ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

INTUITIONISTIC FUZZY IDEALS IN SEMIGROUPS

S.K. Mala [1], S.Shribharathi [2]

Assistant Professor, Department of Mathematics, KG college of arts and science, Coimbatore, India [1]

Email. i.d.: mala.sk@gmail.com [1]

Research Scholar, Department of Mathematics, KG college of arts and science, Coimbatore, India [2]

Email. i.d.: valarshri555@gmail.com2

***Abstract*- In this paper, I consider the Intuitionistic fuzzification of the concept of several ideals in Semigroup S, and investigate some theorems in such ideals.**

Keywords*- *Semigroup, Crisp Set, Fuzzy Set, Intuitionistic Fuzzy ideal, Intuitionistic Fuzzy bi-ideal, Level Subset of Intuitionistic Fuzzy.

I.INTRODUCTION

After the introduction of fuzzy sets by Zadeh in 1965, (see [7]) has achieved great success in a various fields. L.A. Zadeh (see [7]) introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. Also several higher order fuzzy sets, introduced by Atanassov (see [1]). Fuzzy ideals in Semigroup have been first studied by N.edekind for the theory of algebraic numbers, was generalized by Emmy Noether for associative rings.

Since then many papers on ideals for rings and Semigroup appeared showing the importance of the concept [A.H. Clifford, S. Lajos and many others]. Further generalization of ideals by lattice-theoretical methods was given by G.Birkhoff, O.Steinfeld, and N.Kehayopulu.In this paper, we discuss further properties of Intuitionistic fuzzy sets of ideals in Semigroup are discussed.

II. PRELIMINARIES

Definition 2.1 – Crisp Sets:

Either the elements belongs to the set or does not belongs to the set is called crisp set. In this set the membership function has takes only the values 0(false) and 1(true).

Definition 2.2 – Fuzzy Set:

Let S be a Semigroup and F be a "fuzzy" and let f be a subsemigroup. A function f from S to the unit interval $[0,1]$ is called a fuzzy set of S. Let F(S) denote the set of all fuzzy sets in S.

(or)

Non crisp sets are called fuzzy set. In Fuzzy set the membership function takes the value[0,1]

Definition 2.3 – Intuitionistic Fuzzy sets:

Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership function. An Intuitionistic fuzzy set A is a non empty set X is an object having the form

$A = \{\langle x, \mu_A(x), \nu_A(x) \mid x \in E \rangle\}$ and satisfies

$0 \le \mu_A(x) + \nu_A(x) \le 1.$

Definition 2.4- Intuitionistic Fuzzy Set Operation:

For Intuitionistic fuzzy set, for any

$A(\mu_A, \nu_A), B(\mu_B, \nu_B) \in F(S), A \subseteq B$ is defined by

$\mu_A \le \mu_B$ and $\nu_A \le \nu_B$

For $\forall x \in S$, respectively.

$(A \cap B)(x) = \min((\mu_A(x), \nu_A(x)), (\mu_B(x), \nu_B(x)))$

$= (\mu_A(x), \nu_A(x)) \wedge (\mu_B(x), \nu_B(x))$

$(A \cup B)(x) = \max((\mu_A(x), \nu_A(x)), (\mu_B(x), \nu_B(x)))$

$= (\mu_A(x), \nu_A(x)) \vee (\mu_B(x), \nu_B(x))$

For $\forall x \in S$, respectively

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

Definition2.5-Intuitionistic Fuzzy ideal of semigroup:

A fuzzy subsemigroup $A(\mu_A, \nu_A)$ of a Semigroup S is called an Intuitionistic fuzzy ideal of S if

$$\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$$
$$\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$$

For all $x, y \in S$.

Definition2.6-Intuitionistic Fuzzy bi-ideal of Semigroup:

A fuzzy subsemigroup $A(\mu_A, \nu_A)$ of a Semigroup S is called an Intuitionistic fuzzy ideal of S if

$$\mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(z)\}$$
$$\nu_A(xyz) \leq \max\{\nu_A(x), \nu_A(z)\}$$

For all $x, y, z \in S$.

Definition 2.7- Level Subset of Intuitionistic Fuzzy:

Let $A(\mu_A, \nu_A) \in F(S)$, then for $t \in [0,1]$ the set

$$U_t = \{x / \mu_A(x) \geq t\}$$
$$L_t = \{x / \nu_A(x) \leq t\}$$

Is called a level subset of A.

