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Multiple Numerical Solutions for Intuitionistic Fuzzy Differential Equations

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Abstract: In this paper, the first order intuitionistic fuzzy differential equation with intuitionistic initial value is studied under generalised differentiability concept. With the help of the $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cut representation of an intuitionistic fuzzy set, the first order intuitionistic fuzzy differential equation is transformed into four systems of simultaneous ordinary differential equations where each system consists of a set of four equations. Then each system is solved numerically by Euler method.

Keywords: Intuitionistic Fuzzy Differential Equation, Numerical methods, Euler method, Generalised differentiability.

I. INTRODUCTION

The intuitionistic fuzzy set is an extension of fuzzy set which was introduced by Zadeh[1]. The concept of intuionistic fuzzy set was first proposed by Atanassov[2].Fuzzy set is characterized by the degree of belongingness whereas intuitionistic fuzzy set is characterized by belongingness as well as non-belongingness. Therefore the sum of the values of membership and non-membership is less than one [3,4]. In the literature, it is only few investigations on the intuitionistic fuzzy differential equations. Earlier, Melliani and Chadli[5,6] have discussed differential and partial differential equations under intuitionistic fuzzy environment respectively. Abbasbandy and Allahviranloo[7] obtained numerical solution of intuitionistic fuzzy differential equation by Runge-Kutta method. Lata and Kumar[8] studied Nth-order time dependent intuitionistic fuzzy linear differential equation, where initial values were described by trapezoidal intuitionistic fuzzy numbers. Recently, Mondal, et al.[9] studied strong and weak solution of first order homogeneous intuitionistic fuzzy differential equation, subsequently, who studied system of differential equation in literature [10]. Melliani, et al.[11] discussed the existence and uniqueness of the solution of the intuitionistic fuzzy differential equation using the successive approximation method. Based on the α -cut of an intuitionistic fuzzy set, Nirmala and Chenthur Pandian[12]discussed the numerical solution of intuitionistic fuzzy differential equation by Euler method.Nirmala et al [13,14] have discussed numerical solution of IFDE by Modified Euler method and by fourth order Runge-Kutta method respectively.Parimala et al[15,16] have studied numerical solutions of intuitionistic fuzzy differential equations by Milne's predictor-corrector method and by Adam's predictor corrector methods,respectively. Wang and Guo [17]have studied multiple solutions of intuitionistic fuzzy differential equations based (α, β)-cut of an intuitionistic fuzzy set.

This paper is arranged as follows: Section 2 consists of basic definitions related to intuitionistic fuzzy set theory. Differentiabilty of Intuitionistic Fuzzy Number valued functions is discussed in section 3. Intuitionistic fuzzy Cauchy problem is given in section 4. Euler's method is presented in section 5.Section 6 consists of a numerical example and conclusion of the paper is in section 7.

II. BASIC CONCEPTS

Definition 2.1[3]: Let X be a universe of discourse. An IFS " A " in X is an object having the form: $A = \{(x, \mu_A(x), \nu_A(x)) \backslash x \in X\}$ which is characterized

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by a membership function $\mu_A: X \to [0,1]$ and a non-membership function $\nu_A: X \to [0,1]$ with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$,for all $x \in X$.

For each x the numbers $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of the element $x \in X$ to $A \subset X$, respectively.

For each IFS A in X,if $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, $\pi_A(x)$ is called the degree of indeterminacy or hesitancy of x to "A" .Especially ,if $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = 0$, for each $x \in X$ then the IFS is reduced to a fuzzy set.

Definition 2.2[18]: An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x))\backslash x \in R\}$ such that $\mu_A(x)$ and $(1 - \nu_A)(x) = 1 - \nu_A(x), \forall x \in R$ are fuzzy numbers, is called an intuitionistic fuzzy number.

Therefore IFS $A = \{(x, \mu_A(x), \nu_A(x))\backslash x \in R\}$ is a conjection of two fuzzy numbers A^+ with a membership function $\mu_{A^+}(x) = \mu_A(x)$ and A^- with a membership function $\mu_{A^-}(x) = 1 - \nu_A(x)$.

Definition 2.3[19]: A set of (α, β) cut set. Generated by intuitionistic fuzzy set, where α, β are fixed numbers such that $0 \le \alpha + \beta \le 1$ is defined as $A_{(\alpha,\beta)} = \{(x, \mu_A(x), \nu_A(x))\backslash x \in R, \mu_A(x) \ge \alpha \text{ and } \nu_A(x) \le \beta\}, \forall \alpha, \beta \in [0,1]$.

