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Numerical Solutions Of Fuzzy Differential Equations By Comparison Of Adam's Fifth Order Predictor Corrector Method And Runge-Kutta Method Of Order Five

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Abstract: In this paper, we interpret a fuzzy differential equations by using Seikkala derivative of fuzzy process. We investigate the problem of finding a numerical approximation of solutions. Adam's fifth order predictor-corrector method and Runge-Kutta method of order five are implemented and their analysis which guarantees pointwise convergence is discussed. These methods are illustrated by solving example. Finally, we compare the solutions obtained by Adam's fifth order predictor corrector and Runge-Kutta method of order five.

Keywords:Fuzzy differential equations, Fuzzy sets, Runge-Kutta fifth order, Adam's fifth order predictor corrector method.

I. Introduction

Fuzzy differential equations (FDE's) are utilized for the purpose of the modeling problems in science and engineering. The most of the problems in science and engineering require the solutions of a fuzzy differential equation(FDE) which are satisfied in Fuzzy initial conditions. Therefore a fuzzy initial value problem is occurs, and should be solved. It is too complicated to obtain the exact solution of FDE which models the mentioned problem. The concept of fuzzy derivative was first introduced by Chang Zadeh in [7], it was followed up by Duboi's, prade in [8], who defined and used the extension principle. The fuzzy differential equation and the initial value problem where regularly treated by Kaleva in [9],[10] and by Seikkala in [11]. The numerical method for solving fuzzy differential is introduced by Ma, Friedman and Kandel in [12].

In this paper, we develop numerical solution of fuzzy differential equation by an application of the fifth order predictor corrector method.

II. DEFINITIONS AND BASIC PROPERTIES

A. *Fuzzy sets*

The idea of fuzzy set was introduced by Lotfi Zadeh in 1965 as a means of handling uncertainty that is due to imprecision or vagueness rather than to randomness. Fuzzy sets were taken up with interests by engineers, computer scientists and operations researchers. While mathematicians have been involved with the development of fuzzy sets from the very beginning, it has really been in recent years only that fuzzy sets have started receiving serious consideration from a wider mathematical community. Many interesting mathematical problems are coming and the mathematical foundations of the subject are firmly established and now it has emerged as an independent branch of applied Fuzzy sets are considered with respect to a nonempty base set X of elements of interest. The essential idea is that each element $x \in X$ is assigned a membership grade u(x) taking values in [0,1], with u(x) = 0 corresponding to non-membership, 0 <

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u(x) < 1 to partial membership, and = 1 to full membership. According to Zadeh a fuzzy subset of X is a nonempty subset $\{(x, u(x)) : x \in X\}$ of $X \times [0,1]$ for some function $u: X \to [0,1]$. The function u itself used for the fuzzy set.

B. Fuzzy Cauchy Problem

Consider the first order fuzzy differential equation y' = f(t, y) where y is a fuzzy function of f(t, y) is a fuzzy function of t, f(t, y) is a fuzzy function of crisp variable y, and y' is Hukuhara or Seikala fuzzy derivative of y. If an initial value $y(t_0) = \alpha_0$ is given, a fuzzy Cauchy problem of first order will be obtained as follows:

$$y'(t) = f(t, y(t)), t_0 \le t \le t,$$
$$\tilde{y}(t_0) = \tilde{\alpha}_0,$$

Sufficient conditions for the existence of a unique solution to equation 1 are

(i) Continuity of f,
(ii) Lipschitz condition

$$d_\infty(f(t,x), f(t,y)) \le L d_\infty(x,y), L > 0.$$

C. Adam's-Bashforth Five Step Method

$$y_1 = \alpha_1, y_2 = \alpha_2, y_3 = \alpha_3, y_4 = \alpha_4$$

$$y_{i+1} = y_i + \frac{h}{720}[1901 f(t_{i+1}, y_{i+1}) - 2774 f(t_{i-1}, y_{i-1}) + 2664(f(t_{i-2}, y_{i-2}) - 1274 f(t_{i-3}, y_{i+1}) + 251 f(t_{i-4}, y_{i-4}),$$

where i= 4,5,….N-1.