III.INTUITIONISTIC FUZZY IDEALS GENERATED BY INTUITIONISTIC FUZZY SETS

Lemma 3.1:

Let S be a Semigroup. Then FI(S) is a complete completely distributive a lattice with respect to the meet "I" and the union "Y" defined as follows:

(i) $\underset{\alpha\in\Gamma}{\mathrm{Y}} A_\alpha = (\underset{\alpha\in\Gamma}{\vee} \mu_{A_\alpha}, \underset{\alpha\in\Gamma}{\wedge} \nu_{A_\alpha})$

(ii) $\underset{\alpha\in\Gamma}{\mathrm{I}} A_\alpha = (\underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}, \underset{\alpha\in\Gamma}{\vee} \nu_{A_\alpha})$

For all $x \in S$.

Proof:

Since [0,1] is a complete completely distributive lattice with respect to the usual ordering In [0,1] . To show this lemma it is sufficient to show that $\underset{\alpha\in f}{\mathrm{Y}} A_\alpha$ and $\underset{\alpha\in f}{\mathrm{I}} A_\alpha$ is a Intuitionistic fuzzy ideals of S for a family of Intuitionistic fuzzy ideals $\{A_\alpha\}_{\alpha\in\Gamma}$. In fact

(i) $\underset{\alpha\in\Gamma}{\mathrm{Y}} A_\alpha(xy) = (\underset{\alpha\in\Gamma}{\vee} \mu_{A_\alpha}, \underset{\alpha\in\Gamma}{\wedge} \nu_{A_\alpha})(xy)$

$$\underset{\alpha\in\Gamma}{\vee} \mu_{A_\alpha}(xy) \leq \underset{\alpha\in\Gamma}{\vee}\left\{\underset{\alpha\in\Gamma}{\max}\{\mu_{A_\alpha}(x), \mu_{A_\alpha}(y)\}\right\}$$
$$= \underset{\alpha\in\Gamma}{\max}\left\{\underset{\alpha\in\Gamma}{\max}\{\mu_{A_\alpha}(x), \mu_{A_\alpha}(y)\}\right\}$$

$$= \underset{\alpha\in\Gamma}{\max}\left\{\underset{\alpha\in\Gamma}{\max}\{\mu_{A_\alpha}(x)\}, \underset{\alpha\in\Gamma}{\max}\{\mu_{A_\alpha}(y)\}\right\}$$

$$\vee \mu_{A_\alpha}(xy) = \underset{\alpha\in\Gamma}{\max}\left\{\underset{\alpha\in\Gamma}{\vee} \mu_{A_\alpha}(x), \underset{\alpha\in\Gamma}{\vee} \mu_{A_\alpha}(y)\right\}$$

$$\underset{\alpha\in\Gamma}{\wedge} \nu_{A_\alpha}(xy) \geq \underset{\alpha\in\Gamma}{\wedge}\left\{\underset{\alpha\in\Gamma}{\min}\{\nu_{A_\alpha}(x), \nu_{A_\alpha}(y)\}\right\}$$
$$= \underset{\alpha\in\Gamma}{\min}\left\{\underset{\alpha\in\Gamma}{\min}\{\nu_{A_\alpha}(x), \nu_{A_\alpha}(y)\}\right\}$$

$$= \underset{\alpha\in\Gamma}{\min}\left\{\underset{\alpha\in\Gamma}{\min}\{\nu_{A_\alpha}(x)\}, \underset{\alpha\in\Gamma}{\min}\{\nu_{A_\alpha}(y)\}\right\}$$

$$\wedge \nu_{A_\alpha}(xy) = \underset{\alpha\in\Gamma}{\max}\left\{\underset{\alpha\in\Gamma}{\wedge} \nu_{A_\alpha}(x), \underset{\alpha\in\Gamma}{\wedge} \nu_{A_\alpha}(y)\right\}$$

