



# Operation on Disjunctive Sum and Difference of Intuitionistic Fuzzy Soft Sets

R. Buvaneswari<sup>1</sup>, A. Nithya<sup>2</sup>

Assistant Professor, Department of Mathematics, KG College of Arts and Science, Coimbatore, India<sup>1</sup>

Research Scholar, Department of Mathematics, KG College of Arts and Science, Coimbatore, India<sup>2</sup>

[buvaneswari.r@kgcas.com](mailto:buvaneswari.r@kgcas.com)<sup>1</sup>, [nithya9517@gmail.com](mailto:nithya9517@gmail.com)<sup>2</sup>

**Abstract:** In this paper, disjunctive sum and difference of two intuitionistic fuzzy soft sets are defined. Properties of disjunctive sum and difference are studied.

**Keywords:** Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Soft Set, Disjunctive Sum, Difference.

## I. Introduction

Most of the real life problems have various uncertainties. The theory of probability, evidence theory, rough set theory etc. are mathematical tools to deal with such problems. In 1999, Molodtsov[8] introduced the theory of soft set and established the fundamental results related to this theory. In comparison, this theory can be seen free from the inadequacy of parameterization tool. It is a general mathematical tool for dealing with problems in the fields of social science, economics, medical sciences etc. In 2003, the authors in [5] studied the theory of soft sets initiated by Molodtsov[8] defined equality of two soft sets, subset and super sets of a soft set. Soft binary operations like AND, OR also the operations of union, intersection were also defined by the same author. In recent times, researchers have contributed a lot towards fuzzification of soft set theory. Combining fuzzy sets with soft sets, Maji et al. introduced the notion of fuzzy soft sets in [4]. The authors studied some properties regarding fuzzy soft set such as union, intersection etc. These results further revised and improved by Ahmad and Kharal in [1]. The authors defined operations and laws on arbitrary fuzzy soft set. Maji et al. [6] extended soft sets to intuitionistic fuzzy soft sets. Intuitionistic fuzzy soft set theory is a combination of soft sets and intuitionistic fuzzy soft sets initiated by Atanassov. In [9], Neog and Sut have defined disjunctive sum and difference of two fuzzy soft sets. The notions of  $\alpha$ -cut soft set and  $\alpha$ -cut strong soft set of a fuzzy soft set have been put forward in their work. In [7], Manoj et al. extended the same properties defined by the authors in [9] to intuitionistic fuzzy soft sets. In [3], Karunambigai and Parvathi introduced new definition for intuitionistic fuzzy graph and discussed some of their properties. In this paper, the authors defined disjunctive sum and difference of two intuitionistic fuzzy soft sets and

developed some properties on intuitionistic fuzzy soft sets.

## II. Preliminaries

### Definition 2.1[7]

A pair  $(F, E)$  is called a soft set (over  $U$ ) if and only if  $F$  is a mapping  $E$  into the set of all subsets of the set  $U$ .

In other words, the soft set is a parameterized family of subsets of the set  $U$ . Every set  $F(\varepsilon)$ ,  $\varepsilon \in E$ , from this family may be considered as the set of  $\varepsilon$ -elements of the set  $(F, E)$ , are as the set of  $\varepsilon$ -approximate elements of the soft set.

### Definition 2.2[7]

An intuitionistic fuzzy set  $A$  over the universe  $U$  can be defined as follows-  $A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$ , where  $\mu_A(x) : U \rightarrow [0, 1]$ ,  $\nu_A(x) : U \rightarrow [0, 1]$  with the property  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for all  $x \in U$ . The values  $\mu_A(x)$  and  $\nu_A(x)$  represent the degree of membership and non-membership of  $x$  to  $A$  respectively.

$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$  is called the intuitionistic fuzzy hesitation index.

### Definition 2.3[7]

Let  $U$  be an initial universe set and  $E$  be the set of parameters. Let  $IF^U$  denote the collection of all intuitionistic fuzzy subsets of  $U$ . Let  $A \subseteq E$ . A pair  $(F, A)$  is called an intuitionistic fuzzy soft set over  $U$  where  $F$  is a mapping given by  $F : A \rightarrow IF^U$ .