D. Adam's-Moulton Four Step Method

$$y_0 = \alpha_0, y_1 = \alpha_1, y_2 = \alpha_2, y_3 = \alpha_3,$$

$$y_{i+1} = y_i + \frac{h}{720}[251 f(t_{i+1}, y_{i+1}) + 646 f(t_i, y_i) - 264(f(t_{i-1}, y_{i-1}) + 106 f(t_{i-2}, y_{i-2}) - 19 f(t_{i-3}, y_{i-3}),$$

where *i=4,………,N-1.*

E. Definition

Associated with the difference equation

$$y_{i+1} = a_{m-1} y_i + a_{m-2} y_{i-1} + \mathrm{K} + a_0 y_{i+1-m} + hF(t_i, h, y_{i+1}, y_i, \mathrm{K}, y_{i+1-m}),$$
$$y_0 = \alpha, y_1 = \alpha_1, \mathrm{K}, y_{m-1} = a_{m-1},$$

is a polynomial, called the characteristic polynomial of the method given by

$$p(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \mathrm{K}, a_1\lambda - a_0.$$

If $|\lambda_i| \le 1$ for each i=1,2,3,………,m, and all roots with absolute value 1 are simple roots, then the difference method is said to satisfy the root condition.

F. Interpolation of Fuzzy Number

The problem of interpolation for fuzzy sets is as follows: Suppose that at various time instant t information f(t) is presented as fuzzy set. The aim is to approximate the function f(t) , for all t in the domain of f.

Let $t_0 < t_1 < \mathrm{K} < t_n$ be n+1 distinct points in R and let $u_0, u_1, \mathrm{K}\,\mathrm{K}, u_n$ be n+1 fuzzy sets in E.

A fuzzy polynomial interpolation of the data is a fuzzy value continuous function f: R → E satisfying:

(i) $f(t_i) = \tilde{u}_i$, i= 1,…..,n.
(ii) If the data is crisp, then the interpolation f is a crisp polynomial.

A function f which fulfilling these condition may be constructed as follows.

Let $C^i_\infty = [u_i]$ for any $\alpha \in [0,1]$, i=0,1,2,….,n. For each $x = (x_0, x_1, \mathrm{K}, x_n) \in R^{n+1}$ ∟ the unique polynomial of degree ≤ n denoted by P_X such that

$$P_{X_i}(t) = x_i, i = 1,2,....5.$$

$$P_X(t) = \sum_{i=0}^{n} x_i \left(\prod_{i \ne j} \frac{t - t_j}{t_i - t_j} \right).$$

Finally, for each $t \in R$ and all $\xi \in R$ is defined by $f(t) \in E$ by

$$(f(t))(\xi) = \sup\{\alpha \in [0,1] : \exists X \in C^0_\alpha \times \dots\dots\dots \times C^n_\alpha$$

such that $P_X(t) = \xi$ }.

The interpolation polynomial can be written level set wise as

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$$[f(t)]^{\alpha} = \{ y \in R : Y = P_X(t), \quad x \in [u_i]^{\alpha}, i = 1,2,....,n \},$$

for $0 \le \alpha \le 1$.

When the data u_i presents as triangular fuzzy numbers, values of the interpolation polynomial are triangular fuzzy numbers. Then f(t) has a particular simple form that is well situated to computation.

