y)$ (resp. $\mu(x \alpha y) \geqslant \mu(x)$.

for all x, y ∈ M and α ∈ Γ. μ is called a fuzzy ideal of M if μ is both a fuzzy left and right ideal of M.

E. *Definition 2.8[6]*

A fuzzy set μ of G is called a fuzzy MΓ-subgroup of G if these two conditions are satisfied:

(i) $\mu(x - y) = \min\{\mu(x), \mu(y)\}$ and
(ii) $\mu(a \alpha y) = \mu(y)$ for all x, y ∈ G, a ∈ M, and α ∈ G.

Here, M represents a gamma near ring, and G denotes an MΓ-group..

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F. Definition 2.9[1]

Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is an object having the form $A = \{[x, \mu A(x), \nu A(x)] / x \in X\}$, where the functions $\mu A: X \to [0, 1]$ and $\nu A: X \to [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A, respectively, and $0 = \mu(x) = 1$ for every $x \in X$.The intuitionistic fuzzy set (IFS in short) is given as $A = \{[x, \mu A(x), \nu A(x)] / x \in X\}$ by $A = (\mu A, \nu A)$.

G. Definition 2.10[15]

Let A be an IF ideal of M and $x \in M$. Then an IF co-set of A is given by the IF subset x + A defined by $(x + \mu A)(y) = \mu A(y - x)$ and $(x + \nu A)(y) = \nu A(y - x)$ for all $y \in M$.

III. THEROEMS ON MΓ GROUP OF INTUITIONISTIC FUZZY IDEALS IN NEAR RINGS

A. Definition 3.1

An IFS $A = (\mu A, \gamma A)$ in M is called an intuitionistic fuzzy left (resp. right) ideal of a Γ - ring M if $\mu A(x-y) \geqslant \mu A(x) \wedge \mu A(y)$ and $\mu A(x \alpha y) \geqslant \mu A(y)$(resp. $\mu A(x \alpha y) \geqslant) \mu A(x)$),

$\gamma A(x-y) \leqslant \{\gamma A(x) \vee \gamma A(y)\}$ and $\gamma A(x\alpha y) \leqslant \gamma A(y)$(resp. $\gamma A(x\alpha y) \leqslant \gamma A(x)$), for all $x,y \in M$ and $\alpha \in \Gamma$.

B. Proposition 3.2

Suppose that G is an MΓ group, A be an IF ideal and $x, y \in G$ then,

$x+\mu A = y+\mu A$ and $x+\gamma A = y+ \gamma A$ if and only if $\mu A(x-y) = \mu A(0)$ and $\gamma A(x-y) = \gamma A(0)$.

C. Theorem 3.3

Let A (μA, γA) and B (μB, γB) are any two intuitionistic fuzzy of G such that $A \subseteq B$ and $\mu A(0) = \mu B(0)$ and $\gamma A(0) = \gamma B(0)$ then the mapping $hB : G/A \to [0,1]$ defined by

$$hB(x + \mu A) = \mu B(x) \text{ and}$$
$$hB(x + \gamma A) = \gamma B(x) \text{ for all } x + A \in G/A$$

is a fuzzy ideal.

Proof

First, we verify that hB is well defined. Let $x + \mu A$, $y+\mu A$, $x+\gamma A$, $y+\gamma A \in G/A$ such that

$$x + \mu A = y + \mu A$$
$$\Rightarrow \mu A(0) = \mu A(x-y) \text{ by 3.2}$$

We have $\mu B(0) \geqslant \mu B(x-y)$ since $B \supset A$
$\geqslant \mu A(x - y)$
$= \mu A(0) = \mu B(0)$ by given data
$= \mu B(x - y)$

i.e. $\mu B(0) = \mu B(x-y)$
$\Rightarrow x + \mu B = y + \mu B$ by 3.2
$\Rightarrow (x + \mu B)(0) = (y + \mu B)(0)$
$\Rightarrow \mu B(0 - x) = \mu B(0 - y)$ by 3.2
$\Rightarrow \mu B(-x) = \mu B(-y)$
$\Rightarrow \mu B(x) = \mu B(y)$ since B is an intuitionistic fuzzy ideal.