By denoting $A_{(\alpha,\beta)} = \{A^1_\alpha, A^2_\beta\}$ we mean that the crisp set of elements x which belong to at least to the degree α and which does not belong to at most to the degree β.

Definition 2.4[19]: An intuitionistic fuzzy subset of the real line R is called an intuitionistic fuzzy number if the following holds:

(i) There exist $m \in R$, $\mu_A(m) = 1$ and $\nu_A(m) = 0$ (m is called the mean value of A);

(ii) μ_A is a continuous mapping from R to the closed interval $[0,1]$ and for all $x \in R$, the relation $0 \le \mu_A(x) + \nu_A(x) \le 1$ holds:

(iii) The membership and non-membership functions of A are given as follows:

$$\mu_A(x) = \begin{cases} 0, -\infty < x \le a_1, \\ f_1(x), x \in [a_1, m], \\ 1, x = m \\ h_1(x), x \in [m, a_2], \\ 0, a_2 \le x < \infty \end{cases}$$, where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing functions in $[a_1, m]$ and $[m, a_2]$ respectively.

And $\nu_A(x) =$

$$\begin{cases} 1, -\infty < x \le a_1', \\ f_2(x), x \in [a_1', m]; 0 \le f_1(x) + f_2(x) \le 1, \\ 0, x = m \\ h_2(x), x \in [m, a_2']; 0 \le h_1(x) + h_2(x) \le 1 \\ 1, a_2' \le x < \infty \end{cases},$$

where $a_1' \le a_1 \le m \le a_2 \le a_2'$.

The (α, β)-cut representation of IFN "A" generates the following pair of intervals and is denoted by $[A]_{(\alpha,\beta)} = \{[A_L^+(\alpha), A_U^+(\alpha)], [A_L^-(\beta), A_U^-(\beta)], 0 \le \alpha + \beta \le 1\}$
Where $A_L^+(\alpha) = \inf\{x \in R/\mu_A(x) \ge \alpha\}$,
$A_U^+(\alpha) = \sup\{x \in R/\mu_A(x) \ge \alpha\}$,
$A_L^-(\beta) = \inf\{x \in R/\nu_A(x) \le \beta\}$,
$A_U^-(\beta) = \sup\{x \in R/\nu_A(x) \le \beta\}$,
where $[A^+]_\alpha = [A_L^+(\alpha), A_U^+(\alpha)]$ and $[A^-]_\beta = [A_L^-(\beta), A_U^-(\beta)]$ are fuzzy numbers with membership functions μ_A and $1 - \nu_A$ respectively.

Definition 2.5[20]: A Triangular Intuitionistic Fuzzy Number(TIFN) A is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$ given as follows:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{m-a_1} \text{ for } a_1 \le x \le m \\ \frac{a_2-x}{a_2-m} \text{ for } m \le x \le a_2 \\ 0 \text{ otherwise} \end{cases} \text{ and}$$

$$\nu_A(x) = \begin{cases} \frac{m-x}{m-a_1'} \text{ for } a_1' \le x \le m \\ \frac{x-m}{a_2'-m} \text{ for } m \le x \le a_2' \\ 1 \text{ otherwise} \end{cases}$$

where $a_1' \le a_1 \le m \le a_3 \le a_3'$ and TIFN is denoted by A= ($a_1, m, a_3; a_1', m, a_3'$).

Let IFN(X) denote the set of all intuitionistic fuzzy set on X.

Definition 2.6[17]: Let $I = [a, b] \subseteq R$.Then $f: I \to IFN(X)$ is called an intuitionistic fuzzy number-valued function on I. If $: I \to [IFN(X)]^n$, "f" then is called an "n" dimensional vector of intuitionistic fuzzy number-valued function on I.

III. Differentiability of Intuitionistic Fuzzy Number Valued Functions

Definition 3.1[17]: Let $A = \{(x, \mu_A(x), \nu_A(x))\}$ and $B = \{(x, \mu_B(x), \nu_B(x))\}$ where $x \in X$ be two intuitionistic fuzzy sets. Let $[A]_{(\alpha,\beta)} = \{[A_L^+(\alpha), A_U^+(\alpha)], [A_L^-(\beta), A_U^-(\beta)]\}$ and $[B]_{(\alpha,\beta)} = \{[B_L^+(\alpha), B_U^+(\alpha)], [B_L^-(\beta), B_U^-(\beta)]\}$, where

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$0 \leq \alpha + \beta \leq 1$ and $\alpha, \beta \in [0,1]$, then the distance between the intuitionistic fuzzy numbers "A" and "B" is defined as

$$d(A,B) = \frac{1}{4}\Big[\int_0^1 |A_L^+(\alpha) - B_L^+(\alpha)|d\alpha + \int_0^1 |A_U^+(\alpha) - B_U^+(\alpha)|d\alpha + \int_0^1 |A_L^-(\beta) - B_L^-(\beta)|d\beta + \int_0^1 |A_U^-(\beta) - B_U^-(\beta)|d\beta\Big].$$

Then, clearly $(IFN(X), d)$ is a metric space.