### Definition 2.4[7]

A soft set  $(F, A)$  over  $U$  is said to be null intuitionistic fuzzy soft set denoted by  $\phi$  if for



all  $\varepsilon \in A$ ,  $F(\varepsilon)$  is the null intuitionistic fuzzy set  $\bar{0}$  of  $U$  where  $\bar{0}(x) = 0$  for all  $x \in U$ .

Notation:  $(F, \phi)$  is to represent the intuitionistic fuzzy soft null set with respect to the set of parameters  $A$ .

**Definition 2.5[7]**

A soft set  $(F, A)$  over  $U$  is said to be absolute intuitionistic fuzzy soft set denoted by  $\tilde{A}$  if for all  $\varepsilon \in A$ ,  $F(\varepsilon)$  is the absolute intuitionistic fuzzy set  $\bar{1}$  of  $U$  where  $\bar{1}(x) = 1$  for all  $x \in U$ . Notation:  $(U, A)$  is to represent the intuitionistic fuzzy soft null set with respect to the set of parameters  $A$ .

**Definition 2.6[7]**

For two intuitionistic fuzzy soft sets  $(F, A)$  and  $(G, B)$  over  $(U, E)$  we say that  $(F, A)$  is an intuitionistic fuzzy soft subset of  $(G, B)$ ,

- (i)  $A \subseteq B$
- (ii) For all,  $\varepsilon \in A$ ,  $F(\varepsilon) \subseteq G(\varepsilon)$  and is written as  $(F, A) \subseteq (G, B)$ .

**Definition 2.7[7]**

Union of two intuitionistic fuzzy soft sets  $(F, A)$  and  $(G, B)$  over  $(U, E)$  is an intuitionistic fuzzy soft set  $(H, C)$  where  $C = A \cup B$  and for all  $\varepsilon \in C$ ,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ H(\varepsilon), & \text{if } \varepsilon \in A \cup B \end{cases}$$

and is written as  $(F, A) \cup (G, B) = (H, C)$ .

**Definition 2.8[7]**

Let  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ . Then intersection  $(F, A)$  and  $(G, B)$  is an intuitionistic fuzzy soft set  $(H, C)$  where  $C = A \cap B$  and for all  $\varepsilon \in C$ ,  $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$ . We write  $(F, A) \cap (G, B) = (H, C)$ .

**Definition 2.9[7]**

Let  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets in a soft class  $(U, E)$  with  $A \cap B \neq \phi$ . Then intersection of two fuzzy soft  $(F, A)$  and  $(G, B)$  in a soft class  $(U, E)$  is a fuzzy soft set  $(H, C)$  where

$C = A \cap B$  and for all  $\varepsilon \in C$ ,  $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$ . We write  $(F, A) \cap (G, B) = (H, C)$ .

**Definition 2.10[7]**

The complement of an intuitionistic fuzzy soft set  $(F, A)$  is denoted by  $(F, A)^c$  and is defined by  $(F, A)^c = (F^c, A)$ , where  $F^c : A \rightarrow IF^U$  is a mapping given by  $F^c(\varepsilon) = [F(\varepsilon)]^c$  for all  $\varepsilon \in A$ . Thus if  $F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U\}$ , then for all  $\varepsilon \in A$ ,  $F^c(\varepsilon) = [F(\varepsilon)]^c = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U\}$

**Definition 2.11[7]**

If  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets, then " $(F, A)$  AND  $(G, B)$ " is an intuitionistic fuzzy soft set denoted by  $(F, A) \wedge (G, B)$  and is defined by  $(F, A) \wedge (G, B) = (H, A \times B)$ , where  $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ , for all  $\alpha \in A$  and for all  $\beta \in B$ , where  $\cap$  is the intersection of two intuitionistic fuzzy sets.