Which implies $hB(x + \mu A) = hB(y + \mu A)$
i.e. if $x + \mu A = y + \mu A$ then
$hB(x + \mu A) = hB(y + \mu A)$

similarly if $x + \gamma A = y + \gamma A$ then $hB(x + \gamma A) = hB(y + \gamma A)$.

Now we verify that hB is a intuitionistic fuzzy ideal of G/A.

Consider $x + A$, $y + A$, $a + A \in G/A$ and $m \in M$. Now we check the axioms of an intuitionistic fuzzy ideal for G/A.

(i) $hB[(x + \mu A) - (y + \mu A)] = hB[(x - y) + \mu A]$
$= \mu B(x - y)$ by definition of hB $= \mu B(x + (-y))$
$\geqslant \min\{\mu B(x), \mu B(-y)\}$ since B is an intuitionistic fuzzy ideal.
$= \min\{\mu B(x), \mu B(y)\}$ since $\mu B(y) = \mu B(-y)$
$= \min\{hB(x + \mu A), hB(y + \mu A)\}$ by definition of hB.

Therefore, $hB[(x + \mu A) - (y + \mu A)] \geqslant \min\{hB(x + \mu A), hB(y + \mu A)\}$

Similarly, $hB[(x + \gamma A) - (y + \gamma A)] \leqslant \max\{hB(x + \gamma A), hB(y + \gamma B)\}$

(ii) $hB[(x + \mu A) + (y + \mu A) - (x + \mu A)] = hB[(x + y - x) + \mu A]$
$= \mu B(x + y - x)$ by definition of hB
$\geqslant \mu B(y)$ since B is an intuitionistic fuzzy ideal.
$= hB(y + \mu A)$ by definition of hB.

Therefore, $[(x + \mu A) + (y + \mu A) - (x + \mu A)] \geqslant hB(y + \mu A)$

Similarly, $hB(x + \gamma A) + (y + \gamma A) - (x + \gamma A)] \leqslant hB(y + \gamma A)$.

(iii) $hB(-x + \mu A) = \mu B(-x)$ by definition of hB.
$= \mu B(x)$ since B is a intuitionistic fuzzy ideal.
$= hB(x + \mu A)$

Therefore, $hB(-x + \mu A) = hB(x + \mu A)$.

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Similarly, hB (- x + γA) = hB (- x + γA)

(iv) hB (mγ ((a + μA) + (x + μA)) – mγ (a + μA))

= hB (mγ ((a + x) + μA) – mγ (a + μA)) by definition of addition in G/A.

= hB [(mγ (a + x) – mγa) + μA]

= μB(mγ (a + x) – mγa] by definition of hB.

⩾ μB(x) since B is an intuitionistic fuzzy ideal of G.

= hB (x + μA) by definition of hB.

Therefore, hB [m γ ((a + μA) + (x + μA)) – m γ (a + μA)] ⩾ hB (x + μA)

Similarly, hB [m γ (a + γ A) + (x + γ A) – m γ (a + γ A)] ⩽ hB (x + γ A).

Therefore, by definition of intuitionistic fuzzy ideal, hB is an intuitionistic fuzzy ideal of G/A.

D. *Preposition 3.4*

Let A : G → [0 ,1] is an intuitionistic fuzzy ideal of the MΓ group G.

(i) The mapping ϕ (x) : G → G/A is defined by ϕ (x) = x + A is an onto homomorphism with Ker ϕ = GA = { x ϵ G / μA (x) = μA (0) }.

Hence the MΓ group G/A is isomorphic to the MΓ group G/GA under the mapping

f : G/GA → G / A defined by f (x + GA) = x + A.

(ii) Suppose A and B are two intuitionistic fuzzy ideals of the MΓ group G such that

GA = GB. Then the mapping g : G / A → G / B defined by

g (x + A) = x + B is an isomorphism.

(iii) if G / A ≅ G / B under the isomorphism

g (x + A) = x + B then GA = GB.

E. *Theorem 3.5*

Let X&Y be two nonempty IF sets and Let f be a function of X into Y. Let A be a IF subset of Y. Then f-1 (A),the pre image of A under f is a IF subset of X defined by (f-1(A)) for all x ϵX.

F. *Preposition 3.6*

Let A : G → [0 , 1] is a intuitionistic fuzzy ideal of the MΓ group G.

(i) The mapping ϕ : G → G / A is defined by

Φ (x) = x + A is an onto homomorphism with ker ϕ = GA

= { x ϵ G / μA (x) = μA (0), γA (x) = γA (0) }.