Definition 3.2[17]: Let $x, y \in IFN(X)$. If there exists $z \in IFN(X)$ such that $x = y + z$, then z is called the intuitionistic H-difference of x and y.

Definition 3.3[17]: Let $F: [a,b] \to IFN(X)$ and $x_0 \in [a,b]$. We say that F is differentiable at x_0, if there exists an element $F'(x_0) \in IFX(N)$, such that

(i) for all $h > 0$ sufficiently small, $\exists F(x_0 + h) \, \underline{IH} \, F(x_0), \exists F(x_0) \, \underline{IH} \, F(x_0 - h)$ and the limits (in the metric d)

$$\lim_{h\to 0} \frac{F(x_0+h) \, \underline{IH} \, F(x_0)}{h} = \lim_{h\to 0} \frac{F(x_0) \, \underline{IH} \, F(x_0-h)}{h} = F'(x_0),$$

(or)

(ii) for all $h > 0$ sufficiently small, $\exists F(x_0) \, \underline{IH} \, F(x_0 + h)$, $\exists F(x_0 - h) \, \underline{IH} \, F(x_0)$ and the limits

$$\lim_{h\to 0} \frac{F(x_0) \, \underline{IH} \, F(x_0+h)}{(-h)} = \lim_{h\to 0} \frac{F(x_0-h) \, \underline{IH} \, F(x_0)}{(-h)} = F'(x_0),$$

(or)

(iii) for all $h > 0$ sufficiently small, $\exists F(x_0 + h) \, \underline{IH} \, F(x_0)$, $\exists F(x_0 - h) \, \underline{IH} \, F(x_0)$ and the limits

$$\lim_{h\to 0} \frac{F(x_0+h) \, \underline{IH} \, F(x_0)}{h} = \lim_{h\to 0} \frac{F(x_0-h) \, \underline{IH} \, F(x_0)}{(-h)} = F'(x_0),$$

(or)

(iv) for all $h > 0$ sufficiently small, $\exists F(x_0) \, \underline{IH} \, F(x_0 + h)$, $\exists F(x_0) \, \underline{IH} \, F(x_0 - h)$ and the limits

$$\lim_{h\to 0} \frac{F(x_0) \, \underline{IH} \, F(x_0+h)}{(-h)} = \lim_{h\to 0} \frac{F(x_0) \, \underline{IH} \, F(x_0-h)}{h} = F'(x_0),$$

For the sake of simplicity, we say that the intuitionistic fuzzy-valued function "F" is (i)-differentiable if it satisfies in Definition (3.3) case (i), and we say intuitionistic fuzzy-valued function "F" is (ii)-differentiable if it satisfies in Definition (3.3) case (ii).

Theorem 3.1[17]: Let $F: [a,b] \to IFN(X)$ be an intuitionistic fuzzy-valued function and denote

$$[F(t)]_{(\alpha,\beta)} = \{[F_L^+(t,\alpha), F_U^+(t,\alpha)], [F_L^-(t,\beta), F_U^-(t,\beta)]\},$$

where $0 \leq \alpha + \beta \leq 1$ and $\alpha, \beta \in [0,1]$. Then:

(1) If F is (i)- differentiable, then $F_L^+(t,\alpha), F_U^+(t,\alpha), F_L^-(t,\beta)$, and $F_U^-(t,\beta)$ are differentiable functions and $[F'(t)]_{(\alpha,\beta)} = \{[F'^+_L(t,\alpha), F'^+_U(t,\alpha)], [F'^-_L(t,\beta), F'^-_U(t,\beta)]\}$.

(2) If F is (ii)-differentiable, then $F_L^+(t,\alpha), F_U^+(t,\alpha), F_L^-(t,\beta)$, and $F_U^-(t,\beta)$ are differentiable functions and $[F'(t)]_{(\alpha,\beta)} = \{[F'^+_U(t,\alpha), F'^+_L(t,\alpha)], [F'^-_U(t,\beta), F'^-_L(t,\beta),]\}$.

Proof: See [17]

Remark 3.1: In general, if "F^+" is differentiable in the sense of case (m), $m \in \{1,2\}$ of Definition 3[21] and "F^-" is differentiable in the sense of case(n), $n \in \{1,2\}$ of Definition 3[21], then "F" is (m, n)-diferentaible, denoted by $D^{(m,n)}F$, $m, n \in \{1,2\}$.

