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INTUITIONISTIC FUZZY BI-IDEALS IN GAMMA NEAR-RINGS

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***Abstract*- In this paper, we introduce the concept of intuitionistic fuzzy bi-ideals in Γ-near rings, Gamma near-rings were defined by Satyanarayana and the ideal theory in gamma near ringwas studied by Satyanarayana and Booth. Also, we proved some of the theorems in intuitionistic fuzzy bi-ideal.**

***Keywords*- Gamma bi-ideals, intuitionistic fuzzy bi-ideal, normal intuitionistic fuzzy bi-ideal.**

Introduction

Following the introduction of fuzzy sets by zadeh [6], the fuzzy set theory developed by zadeh and others has found many applications in mathematics. Gamma near-rings were defined by bh.satyanarayana [4] and g.l. booth [2]. K.p. shum and m.akram [5] studied about intuitionistic fuzzy ideals of near-rings. In this paper, we introduce fuzzy bi-ideals in gamma-near rings and proved some of the fuzzy bi-ideals theorem in intuitionistic fuzzy bi-ideal.

II. PRELIMINARIES

Definition 2.1 – Near-Ring:

A non-empty set N with two binary operations + & .is called a near-ring if it satisfies the following axioms :

(i) (N,+) is a group
(ii) (N,.) is a semi-group
(iii) (x+y).z = x.z+y.z $\forall x, y, z \in N$

Definition 2.2 – Fuzzy bi-ideals in $\Gamma-$near-rings:

A fuzzy set μ in M is called a fuzzy bi-ideal of M if

(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\} \quad \forall x, y \in M$
(ii) $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(y)\}$
$\forall x, y, z \in M \;\&\; \alpha, \beta \in \Gamma$

Definition 2.3 –Intuitionistic fuzzy bi-ideals in $\Gamma-$near-rings:

A fuzzy set A (μ_A, ν_A) in M is called Intuitionistic fuzzy bi-ideals of M if

(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\} \;\forall x, y \in M$
$\nu(x-y) \leq \max\{\nu(x), \nu(y)\} \forall x, y \in M$
(ii) $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(y)\}$
$\forall x, y, z \in M \;\&\; \alpha, \beta \in \Gamma$
$\nu(x\alpha y\beta z) \leq \max\{\nu(x), \nu(y)\}$
$\forall x, y, z \in M \;\&\; \alpha, \beta \in \Gamma$

Definition 2.4- Normal fuzzy bi-ideal:

A fuzzy bi-ideal μ of a Γ near-ring M is said to be normal if $\mu(0) = 1$.

Definition 2.5- Normal Intuitionistic fuzzy bi-ideal:

An intuitionistic fuzzy bi-ideal A (μ_A, ν_A) of a Γ near-ring M is said to be normal if $\mu_A(0) = 1$ and $\nu_A(1) = 0$.

Definition 2.6- Characteristic function:

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$$\chi_A(\langle x,a,b\rangle)=\begin{cases}1, if\mu_A(x)=a,\nu_A(x)=b\\0, otherwise\end{cases}$$

III. INTUITIONISTIC FUZZY BI-IDEALS IN GAMMA NEAR-RINGS

Lemma 3.1:

Let B be a bi-ideal of a Γ near-ring M. For any 0<t<1 there exists an intuitionistic fuzzy bi-ideal $A(\mu_A,\nu_A)$ of M such that A=B.

Proof:

Let B be a bi-ideal of a Γ near-ring M.
Define $A(\mu_A,\nu_A)$: M ⟶ [0,1] by

$$\mu_A(x)=\begin{cases}t, if x\in B\\0, if x\notin B\end{cases}$$ And

$$\nu_A(x)=\begin{cases}s, if x\in B\\0, if x\notin B\end{cases}$$

Where s, t ∈ (0,1). clearly $A(\mu_A(x),\nu_A(x))=B$ i.e., A=B.

Let x, y ∈ M. If x, y ∈B then

$\mu_A(x-y)=t=\min\{\mu_A(x),\mu_A(y)\}$

and $\nu_A(x-y)=s=\max\{\nu_A(x),\nu_A(y)\}$. If atleast one of x and y is not in B then x-y ∉ B and so,

$\mu_A(x-y)=0=\min\{\mu_A(x),\mu_A(y)\}$

and $\nu_A(x-y)=0=\max\{\nu_A(x),\nu_A(y)\}$.

Let x, y, z ∈ M and $\alpha,\beta\in\Gamma$. If x, z ∈B then $\mu_A(x)=t=\mu_A(z)$; $\nu_A(x)=s=\nu_A(z)$. Also,

$\mu_A(x\alpha y\beta z)=t\geq\min\{\mu_A(x),\mu_A(z)\}$;

$\nu_A(x\alpha y\beta z)=s\leq\max\{\nu_A(x),\nu_A(z)\}$. If atleast one of x and z not in B then

$\mu_A(x\alpha y\beta z)\geq 0=\min\{\mu_A(x),\mu_A(z)\}$;

$\nu_A(x\alpha y\beta z)\leq 0=\max\{\mu_A(x),\mu_A(z)\}$.

Thus, $A(\mu_A,\nu_A)$ is a intuitionistic fuzzy bi-ideal of M. Hence it completes the proof.

Lemma 3.2:

Let B be a non-empty subset of a Γ near-ring M. then B is a bi-ideal of M if and only if χ_B is an intuitionistic fuzzy bi-ideal of M.