Hence the MΓ group G / A is isomorphic to the MΓ group G / GA under the mapping

f : G / GA →G / A defined by f (x + GA) = x + A .

(ii) Suppose A and B are two intuitionistic fuzzy ideals of the MΓgroup G such that

GA = GB then the mapping g : G / A → G / B defined by g (x + A) = x + B is an isomorphism.

(iii) If G / A ≅ G / B under the isomorphism

g (x + A) = x + B then GA = GB.

Proof

Define ϕ : G →G / A by Φ (x) = x + A for all x ϵ G.

Φ is well defined as x ϵ G are all distinct.

Let x , y ϵ G and m ϵ M . then ϕ (x + y) = (x + y) + A by definition of A .

= (x + A) + (y + A)

= ϕ (x) + ϕ (y)

i.e. ϕ (x + y) = ϕ (x) + ϕ (y).

And ϕ (mαx) = mαx + A for all m ϵ M and α ϵ Γ

= mα (x + A) by definition of G / A.

= mα ϕ (x) by definition of ϕ.

i.e. ϕ (mαx) = mα ϕ (x).Therefore, ϕ is an MΓ group homomorphism.

To prove ϕ is onto,

Consider an element x + A ϵ G / A.

As x ϵ G and by definition of ϕ, we have Φ (x) = x + A

Hence ϕ is onto. So ϕ is an MΓ group epimorphism.

Therefore by fundamental theorem of homomorphism (theorem 3.5)

G / ker ϕ ≅ ϕ (G)Where ϕ (G) = G /A and hence

G / ker ϕ ≅ G \ A.

To prove:

Ker ϕ = GA, let us consider x ϵ ker ϕ

⟹ ϕ (x) = 0 , 0 is the zero element of G / A.

⟹ x + A = 0 + A (=A)

⟹ μA(x) = μA (0) (by preposition 3.2)

⟹x ϵ GA by definition of GA.

Therefore, ker ϕ = GA.

Hence G / GA≅ G / A.

(ii) Given GA = GB⟹ G / GA= G / GB

Using (I) we have

G/ A ≅ G / B under the isomorphism defined by g (x + A) = x + B .

(iii) Assume G / A ≅ G/ B under the isomorphism of defined by g (x + A) = x + B .

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We have to prove that GA = GB.
Let $x \in GA \Longrightarrow \mu A(x) = \mu A(0)$
$\Longrightarrow \mu A(x - 0) = \mu A(0)$ by preposition 3.2
$\Longrightarrow x + A = 0 + a$
Operating g on both sides
$g(x + A) = g(0 + B)$
$\Longrightarrow x + B = 0 + B$
$\Longrightarrow \mu B(x) = \mu B(0)$
$\Longrightarrow x \in GB$ (by definition of GB)
$\Longrightarrow GA \subseteq GB$
Similar proof gives $GB \subseteq GA$ and so GA = GB.

IV. Conclusion

We have considered IF ideals of $M\Gamma$ groups in near rings. We proved few important theorems on IF ideals of $M\Gamma$ group in near rings.

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BIOGRAPHY

Mrs Mala. S.K is currently working as Assistant Profesor, Deaprtment of Mathematics, KG College of Arts and Science, Coimbatore. She received her MSc. and M.Phil. from Madras University. She is currently pursuing her Ph.D in Intuitionistic Fuzzy set from Karpagam University. She has teaching experience of more than 20 years both in college and school. She has published many papers in international journals and national journals. Her area of interest in research includes Functional analysis, Topology and Intuitionistic Fuzzy set. She is one of the author for the book - Mathematical structures for computer science. She was a key participant of ICIFSS 2018.

Dr. M.M. Shanmuga priya is currently working as Assistant Profesor, Deaprtment of Mathematics, Karpagam University, Coimbatore. She did her M.Phil. from Madurai Kamarjar University and Ph.D in Karpagam University. Her major research interest is Fuzzy set. She has intensive experience in lecturing and has been invited as guest lecuturer in many colleges. Her programming skills are her additional qualifictions extending her expertise in computer application oriented fields. She has been Head(i/c) of Mathematics in Karpagam Academy of Higher Education. She is an active researcher and have published many articles in various journals and attended many conferences.