**Definition 2.12[7]**

If  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets, then " $(F, A)$  OR  $(G, B)$ " is an intuitionistic fuzzy soft set denoted by  $(F, A) \vee (G, B)$  and is defined by  $(F, A) \vee (G, B) = (K, A \times B)$ , where  $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$ , for all  $\alpha \in A$  and for all  $\beta \in B$ , where  $\cup$  is the union of two intuitionistic fuzzy sets.

### III. Disjunctive of Intuitionistic Fuzzy Soft Sets

**Definition 3.1**

Let  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ . The disjunctive sum of  $(F, A)$  and  $(G, B)$  is defined as the intuitionistic fuzzy soft set  $(H, C)$  over  $(U, E)$ . In symbol  $(F, A) \oplus (G, B) = (H, C)$ , where

$$C = A \cap B \neq \phi \text{ and for all } \varepsilon \in C, x \in U, \\ \mu_{H(\varepsilon)}(x) = \min(\max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))) \\ \nu_{H(\varepsilon)}(x) = \max(\min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)))$$

**Example 3.2**

Let  $U = \{a, b, c\}$  and  $E = \{e_1, e_2, e_3, e_4\}$ ,  $A = \{e_1, e_2, e_4\} \subseteq E$ ,  $B = \{e_1, e_2, e_3\} \subseteq E$

$$(F, A) = \begin{cases} F(e_1) = \{(a, 0.2, 0.6), (b, 0.5, 0.4), (c, 0.8, 0.1)\} \\ F(e_2) = \{(a, 0.7, 0.1), (b, 0.2, 0.3), (c, 0.1, 0.3)\} \\ F(e_4) = \{(a, 0.6, 0.3), (b, 0.5, 0.2), (c, 0.4, 0.2)\} \end{cases}$$



$$(G, B) = \begin{cases} G(e_1) = \{(a, 0.4, 0.2), (b, 0.1, 0.3), (c, 0.7, 0.2)\} \\ G(e_2) = \{(a, 0.1, 0.4), (b, 0.2, 0.2), (c, 0, 0.5)\} \\ G(e_3) = \{(a, 0.5, 0.2), (b, 0.4, 0.3), (c, 0.2, 0.6)\} \end{cases}$$

Then  $(F, A) \tilde{\oplus} (G, B) = (H, C)$  where  $C = A \cap B = \{e_1, e_2\}$  and

$$(H, C) = \begin{cases} H(e_1) = \left\{ \begin{aligned} &(a, \min(\max(0.2, 0.2), \max(0.6, 0.4)), \max(\min(0.6, 0.4), \min(0.2, 0.2))) \\ &(b, \min(\max(0.5, 0.3), \max(0.4, 0.1)), \max(\min(0.4, 0.1), \min(0.5, 0.3))) \\ &(c, \min(\max(0.8, 0.2), \max(0.1, 0.7)), \max(\min(0.1, 0.7), \min(0.8, 0.2))) \end{aligned} \right\} \\ H(e_2) = \left\{ \begin{aligned} &(a, \min(\max(0.7, 0.4), \max(0.1, 0.1)), \max(\min(0.7, 0.4), \min(0.1, 0.1))) \\ &(b, \min(\max(0.2, 0.2), \max(0.3, 0.2)), \max(\min(0.3, 0.2), \min(0.2, 0.2))) \\ &(c, \min(\max(0.1, 0.5), \max(0.3, 0)), \max(\min(0.3, 0), \min(0.1, 0.5))) \end{aligned} \right\} \end{cases}$$

$$(H, C) = \begin{cases} H(e_1) = \left\{ \begin{aligned} &(a, \min(0.2, 0.6), \max(0.4, 0.2)) \\ &(b, \min(0.5, 0.4), \max(0.1, 0.3)) \\ &(c, \min(0.8, 0.7), \max(0.1, 0.2)) \end{aligned} \right\} \\ H(e_2) = \left\{ \begin{aligned} &(a, \min(0.7, 0.1), \max(0.4, 0.1)) \\ &(b, \min(0.2, 0.2), \max(0.3, 0.2)) \\ &(c, \min(0.5, 0.3), \max(0, 0.1)) \end{aligned} \right\} \end{cases}$$

$$(H, C) = \begin{cases} H(e_1) = \{(a, 0.2, 0.4), (b, 0.4, 0.3), (c, 0.7, 0.2)\} \\ H(e_2) = \{(a, 0.1, 0.4), (b, 0.2, 0.3), (c, 0.3, 0)\} \end{cases}$$