(ii) $\underset{\alpha\in\Gamma}{\mathrm{I}} A_\alpha(xy) = (\underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}, \underset{\alpha\in\Gamma}{\vee} \nu_{A_\alpha})(xy)$

$$\underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}(xy) \geq \underset{\alpha\in\Gamma}{\wedge}\left\{\underset{\alpha\in\Gamma}{\min}\{\mu_{A_\alpha}(x), \mu_{A_\alpha}(y)\}\right\}$$

$$= \underset{\alpha\in\Gamma}{\min}\left\{\underset{\alpha\in\Gamma}{\min}\{\mu_{A_\alpha}(x), \mu_{A_\alpha}(y)\}\right\}$$

$$\wedge \mu_{A_\alpha}(xy) = \underset{\alpha\in\Gamma}{\min}\left\{\underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}(x), \underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}(y)\right\}$$

$$\underset{\alpha\in\Gamma}{\vee} \nu_{A_\alpha}(xy) \leq \underset{\alpha\in\Gamma}{\vee}\left\{\underset{\alpha\in\Gamma}{\max}\{\nu_{A_\alpha}(x), \nu_{A_\alpha}(y)\}\right\}$$
$$= \underset{\alpha\in\Gamma}{\max}\left\{\underset{\alpha\in\Gamma}{\max}\{\nu_{A_\alpha}(x), \nu_{A_\alpha}(y)\}\right\}$$

$$\underset{\alpha\in\Gamma}{\mathrm{I}} A_\alpha(xy) = \begin{pmatrix} \underset{\alpha\in\Gamma}{\min}\left\{\underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}(x), \underset{\alpha\in\Gamma}{\wedge} \mu_{A_\alpha}(y)\right\}, \\ \underset{\alpha\in\Gamma}{\max}\left\{\underset{\alpha\in\Gamma}{\vee} \nu_{A_\alpha}(x), \underset{\alpha\in\Gamma}{\vee} \nu_{A_\alpha}(y)\right\} \end{pmatrix}$$

Hence it completes the proof.

Lemma 3.2:

Let $A(\mu_A, \nu_A) \in F(S)$. Then

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

$$\mu_A(x) = \sup\{t / x \in U_t\}$$
$$\nu_A(x) = \inf\{t / x \in L_t\}$$

Where U_t and L_t are level subsets of S as in def (2.3)

Proof:

Let $\alpha = \sup\{k / x \in f_K\}$ where f_K is a Intuitionistic fuzzy ideal and let $\in > 0$.

Then

$\sup\{k / x \in f_K\} > \alpha - \in$

And so there exist $t \in \{k / x \in f_K\}$ such that t $> \alpha - \in$.Since $x \in \{U_t, L_t\}$, by def (2.3) we have $\{\mu_A(x), \nu_A(x)\} \geq t.$

Thus $\{\mu_A(x), \nu_A(x)\} \geq t - \in$. Since $\in$ is an arbitrary real number. Then $\{\mu_A(x), \nu_A(x)\} \geq t.$

On the other hand, Let $t = \{\mu_A(x), \nu_A(x)\}$.

Then $x \in (U_t, L_t)$, and $t \in \{k / x \in f_K\}$

So that $\{\mu_A(x), \nu_A(x)\} = t.$

$$\therefore \quad \begin{array}{l} \mu_A(x) = t \leq \sup\{t / x \in U_t\} \\ \nu_A(x) = t \geq \inf\{t / x \in L_t\} \end{array}$$

Hence it completes the proof.

Lemma 3.3:

An Intuitionistic fuzzy set $A(\mu_A, \nu_A)$ in S is an Intuitionistic fuzzy bi-ideal of S if and only if for any $\lambda \in (0,1]$, if $(f_\lambda \neq \Phi)$, and then f_λ is a subsemigroup of S is a bi-ideal of S.

Proof:

Let $A(\mu_A, \nu_A)$ be an Intuitionistic fuzzy bi-ideal and $\lambda \in (0,1]$. For any $x, y \in f_\lambda (f_\lambda \neq \Phi)$, if we $\{\mu_A(x), \mu_A(y)\} \geq \lambda$, $\{\nu_A(x), \nu_A(y)\} \leq \lambda.$

Then

$$\left.\begin{array}{l} \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \lambda \\ \nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \lambda \end{array}\right\} \Rightarrow (1)$$

So $x, y \in f_\lambda$ .Moreover, let $\forall z \in S$. since

$$\left.\begin{array}{l} \mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(z)\} \geq \lambda \\ \nu_A(xyz) \leq \max\{\nu_A(x), \nu_A(z)\} \leq \lambda \end{array}\right\} \Rightarrow (2)$$

Then from (1) and (2) we have f_λ as it satisfies the definition of (2.6).

Thus f_λ is a bi-ideal of S.