III. ADAM'S METHOD

A. *Adam's-Bashforth methods*

Now we are going to solve fuzzy initial value problem $y'(t) = f(t, y(t))$ by Adams-Bashforth five step method. Let the fuzzy initial values be

i.e. $\tilde{f}(t_{i-1}, y(t_{i-1})), \tilde{f}(t_i, y(t_i)), \tilde{f}(t_{i+1}, y(t_{i+1})), \tilde{f}(t_{i+2}, y(t_{i+2})), \tilde{f}(t_{i+3}, y(t_{i+3})),$

which are triangular fuzzy numbers and are shown by

$$\{f^l(t_{i-1}, y(t_{i-1})), f^c(t_{i-1}, y(t_{i-1})), f^r(t_{i-1}, y(t_{i-1}))\},$$
$$\{f^l(t_i, y(t_i)), f^c(t_i, y(t_i)), f^r(t_i, y(t_i))\},$$
$$\{f^l(t_{i+1}, y(t_{i+1})), f^c(t_{i+1}, y(t_{i+1})), f^r(t_{i+1}, y(t_{i+1}))\},$$
$$\{f^l(t_{i+2}, y(t_{i+2})), f^c(t_{i+2}, y(t_{i+2})), f^r(t_{i+2}, y(t_{i+2}))\},$$
$$\{f^l(t_{i+3}, y(t_{i+3})), f^c(t_{i+3}, y(t_{i+3})), f^r(t_{i+3}, y(t_{i+3}))\},$$

Also,

$$\tilde{y}(t_{i+4}) = \tilde{y}(t_{i+3}) + \int_{t_{i+3}}^{t_{i+4}} \tilde{f}(t, y(t))dt.$$

By fuzzy interpolation:

$\tilde{f}(t_{i-1}, y(t_{i-1})), \tilde{f}(t_i, y(t_i)), \tilde{f}(t_{i+1}, y(t_{i+1})), \tilde{f}(t_{i+2}, y(t_{i+2})), \tilde{f}(t_{i+3}, y(t_{i+3})),$

We have:
$$f^l(t, y(t)) = \sum_{j=i-1, l_j(t)\ge 0}^{i+3} l_j(t) f^l(t_j, y(t_j)) + \sum_{j=i-1, l_j(t)<0}^{i+3} l_j(t) f^r(t_j, y(t_j)),$$

$$f^c(t, y(t)) = \sum_{j=i-1}^{i+3} l_j(t) f^c(t_j, y(t_j)),$$

$$f^r(t, y(t)) = \sum_{j=i-1, l_j(t)\ge 0}^{i+3} l_j(t) f^r(t, y(t_j)) + \sum_{j=i-1, l_j(t)<0}^{i+3} l_j(t) f^l(t_j, y(t_j)),$$

For $t_{i+3} \le t \le t_{i+4}$:

$$l_{i-1}(t) = \frac{(t-t_i)(t-t_{i+1})(t-t_{i+2})(t-t_{i+3})}{(t_{i-1}-t_i)(t_{i-1}-t_{i+1})(t_{i-1}-t_{i+2})(t_{i-1}-t_{i+3})} \ge 0$$
,
$$l_i(t) = \frac{(t-t_i)(t-t_{i+1})(t-t_{i+2})(t-t_{i+3})}{(t_i-t_i-1)(t_i-t_{i+1})(t_i-t_{i+2})(t_{i-1}-t_{i+3})} \le 0,$$

$$l_{i+1}(t) = \frac{(t-t_i)(t-t_{i+1})(t-t_{i+2})(t-t_{i+3})}{(t_{i+1}-t_i)(t_{i+1}-t_i)(t_{i+1}-t_{i+2})(t_{i+1}-t_{i+3})} \ge 0,$$

$$l_{i+2}(t) = \frac{(t-t_{i-1})(t-t_i)(t-t_{i+1})(t-t_{i+3})}{(t_{i+2}-t_{i-1})(t_{i+2}-t_i)(t_{i+2}-t_{i+1})(t_{i+2}-t_{i+3})} \le 0,$$

$$l_{i+3}(t) = \frac{(t-t_{i-1})(t-t_i)(t-t_{i+1})(t-t_{i+2})}{(t_{i+3}-t_{i-1})(t_{i+3}-t_i)(t_{i+3}-t_{i+1})(t_{i+3}-t_{i+2})} \ge 0,$$

Therefore the following results will be obtained.