**Proposition 3.3**

Let  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ . Then  $(F, A) \tilde{\oplus} (G, B) = (G, B) \tilde{\oplus} (F, A)$

**Proof**

Given that  $(F, A)$  and  $(G, B)$  are two intuitionistic fuzzy soft sets.

$$(F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(G, B) = \{ \langle x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in B \}$$

By definition 3.1

Let  $(F, A) \tilde{\oplus} (G, B) = (H, C)$ , where  $C = A \cap B$  and for all  $\varepsilon \in C, x \in U$ ,

$$\mu_{H(\varepsilon)}(x) = \min(\max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)))$$

$$\nu_{H(\varepsilon)}(x) = \max(\min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))) \rightarrow (1)$$

By definition 3.1

Let  $(G, B) \tilde{\oplus} (F, A) = (H, C)$ , where  $D = A \cap B$  and  $\forall \varepsilon \in D, x \in U$ ,

$$\mu_{H(\varepsilon)}(x) = \min(\max(\mu_{G(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)), \max(\nu_{G(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)))$$

$$\nu_{H(\varepsilon)}(x) = \max(\min(\nu_{G(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)), \min(\mu_{G(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)))$$

From (1) and (2), It is verified.

$$\therefore (F, A) \tilde{\oplus} (G, B) = (G, B) \tilde{\oplus} (F, A)$$

**Proposition 3.4**

Let  $(F, A)$ ,  $(G, B)$  and  $(H, C)$  be three intuitionistic fuzzy soft sets over  $(U, E)$ . Then

$$(F, A) \tilde{\oplus} ((G, B) \tilde{\oplus} (H, C)) = ((F, A) \tilde{\oplus} (G, B)) \tilde{\oplus} (H, C)$$

**Proof**

Given that  $(F, A)$ ,  $(G, B)$  and  $(H, C)$  are three intuitionistic fuzzy soft sets over  $(U, E)$ .

$$(F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(G, B) = \{ \langle x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in B \}$$

$$(H, C) = \{ \langle x, \mu_{H(\varepsilon)}(x), \nu_{H(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in C \}$$

Now first let us consider  $(G, B) \tilde{\oplus} (H, C) = (I, D)$ , where  $D = A \cap C$  and for all  $\varepsilon \in D, x \in U$ ,

$$(I, D) = \begin{cases} \mu_{I(\varepsilon)}(x) = \min(\max(\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)), \max(\nu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))) \\ \nu_{I(\varepsilon)}(x) = \max(\min(\nu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)), \min(\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))) \end{cases} \rightarrow (1)$$

$$\text{Therefore } (I, D) = \{ \langle x, \mu_{I(\varepsilon)}(x), \nu_{I(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in D \}$$

By definition 3.1

Let us consider  $(F, A) \tilde{\oplus} (I, D) = (J, E)$ , where  $E = A \cap D$  and for all  $\varepsilon \in E, x \in U$ ,

From the above  $(J, E)$  becomes

$$(J, E) = \begin{cases} \mu_{J(\varepsilon)}(x) = \min(\max(\mu_{F(\varepsilon)}(x), \nu_{I(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \mu_{I(\varepsilon)}(x))) \\ \nu_{J(\varepsilon)}(x) = \max(\min(\nu_{F(\varepsilon)}(x), \mu_{I(\varepsilon)}(x)), \min(\mu_{F(\varepsilon)}(x), \nu_{I(\varepsilon)}(x))) \end{cases} \rightarrow (2)$$