Proof:

Let B be a bi-ideal of a Γ near-ring M. For x,y ∈ B, x-y ∈ B

(i)Let x, y ∈ M and $\chi_B(\langle x,\mu_B(x),\nu_B(x)\rangle)$ is a characteristic function.

a. If x, y ∈ B then $\mu_B(x)=1$ and $\mu_B(y)=1$ thus

$\mu_B(x-y)=1\geq\min\{\mu_B(x),\mu_B(y)\}$

If x, y ∈ B then $\nu_B(x)=1$ and $\nu_B(y)=1$ thus

$\nu_B(x-y)=1\leq\max\{\nu_B(x),\nu_B(y)\}$

b. If x ∈ B & y∉ B then $\mu_B(x)=1$ and $\mu_B(y)=0$ thus

$\mu_B(x-y)=0\geq\min\{\mu_B(x),\mu_B(y)\}$

If x ∈ B & y∉ B then $\nu_B(x)=1$ and $\nu_B(y)=0$ thus

$\nu_B(x-y)=1\leq\max\{\nu_B(x),\nu_B(y)\}$

c. If x ∉ B & y ∈ B then $\mu_B(x)=0$ and $\mu_B(y)=1$ thus

$\mu_B(x-y)=0\geq\min\{\mu_B(x),\mu_B(y)\}$

If x ∉ B & y ∈ B then $\nu_B(x)=0$ and $\nu_B(y)=1$ thus

$\nu_B(x-y)=1\leq\max\{\nu_B(x),\nu_B(y)\}$

d. If x ∉ B & y ∉ B then $\mu_B(x)=0$ and $\mu_B(y)=0$ thus

$\mu_B(x-y)=0\geq\min\{\mu_B(x),\mu_B(y)\}$

If x ∉ B & y ∉ B then $\nu_B(x)=0$ and $\nu_B(y)=0$ thus

$\nu_B(x-y)=0\leq\max\{\nu_B(x),\nu_B(y)\}$

Hence in all cases condition (i) of definition (2.3) is satisfied.

(ii) Let x, y, z ∈ M

a. If x, z ∈ B then $\mu_B(x)=1$ and $\mu_B(z)=1$ thus

$\mu_B(x\alpha y\beta z)=1\geq\min\{\mu_B(x),\mu_B(z)\}$

If x, z∈ B then $\nu_B(x)=1$ and $\nu_B(z)=1$ thus

$\nu_B(x\alpha y\beta z)=1\leq\max\{\nu_B(x),\nu_B(z)\}$

b. If x ∈ B & z∉ B then $\mu_B(x)=1$ and $\mu_B(z)=0$ thus

$\mu_B(x\alpha y\beta z)=0\geq\min\{\mu_B(x),\mu_B(z)\}$

If x ∈ B & z∉ B then $\nu_B(x)=1$ and $\nu_B(z)=0$ thus

$\nu_B(x\alpha y\beta z)=1\leq\max\{\nu_B(x),\nu_B(z)\}$

c. If x ∉ B & z∈ B then $\mu_B(x)=0$ and $\mu_B(z)=1$ thus

$\mu_B(x\alpha y\beta z)=0\geq\min\{\mu_B(x),\mu_B(z)\}$

If x ∉ B & z∈ B then $\nu_B(x)=0$ and $\nu_B(z)=1$ thus

$\nu_B(x\alpha y\beta z)=1\leq\max\{\nu_B(x),\nu_B(z)\}$

d. If x ∉ B & z∉ B then $\mu_B(x)=0$ and $\mu_B(z)=0$ thus

$\mu_B(x\alpha y\beta z)=0\geq\min\{\mu_B(x),\mu_B(z)\}$

If x ∉ B & z∉ B then $\nu_B(x)=0$ and $\nu_B(z)=0$ thus

$\nu_B(x\alpha y\beta z)=0\leq\max\{\nu_B(x),\nu_B(z)\}$

Hence in all cases condition (ii) of definition (2.3) is satisfied.

Thus, χ_B is an intuitionistic fuzzy bi-ideal of M.

Conversely, Suppose χ_B is an intuitionistic fuzzy bi-ideal of M. Then, by Lemma 3.1 we have χ_B is two-valued.

Hence B is a bi-ideal of M.

Hence it completes the proof.

Lemma 3.3:

For a bi-ideal B of a Γ near-ring M, the intuitionistic fuzzy set $A(\mu_A,\nu_A)$ in M is a normal intuitionistic fuzzy bi-ideal of M and $M_A=B$.

Proof:

By lemma 3.2, $A(\mu_A,\nu_A)$ is a fuzzy bi-ideal. Since B is a bi-ideal (0, 1) ∈ B and so

$\mu_A(0)=1$ and $\nu_A(1)=0$. Thus, $A(\mu_A,\nu_A)$ is normal.

Now, $M_A=(\mu_A,\nu_A)$

Where,

$\mu_A=\{x\in M/\mu_A(x)\}=\mu_A(0)=\{x\in M/\mu_A(x)=1\}$

$\nu_A=\{x\in M/\nu_A(x)\}=\nu_A(0)=\{x\in M/\nu_A(x)=0\}$

$=\{x\in M/x\in B\}$=B

i.e., $M_A=B$

Hence it completes the proof.

IV.CONCLUSION

In this paper, basic definition of intuitionistic fuzzy bi-ideal, Normal intuitionistic fuzzy bi-ideal are introduced with some conditions. We plan to extend our research of fuzzy bi-ideal in gamma-near-rings to intuitionistic fuzzy bi-ideal in gamma near-rings.

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