$$f^l(t, y(t)) = l_{i-1}(t) f^l(t_{i-1}, y(t_{i-1})) + l_i(t) f^r(t_i, y(t_i)) + l_{i+1}(t) f^l(t_{i+1}, y(t_{i+1})) + l_{i+2}(t) f^r(t_{i+2}, y(t_{i+2})) + l_{i+3}(t) f^l(t_{i+3}, y(t_{i+3})),$$

It follows that:

$$\tilde{y}^{\alpha}(t_{i+4}) = [\underline{y}^{\alpha}(t_{i+4}), \overline{y}^{\alpha}(t_{i+4})],$$

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$$
\begin{aligned}
f^{c}(t,y(t)) &= l_{i-1}(t)\, f^{c}(t_{i-1}, y(t_{i-1})) \\
&+ l_{i}(t)\, f^{c}(t_{i}, y(t_{i})) + l_{i+1}(t)\, f^{c}(t_{i+1}, y(t_{i+1})) \\
&+ l_{i+2}(t)\, f^{c}(t_{i+2}, y(t_{i+2})) + l_{i+3}(t)\, f^{c}(t_{i+3}, y(t_{i+3})),
\end{aligned}
$$

$$
\begin{aligned}
f^{r}(t,y(t)) &= l_{i-1}(t)\, f^{r}(t_{i-1}, y(t_{i-1})) \\
&+ l_{i}(t)\, f^{l}(t_{i}, y(t_{i})) + l_{i+1}(t)\, f^{r}(t_{i+1}, y(t_{i+1})) \\
&+ l_{i+2}(t)\, f^{l}(t_{i+2}, y(t_{i+2})) + l_{i+3}(t)\, f^{r}(t_{i+3}, y(t_{i+3})).
\end{aligned}
$$

$$
\underline{y}^{\alpha}(t_{i+4}) = \overline{y}^{\alpha}(t_{i+3}) + \int_{t_{i+3}}^{t_{i+4}} \{\alpha\, f^{c}(t, y(t) + (1-\alpha)\, f^{l}(t, y(t))\}dt
$$

$$
\overline{y}^{\alpha}(t_{i+4}) = \overline{y}^{\alpha}(t_{i+3}) + \int_{t_{i+3}}^{t_{i+4}} \{\alpha\, f^{c}(t, y(t) + (1-\alpha) f^{r}(t, y(t))\}dt,
$$

$$
\begin{aligned}
\underline{y}^{\alpha}(t_{i+4}) &= \underline{y}^{\alpha}(t_{i+3}) + \int_{t_{i+3}}^{t_{i+4}} \{\alpha(l_{i-1}(t)\, f^{c}(t_{i-1}, y(t_{i-1})) \\
&+ l_{i}(t)\, f^{c}(t_{i}, y(t_{i})) + l_{i+1}(t)\, f^{c}(t_{i+1}, y(t_{i+1})) \\
&+ l_{i+2}(t)\, f^{c}(t_{i+2}, y(t_{i+2})) + l_{i+3}(t)\, f^{c}(t_{i+3}, y(t_{i+3})) \\
&+ (1-\alpha) l_{i-1}(t)\, f^{l}(t_{i-1}, y(t_{i-1})) + l_{i}(t)\, f^{r}(t_{i}, y(t_{i})) \\
&+ l_{i+1}(t)\, f^{l}(t_{i+1}, y(t_{i+1})) + l_{i+2}(t)\, f^{r}(t_{i+2}, y(t_{i+2})) \\
&+ l_{i+3}(t)\, f^{l}(t_{i+3}, y(t_{i+3}))\}dt,
\end{aligned}
$$