Now let us consider  $(F, A) \tilde{\oplus} (G, B) = (K, M)$ , where

$M = A \cap B$  and  $\forall \varepsilon \in M, x \in U$ ,

From the above  $(K, M)$  becomes

$$(K, M) = \begin{cases} \mu_{K(\varepsilon)}(x) = \min(\max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))) \\ \nu_{K(\varepsilon)}(x) = \max(\min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))) \end{cases} \rightarrow (3)$$

Now let us consider  $(K, M) \tilde{\oplus} (H, C) = (L, N)$ , where

$N = M \cap C$  and  $\forall \varepsilon \in N, x \in U$

From the above  $(L, N)$  becomes

$$(L, N) = \begin{cases} \mu_{L(\varepsilon)}(x) = \min(\max(\mu_{K(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)), \max(\nu_{K(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))) \\ \nu_{L(\varepsilon)}(x) = \max(\min(\nu_{K(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)), \min(\mu_{K(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))) \end{cases} \rightarrow (4)$$

From (2) and (4), It is verified.

$$(J, E) = (L, N)$$

$$\therefore (F, A) \tilde{\oplus} ((G, B) \tilde{\oplus} (H, C)) = ((F, A) \tilde{\oplus} (G, B)) \tilde{\oplus} (H, C)$$





**Proposition 3.5**

Let  $(F, A)$  and  $(\phi, A)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ .  $(F, A) \tilde{\oplus} (\phi, A) = (F, A)^c$

**Proof**

Given that  $(F, A)$  and  $(\phi, A)$  are two intuitionistic fuzzy soft sets.

$$(F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(\phi, A) = \{ \langle 0, 1 \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

Let  $(F, A) \tilde{\oplus} (\phi, A) = (H, A)$  where for all  $\varepsilon \in A, x \in U$ , we have

$$\mu_{H(\varepsilon)}(x) = \min(\max(\mu_{F(\varepsilon)}(x), \nu_{\phi(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \mu_{\phi(\varepsilon)}(x)))$$

$$= \min(\max(\mu_{F(\varepsilon)}(x), 1), \max(\nu_{F(\varepsilon)}(x), 0))$$

$$= \min(1, \nu_{F(\varepsilon)}(x))$$

$$= \nu_{F(\varepsilon)}(x)$$

$$\nu_{H(\varepsilon)}(x) = \max(\min(\nu_{F(\varepsilon)}(x), \mu_{\phi(\varepsilon)}(x)), \min(\mu_{F(\varepsilon)}(x), \nu_{\phi(\varepsilon)}(x)))$$

$$= \max(\min(\nu_{F(\varepsilon)}(x), 0), \min(\mu_{F(\varepsilon)}(x), 1))$$

$$= \max(0, \mu_{F(\varepsilon)}(x))$$

$$= \mu_{F(\varepsilon)}(x)$$

$$\therefore (H, A) = \{ \langle \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x) \rangle, \forall \varepsilon \in A, x \in U \}$$

$$= (F, A)^c$$

Consequently,

$$(F, A) \tilde{\oplus} (\phi, A) = (F, A)^c$$

Hence the Proof

**Proposition 3.6**

Let  $(F, A)$  and  $(U, A)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ .  $(F, A) \tilde{\oplus} (U, A) = (F, A)$

**Proof**

Given that  $(F, A)$  and  $(U, A)$  are two intuitionistic fuzzy soft sets.

$$(F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(U, A) = \{ \langle 1, 0 \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

Let  $(F, A) \tilde{\oplus} (U, A) = (H, A)$  where for all  $\varepsilon \in A, x \in U$ , we have

$$\mu_{H(\varepsilon)}(x) = \min(\max(\mu_{F(\varepsilon)}(x), \nu_{U(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x)))$$

$$= \min(\max(\mu_{F(\varepsilon)}(x), 0), \max(\nu_{F(\varepsilon)}(x), 1))$$

$$= \min(\mu_{F(\varepsilon)}(x), 1) = \mu_{F(\varepsilon)}(x)$$

$$\nu_{H(\varepsilon)}(x) = \max(\min(\nu_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x)), \min(\mu_{F(\varepsilon)}(x), \nu_{U(\varepsilon)}(x)))$$