Theorem 3.2[17]: Let $F: [a,b] \to IFN(X)$ be an intuitionistic fuzzy-valued function and denote

$$[F(t)]_{(\alpha,\beta)} = \{[F_L^+(t,\alpha), F_U^+(t,\alpha)], [F_L^-(t,\beta), F_U^-(t,\beta)]\},$$

where $0 \leq \alpha + \beta \leq 1$ and $\alpha, \beta \in [0,1]$. Then:

(1) If F is (1,1)-differentiable, then $F_L^+(t,\alpha), F_U^+(t,\alpha), F_L^-(t,\beta)$, and $F_U^-(t,\beta)$ are differentiable functions and $[F'(t)]_{(\alpha,\beta)} = \{[F'^+_L(t,\alpha), F'^+_U(t,\alpha)], [F'^-_L(t,\beta), F'^-_U(t,\beta)]\}$.

(2) If F is (1,2)- differentiable, then $F_L^+(t,\alpha), F_U^+(t,\alpha), F_L^-(t,\beta)$, and $F_U^-(t,\beta)$ are differentiable functions and $[F'(t)]_{(\alpha,\beta)} = \{[F'^+_L(t,\alpha), F'^+_U(t,\alpha)], [F'^-_U(t,\beta), F'^-_L(t,\beta)]\}$.

(3) If F is (2,1)- differentiable, then $F_L^+(t,\alpha), F_U^+(t,\alpha), F_L^-(t,\beta)$, and $F_U^-(t,\beta)$ are differentiable functions and $[F'(t)]_{(\alpha,\beta)} = \{[F'^+_U(t,\alpha), F'^+_L(t,\alpha)], [F'^-_L(t,\beta), F'^-_U(t,\beta)]\}$.

4. If F is (2,2)- differentiable, then $F_L^+(t,\alpha), F_U^+(t,\alpha), F_L^-(t,\beta)$, and $F_U^-(t,\beta)$ are differentiable functions and $[F'(t)]_{(\alpha,\beta)} = \{[F'^+_U(t,\alpha), F'^+_L(t,\alpha)], [F'^-_U(t,\beta), F'^-_L(t,\beta)]\}$.

Proof: See [17].

IV. Intuitionistic Fuzzy Cauchy Problem

A first order intuitionistic fuzzy differential equation is a differential equation of the form

$$y' = f(t,y), t \in I = [a,b] \tag{4.1}$$

where (i) y is an intuitionistic fuzzy function of the crisp variable t

(ii) $f(t,y)$ is an intuitionistic fuzzy function of the crisp variable t and the intuitionistic fuzzy variable y and

(iii) y' is the intuitionistic fuzzy derivative.

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If an initial value $y(t_0) = y_0$ {intiutionistic fuzzy number},we get an intuitionistic fuzzy Cauchy problem of first order $y' = f(t, y), y(t_0) = y_0$.
As each intuitionistic fuzzy number is a conjection two fuzzy numbers [18], Equation (4.1) can be replaced by an equivalent system as follows:

$y'(t) = \{[\underline{y'^{+}}(t), \overline{y'^{+}}(t)], [\underline{y'^{-}}(t), \overline{y'^{-}(t)}]\}$,

Where $\underline{y'^{+}}(t) = \min\{f(t,u)\backslash u \in [\underline{y^{+}}, \overline{y^{+}}]\}$

$= F(t, \underline{y'^{+}}(t), \overline{y'^{+}}(t)), \underline{y^{+}}(t_0) = \underline{y_0}^{+}$

(4.2)

$\overline{y'^{+}}(t) = \max\{f(t,u)\backslash u \in [\underline{y^{+}}, \overline{y^{+}}]\}$

$$= G\left(t, \underline{y'^{+}}(t), \overline{y'^{+}}(t)\right), \overline{y^{+}}(t_0) = \overline{y_0}^{+} \tag{4.3}$$

$\underline{y'^{-}}(t) = \min\{f(t,v)\backslash v \in [\underline{y^{-}}, \overline{y^{-}}]\}$

$$= F(t, \underline{y'^{-}}(t), \overline{y'^{-}}(t)), \underline{y^{-}}(t_0) = \underline{y_0}^{-} \tag{4.4}$$

$\overline{y'^{-}}(t) = \max\{f(t,v)\backslash v \in [\underline{y^{-}}, \overline{y^{-}}]\}$

$$= G(t, \underline{y'^{-}}(t), \overline{y'^{-}}(t)), \overline{y^{-}}(t_0) = \overline{y_0}^{-} \tag{4.5}$$