$$
\begin{aligned}
\overline{y}^{\alpha}(t_{i+4}) &= \overline{y}^{\alpha}(t_{i+3}) + \int_{t_{i+3}}^{t_{i+4}} \{\alpha(l_{i-1}(t)\, f^{c}(t_{i-1}, y(t_{i-1})) \\
&+ l_{i}(t)\, f^{c}(t_{i}, y(t_{i})) + l_{i+1}(t)\, f^{c}(t_{i+1}, y(t_{i+1})) \\
&+ l_{i+2}(t)\, f^{c}(t_{i+2}, y(t_{i+2})) + l_{i+3}(t)\, f^{c}(t_{i+3}, y(t_{i+3})) \\
&+ (1-\alpha) l_{i-1}(t)\, f^{r}(t_{i-1}, y(t_{i-1})) + l_{i}(t)\, f^{l}(t_{i}, y(t_{i})) \\
&+ l_{i+1}(t)\, f^{r}(t_{i+1}, y(t_{i+1})) + l_{i+2}(t)\, f^{l}(t_{i+2}, y(t_{i+2})) \\
&+ l_{i+3}(t)\, f^{r}(t_{i+3}, y(t_{i+3}))\}dt.
\end{aligned}
$$

The following results will be obtained by using the method of

$$
\begin{aligned}
\underline{y}^{\alpha}(t_{i+4}) &= \overline{y}^{\alpha}(t_{i+3}) + \frac{1901h}{720}[\alpha f^{c}(t_{i+3}, y(t_{i+3}))) \\
&+ (1-\alpha)\, f^{l}(t_{i+3}, y(t_{i+3}))] - \frac{2774h}{720}[\alpha f^{c}(t_{i+2}, y(t_{i+2})) \\
&+ (1-\alpha)\, f^{r}(t_{i+2}, y(t_{i+2}))] + \frac{2616h}{720}[\alpha f^{c}(t_{i+1}, y(t_{i+1})) \\
&+ (1-\alpha) f^{c}(t_{i+1}, y(t_{i+1}))] - \frac{1274h}{720}[\alpha f^{c}(t_{i}, y(t_{i})) \\
&+ (1-\alpha) f^{r}(t_{i}, y(t_{i}))] + \frac{251h}{720}[\alpha f^{c}(t_{i-1}, y(t_{i-1})) \\
&+ (1-\alpha) f^{l}(t_{i-1}, y(t_{i-1}))],
\end{aligned}
$$

$$
\begin{aligned}
\overline{y}^{\alpha}(t_{i+4}) &= \overline{y}^{\alpha}(t_{i+3}) + \frac{1901h}{720}[\alpha f^{c}(t_{i+3}, y(t_{i+3}))) \\
&+ (1-\alpha)\, f^{r}(t_{i+3}, y(t_{i+3}))] - \frac{2774h}{720}[\alpha f^{c}(t_{i+2}, y(t_{i+2})) \\
&+ (1-\alpha)\, f^{l}(t_{i+2}, y(t_{i+2}))] + \frac{2616h}{720}[\alpha f^{c}(t_{i+1}, y(t_{i+1})) \\
&+ (1-\alpha) f^{r}(t_{i+1}, y(t_{i+1}))] - \frac{1274h}{720}[\alpha f^{c}(t_{i}, y(t_{i})) \\
&+ (1-\alpha) f^{l}(t_{i}, y(t_{i}))] + \frac{251h}{720}[\alpha f^{c}(t_{i-1}, y(t_{i-1})) \\
&+ (1-\alpha) f^{r}(t_{i-1}, y(t_{i-1}))],
\end{aligned}
$$

$$
\begin{aligned}
\underline{y}^{\alpha}(t_{i+4}) &= \underline{y}^{\alpha}(t_{i+3}) + \frac{h}{720}[1901\underline{f}^{\alpha}(t_{i+3}, y(t_{i+3})) \\
&- 2774\overline{f}^{\alpha}(t_{i+2}), y(t_{i+2})) + 2616\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1})) \\
&- 1274(\overline{f}^{\alpha}((t_{i}), y(t_{i})) + 251\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1}))],
\end{aligned}
$$

$$
\begin{aligned}
\overline{y}^{\alpha}(t_{i+4}) &= \overline{y}^{\alpha}(t_{i+3}) + \frac{h}{720}[1901\overline{f}^{\alpha}(t_{i+3}, y(t_{i+3})) \\
&- 2774\underline{f}^{\alpha}(t_{i+2}), y(t_{i+2})) + 2616\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1})) \\
&- 1274(\underline{f}^{\alpha}((t_{i}), y(t_{i})) + 251\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}))].
\end{aligned}
$$