$$= \max(\min(\nu_{F(\varepsilon)}(x), 1), \min(\mu_{F(\varepsilon)}(x), 0))$$

$$= \max(\nu_{F(\varepsilon)}(x), 0) = \nu_{F(\varepsilon)}(x)$$

$$\therefore (H, A) = \{ \langle \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall \varepsilon \in C = A, x \in U \}$$

$$= (F, A)$$

Consequently,

$$(F, A) \tilde{\oplus} (U, A) = (F, A)$$

Hence the Proof

**IV. Difference of intuitionistic fuzzy soft sets**

**Definition 4.1**

Let  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ . We define the difference of  $(F, A)$  and  $(G, B)$  as the intuitionistic fuzzy soft set  $(H, C)$  over  $(U, E)$ , written as  $(F, A) \tilde{\ominus} (G, B) = (H, C)$ , where  $C = A \cap B \neq \emptyset$  and for all  $\varepsilon \in C, x \in U$ ,

$$\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$$

$$\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x))$$

**Example 4.2**

Let  $U = \{a, b, c\}$  and  $E = \{e_1, e_2, e_3, e_4\}$ ,  $A = \{e_1, e_2, e_4\} \subseteq E$ ,

$$B = \{e_1, e_2, e_3\} \subseteq E$$

$$(F, A) = \left\{ \begin{array}{l} F(e_1) = \{ \langle a, 0.2, 0.6 \rangle, \langle b, 0.5, 0.4 \rangle, \langle c, 0.8, 0.1 \rangle \} \\ F(e_2) = \{ \langle a, 0.7, 0.1 \rangle, \langle b, 0.2, 0.3 \rangle, \langle c, 0.1, 0.3 \rangle \} \\ F(e_4) = \{ \langle a, 0.6, 0.3 \rangle, \langle b, 0.5, 0.2 \rangle, \langle c, 0.4, 0.2 \rangle \} \end{array} \right\}$$

$$(G, B) = \left\{ \begin{array}{l} G(e_1) = \{ \langle a, 0.4, 0.2 \rangle, \langle b, 0.1, 0.3 \rangle, \langle c, 0.7, 0.2 \rangle \} \\ G(e_2) = \{ \langle a, 0.1, 0.4 \rangle, \langle b, 0.2, 0.2 \rangle, \langle c, 0, 0.5 \rangle \} \\ G(e_3) = \{ \langle a, 0.5, 0.2 \rangle, \langle b, 0.4, 0.3 \rangle, \langle c, 0.2, 0.6 \rangle \} \end{array} \right\}$$

Then  $(F, A) \tilde{\ominus} (G, B) = (H, C)$  where  $C = A \cap B = \{e_1, e_2\}$  and

$$(H, C) = \left\{ \begin{array}{l} H(e_1) = \left\{ \begin{array}{l} \langle a, \max(0.2, 0.2), \min(0.6, 0.4) \rangle \\ \langle b, \max(0.5, 0.3), \min(0.4, 0.1) \rangle \\ \langle c, \max(0.8, 0.2), \min(0.1, 0.7) \rangle \end{array} \right\} \\ H(e_2) = \left\{ \begin{array}{l} \langle a, \max(0.7, 0.4), \min(0.1, 0.1) \rangle \\ \langle b, \max(0.2, 0.2), \min(0.3, 0.2) \rangle \\ \langle c, \max(0.1, 0.5), \min(0.3, 0) \rangle \end{array} \right\} \end{array} \right\}$$

$$(H, C) = \left\{ \begin{array}{l} H(e_1) = \{ \langle a, 0.2, 0.4 \rangle, \langle b, 0.5, 0.1 \rangle, \langle c, 0.8, 0.1 \rangle \} \\ H(e_2) = \{ \langle a, 0.7, 0.1 \rangle, \langle b, 0.2, 0.2 \rangle, \langle c, 0.5, 0 \rangle \} \end{array} \right\}$$



### Preposition 4.3

Let  $(F, A)$  and  $(U, A)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ . Then  $(F, A) \tilde{\Theta}(\phi, A) = (U, A)$