Conversely, suppose that f is not an Intuitionistic fuzzy bi-ideal of S, then there exist $x_0, y_0, z_0 \in S$ such that

$$\left.\begin{array}{l} \mu_A(x_0 y_0 z_0) < \min\{\mu_A(x_0), \mu_A(z_0)\} \\ \nu_A(x_0 y_0 z_0) > \max\{\nu_A(x_0), \nu_A(z_0)\} \end{array}\right\} \Rightarrow (3)$$

Or there exist $x_0, y_0 \in S$ such that

$$\left.\begin{array}{l} \mu_A(x_0 y_0) < \min\{\mu_A(x_0), \mu_A(y_0)\} \\ \nu_A(x_0 y_0) > \max\{\nu_A(x_0), \nu_A(y_0)\} \end{array}\right\} \Rightarrow (4)$$

We consider above two cases separately.

(A) If (3) holds, define

$$\lambda_0 = \min\left\{\begin{array}{l} \frac{1}{2}\left(\mu_A(x_0 z_0 y_0), \nu_A(x_0 z_0 y_0)\right) + \left(\mu_A(x_0), \nu_A(x_0)\right), \\ \frac{1}{2}\left(\mu_A(x_0 z_0 y_0), \nu_A(x_0 z_0 y_0)\right) + \left(\mu_A(y_0), \nu_A(y_0)\right) \end{array}\right\}$$

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

Then $\lambda_0 \in (0,1]$ and

$$0 \le (\mu_A(x_0 z_0 y_0), \nu_A(x_0 z_0 y_0)) \le \lambda_0 \le 1,$$
$$(\mu_A(y_0), \nu_A(y_0) > \lambda_0 > 0).$$

So that

$x_0 y_0 \in f_{\lambda_0}$. Since f_{λ_0} is a bi-ideal of S, then we have $x_0, y_0, z_0 \in f_{\lambda_0}$. Thus

$$(\mu_A(x_0 z_0 y_0), \nu_A(x_0 z_0 y_0)) \ge \lambda_0$$

(B) If (2) holds, define

$$\lambda_0 = \min\left\{\begin{array}{l} \frac{1}{2}(\mu_A(x_0 y_0), \nu_A(x_0 y_0)) + (\mu_A(x_0), \nu_A(x_0)), \\ \frac{1}{2}(\mu_A(x_0 y_0), \nu_A(x_0 y_0)) + (\mu_A(y_0), \nu_A(y_0)) \end{array}\right\}$$

Then $\lambda_0 \in (0,1]$ and $\begin{array}{l} 0 \le (\mu_A(x_0 y_0), \nu_A(x_0 y_0)) < \lambda_0 \\ \le 1, (\mu_A(x_0), \nu_A(x_0) > \lambda_0 > 0), \\ (\mu_A(y_0), \nu_A(y_0)) > \lambda_0 > 0. \end{array}$

So that $x_0 y_0 \in f_{\lambda_0}$.

Since f_{λ_0} is a subsemigroup of S, then we have $x_0 y_0 \in f_{\lambda_0}$. Thus $f(x_0 y_0) \ge \lambda_0$ which is impossible. Hence $f(x_0 y_0) > \lambda_0$

$\therefore$ By (3) & (4) using definition of bi-ideals we have $A(\mu_A, \mu_A)$ is an Intuitionistic bi-ideal of S.

Hence it completes the proof.

IV. CONCLUSION

Finally, I proved all the theorems in Intuitionistic fuzzification of the concept of several ideals in Semigroup S, and Investigate some properties of such ideals.

REFERENCES

[1] Atanassov, K. T, *Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems*, (1986)., 197

[2]] Birkhoff, G., *Lattice Theory,* Amer. Math. Soc.,Coll. Publ., Vol. XXV, Providence, Rhode Island.1967 .

[3]] Clifford, A.H. and Preston, G.B. *The Algebraic Theory of Semigroups* ,Vol I,Amer. Math. Soc., Math. Surveys 7, Providence, Rhode Island, 1997.

[4] Kim, K.H. and Lee, J.G., On Intuitionistic Fuzzy Bi-ideals of Semigroup29(2005),201-210.

[5] Kim, K.H. and Jun. Y.B., *Intuitionistic fuzzy ideals of Semigrops* 33(4):443-449, April 2002.

[6]Kuroki, N., On fuzzy ideals and fuzzy bi-ideals in Semigroups, Fuzzy sets and Systems 5:203-215(1981).

[7] Zadeh. L. A, *Fuzzy Sets, Information Control*. Vol. 8 (1965).