Therefore Adam's –Bashforth five step method is obtained as follows:

$$
\begin{aligned}
\underline{y}^{\alpha}(t_{i+4}) &= \underline{y}^{\alpha}(t_{i+3}) + \frac{h}{720}[1901\underline{f}^{\alpha}(t_{i+3}, y(t_{i+3})) \\
&- 2774\overline{f}^{\alpha}(t_{i+2}), y(t_{i+2})) + 2616\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1})) \\
&- 1274(\overline{f}^{\alpha}(t_{i}, y(t_{i})) + 251\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1}))],
\end{aligned}
$$

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$$\overline{y}^{\alpha}(t_{i+4}) = \overline{y}^{\alpha}(t_{i+3}) + \frac{h}{720}[1901\overline{f}^{\alpha}(t_{i+3}, y(t_{i+3}))$$
$$-2774\underline{f}^{\alpha}(t_{i+2}, y(t_{i+2})) + 2616\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1}))$$
$$-1274(\underline{f}^{\alpha}(t_i, y(t_i)) + 251\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}))],$$
$$\underline{y}^{\alpha}(t_{i-1}) = \alpha_0, \underline{y}^{\alpha}(t_i) = \alpha_1, \underline{y}^{\alpha}(t_{i+1}) = \alpha_2,$$
$$\underline{y}^{\alpha}(t_{i+2}) = \alpha_3, \underline{y}^{\alpha}(t_{i+3}) = \alpha_4$$
$$\overline{y}^{\alpha}(t_{i-1}) = \alpha_5, \overline{y}^{\alpha}(t_i) = \alpha_6, \overline{y}^{\alpha}(t_{i+1}) = \alpha_7,$$
$$\overline{y}^{\alpha}(t_{i+2}) = \alpha_8, \overline{y}^{\alpha}(t_{i+3}) = \alpha_9.$$

IV . NUMERICAL EXAMPLES

A. *Example 4.1*

Consider the fuzzy initial value problem,

$$y'(t) = y(t), \qquad t \in I=[0,1],$$

$$y(0)= (0.75 + 0,25\ r, 1,125-0.125\ r),$$

$0 < r \leq 1$.

By using the Runge-Kutta method of order 5, we have

$$y_1(t_{n+1}:r) = y_1(t_n;r)\left[1+h+\frac{h^2}{2}+\frac{h^3}{6}+\frac{h^4}{24}+\frac{h^5}{144}\right]$$

$$y_2(t_{n+1};r) = y_2(t_n;r)\left[1+h+\frac{h^2}{2}+\frac{h^3}{6}+\frac{h^4}{24}+\frac{h^5}{144}\right]$$

The exact solution is given by

$$y_1(t;r) = y_1(0;r)e^t,$$
$$y_2(t;r) = y_2(0;r)e^t$$

where at t=1,

$$y_1(1;r) = [(0.75+0.25r)e, (1.125-0.125r)e],$$
$$0 < r \leq 1.$$

By using Runge-Kutta fifth order method the following results are obtained:

TABLE 4.1

α	Rk-Order 5		Exact Solution	
	$\underline{y}(t_i;\alpha)$	$\overline{y}(t_i;\alpha)$	$\underline{Y}(t_i;\alpha)$	$\overline{Y}(t_i;\alpha)$
0.1	0.9078	3.5140	0.9079	3.7886
0.2	0.9584	3.1224	0.9585	3.2851
0.3	1.0128	2.8014	1.0129	2.8994
0.4	1.0714	2.5314	1.0715	2.5918
0.5	1.1344	2.3039	1.1348	2.3419
0.6	1.2034	2.1096	1.2038	2.1330
0.7	1.2785	1.9419	1.2793	1.9568
0.8	1.3610	1.7957	1.3625	1.8051
0.9	1.4524	1.6674	1.4545	1.6732
1.0	1.5537	1.5537	1.5574	1.5574