#### Proof

Given that  $(F, A)$  and  $(\phi, A)$  are two intuitionistic fuzzy soft sets.

$$(F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(\phi, A) = \{ \langle 0, 1 \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

Let  $(F, A) \tilde{\Theta}(\phi, A) = (H, A)$  where  $\forall \varepsilon \in A, x \in U$ , we have

$$\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \nu_{\phi(\varepsilon)}(x))$$

$$= \max(\mu_{F(\varepsilon)}(x), 1) = 1$$

$$\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \mu_{\phi(\varepsilon)}(x))$$

$$= \min(\nu_{F(\varepsilon)}(x), 0)$$

$$= 0$$

$$\therefore (H, A) = \{ \langle 1, 0 \rangle, \forall \varepsilon \in A, x \in U \}$$

$$= (U, A)$$

Consequently,

$$(F, A) \tilde{\Theta}(\phi, A) = (U, A)$$

Hence the Proof

### Proposition 4.4

Let  $(F, A)$  and  $(U, A)$  be two intuitionistic fuzzy soft sets over  $(U, E)$ . Then  $(F, A) \tilde{\Theta}(U, A) = (F, A)$

#### Proof

Given that  $(F, A)$  and  $(U, A)$  are two intuitionistic fuzzy soft sets.

$$(F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(U, A) = \{ \langle 1, 0 \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

Let  $(F, A) \tilde{\Theta}(U, A) = (H, A)$  where  $\forall \varepsilon \in A, x \in U$ , we have

$$\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \nu_{U(\varepsilon)}(x))$$

$$= \max(\mu_{F(\varepsilon)}(x), 0) = \mu_{F(\varepsilon)}(x)$$

$$\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x))$$

$$= \min(\nu_{F(\varepsilon)}(x), 1)$$

$$= \nu_{F(\varepsilon)}(x)$$

$$\therefore (H, A) = \{ \langle \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall \varepsilon \in A, x \in U \}$$

$$= (F, A)$$

Consequently,

$$(F, A) \tilde{\Theta}(U, A) = (F, A)$$

Hence the Proof

### V. Conclusion

In this paper, operations on disjunctive sum and difference of intuitionistic fuzzy soft sets are defined and verified some important properties. The significance of this work is to develop theoretical aspects in intuitionistic fuzzy soft sets. Further, the author proposed to continue this research in theory as well as its applications.

### Reference

- [1] B. Ahmad and A. Kharal, "On Fuzzy Soft Sets", *Advances in Fuzzy Systems*, Volume 2009, pp. 1-6, 2009.
- [2] K. Atanassov, "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems* 20 (1986), 87-96
- [3] M. G. Karunambigai and R. Parvathi, "Intuitionistic Fuzzy Graph", *Proceedings of 9<sup>th</sup> Fuzzy Days International Conference Intelligence, Advances in soft computing: Computational intelligence; Theory and Applications*, Springer-Verlag, 20 (2006), 139-150
- [4] P. K. Maji, R. Biswas and A. R. Roy, "Fuzzy Soft Sets", *Journal of Fuzzy Mathematics*, Vol 9, No. 3, pp. 589-602, 2001.
- [5] P. K. Maji and A. R. Roy, "Soft Set Theory", *Computers and Mathematics with Applications* 45 (2003) 555 – 562.
- [6] P. K. Maji, R. Biswas, A. R. Roy, "Intuitionistic fuzzy soft sets", *The journal of fuzzy mathematics* 9(3)(2001), 677-692
- [7] Manoj Bora, Tridiv Jyoti Neog, and Dusmanta Kumar Sut, "Some New Operations of Intuitionistic Fuzzy Soft Sets", *International Journal of Soft Computing and Engineering (IJSCSE)*, Volume-2, Issue-4, September 2012, ISSN: 2231-2307.
- [8] D. A. Molodtsov, "Soft Set Theory – First Result", *Computers and Mathematics with Applications*, Vol. 37, pp. 19-31, 1999.