B. *Example 4.2*

Consider the fuzzy initial value problem,

$$y'(t) = y(t), t \in I = [0,1],$$
$$y(0) = [0.75+0.25\alpha, 1.125-0.125\alpha], 0 < \alpha \leq 1$$
$$y(0.1) = [(0.75+0.25\alpha)e^{0.1}, (1.125-0.125\alpha)e^{0.1}],$$
$$y(0.2) = [(0.75+0.25\alpha)e^{0.2}, (1.125-0.125\alpha)e^{0.2}],$$
$$y(0.3) = [(0.75+0.25\alpha)e^{0.3}, (1.125-0.125\alpha)e^{0.3}],$$
$$y(0.4) = [(0.75+0.25\alpha)e^{0.4}, (1.125-0.125\alpha)e^{0.4}],$$

The exact solution at t=1 is given by

$$Y(1;\alpha) = [(0.75+0.25\alpha)e, (1.125-0.125\alpha)e],$$
$$0 < \alpha \leq 1.$$

By using Adam's-fifth order predictor corrector method the following results are obtained:

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TABLE 4.2

α	Adam'sOrder-5		ExactSolution	
	$\underline{y}(t_i;\alpha)$	$\overline{y}(t_i;\alpha)$	$\underline{Y}(t_i;\alpha)$	$\overline{Y}(t_i;\alpha)$
0.1	2.1067	3.0241	2.10666841	3.02408853
0.2	2.1746	2.9901	2.17462546	2.99011001
0.3	2.2426	2.9561	2.24258250	2.95613148
0.4	2.3105	2.9222	2.31053955	2.92215296
0.5	2.3785	2.8882	2.37849660	2.88817444
0.6	2.4465	2.8542	2.44645364	2.85419592
0.7	2.5144	2.8202	2.51441069	2.82021739
0.8	2.5824	2.7862	2.58236773	2.78623887
0.9	2.6503	2.7523	2.60532478	2.75226035
1.0	2.7183	2.7183	2.71828182	2.71828182

The error of Runge-Kutta method of order 5 and Adam's method of order 5 are shown in order to show that our proposed method gives better solution.

TABLE 4.3

α	Rk- Order 5		Adam's -5	
	$\underline{y}(t_i;\alpha)$	$\overline{y}(t_i;\alpha)$	$\underline{Y}(t_i;\alpha)$	$\overline{Y}(t_i;\alpha)$
0.1	1.615×10^{-4}	2.391×10^{-6}	8.7741×10^{-8}	1.2595×10^{-7}
0.2	1.708×10^{-4}	2.193×10^{-6}	9.0571×10^{-8}	1.2453×10^{-7}
0.3	1.726×10^{-4}	2.266×10^{-6}	9.3401×10^{-8}	1.2312×10^{-7}
0.4	1.772×10^{-4}	2.241×10^{-6}	9.6231×10^{-8}	1.2171×10^{-7}
0.5	1.824×10^{-4}	2.215×10^{-6}	9.9062×10^{-8}	1.2028×10^{-7}
0.6	1.876×10^{-4}	2.189×10^{-6}	1.0189×10^{-7}	1.1887×10^{-7}
0.7	1.928×10^{-4}	2.163×10^{-6}	1.0472×10^{-7}	1.1745×10^{-7}
0.8	1.980×10^{-4}	2.136×10^{-6}	1.0755×10^{-7}	1.1604×10^{-7}
0.9	2.033×10^{-4}	2.116×10^{-6}	1.1038×10^{-7}	1.1462×10^{-7}
1.0	2.084×10^{-4}	2.084×10^{-6}	1.1321×10^{-7}	1.1321×10^{-7}

IV. CONCLUSION

In this Paper, we have Applied Iterative Solution of Adam's Predictor Corrector Fifth Order method for finding the Numerical Solution of Fuzzy Differential Equations. Comparison of Solution of Example 1 And 2 Shows that our Proposed method gives better Solution than Runge-Kutta Fifth Order method.

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