The system of equations given in (4.2) and (4.3) will have unique solution $[\underline{y^{+}}, \overline{y^{+}}] \in B = \bar{C}[0,1]X\bar{C}[0,1]$ and the system of equations given in (4.4) and(4.5) will have unique solution $[\underline{y^{-}}, \overline{y^{-}}] \in B = \bar{C}[0,1]X\bar{C}[0,1]$.Therefore the system given from(4.2) to (4.5) possesses unique solution $y'(t) = \{[\underline{y'^{+}}(t), \overline{y'^{+}}(t)], [\underline{y'^{-}}(t), \overline{y'^{-}(t)}]\} \in BXB$ which is an intuitionistic fuzzy function.
(i.e) for each t,
$y(t; r) = \{[\underline{y^{+}}(t;\alpha), \overline{y^{+}}(t;\alpha)], [\underline{y^{-}}(t;\beta), \overline{y^{-}}(t;\beta)]\}$, where $0 \le \alpha + \beta \le 1$ and $\alpha, \beta \in [0,1]$. is an intuitionistic fuzzy number.
The parametric form of the system of equations (4.2) to (4.5) is given by $\underline{y'^{+}}(t;\alpha) = F(t, \underline{y^{+}}(t;\alpha), \overline{y^{+}}(t;\alpha))$,

$$\underline{y^{+}}(t_0;\alpha) = \underline{y_0}^{+}(\alpha) \tag{4.6}$$

$\overline{y'^{+}}(t;\alpha) = G(t, \underline{y^{+}}(t;\alpha), \overline{y^{+}}(t;\alpha))$,

$$\overline{y^{+}}(t_0;\alpha) = \overline{y_0}^{+}(\alpha) \tag{4.7}$$

$\underline{y'^{-}}(t;\beta) = H(t, \underline{y^{-}}(t;\beta), \overline{y^{-}}(t;\beta))$,

$$\underline{y^{-}}(t_0;\beta) = \underline{y_0}^{-}(\beta) \tag{4.8}$$

$\overline{y'^{-}}(t;\beta) = I(t, \underline{y^{-}}(t;\beta), \overline{y^{-}}(t;\beta))$,

$$\overline{y^{-}}(t_0;\beta) = \overline{y_0}^{-}(\beta), \tag{4.9}$$

for $\alpha, \beta \in [0,1]$ and $0 \le \alpha + \beta \le 1$.

V. The Euler Method

In this section, we will present the Euler formula for the intuitionistic fuzzy differential equations. To solve the intuitionistic fuzzy system of ordinary differential system in $[t_0, t_1], [t_1, t_2], \dots, [t_k, t_{k+1}], \dots$,for $r \in [0,1]$,we will replace each interval $[t_k, t_{k+1}]$by a set of $N_k + 1$ regularly spaced points. The grid points on $[t_k, t_{k+1}]$ will be $t_{k,n} = t_k + nh_k$ where $h_k = \frac{t_{k+1} - t_k}{N_k}$ and $0 \le n \le N_k$.Now we will give algorithm to numerically solve the system in $[t_0, t_1], [t_1, t_2], \dots, [t_k, t_{k+1}], \dots$
Algorithm: To approximate the solution of the fuzzy initial value problem given by the system of equations in Eqs (4.6) to (4.9).

Case:1 (1,1)-Differentiability

Step: 1 Let $h_k = \frac{t_{k+1} - t_k}{N_k}$,$\underline{w_r^{+}}(t_{k,0}) = a_0$,

$\overline{w_r^{+}}(t_{k,0}) = a_1$, $\underline{w_r^{-}}(t_{k,0}) = b_0$ and $\overline{w_r^{-}}(t_{k,0}) = b_1$

Step: 2 Let i=1.

Step:3 Let

$\underline{w_r^{+}}(t_{k,i}) = \underline{w_r^{+}}(t_{k,i-1}) + hF_r(t_{k,i-1}, w_r^{+}(t_{k,i-1}))$;

$\overline{w_r^{+}}(t_{k,i}) = \overline{w_r^{+}}(t_{k,i-1}) + hG_r(t_{k,i-1}, w_r^{+}(t_{k,i-1}))$;

$\underline{w_r^{-}}(t_{k,i}) = \underline{w_r^{-}}(t_{k,i-1}) + hH_r(t_{k,i-1}, w_r^{-}(t_{k,i-1}))$;

$\overline{w_r^{-}}(t_{k,i}) = \overline{w_r^{-}}(t_{k,i-1}) + hI_r(t_{k,i-1}, w_r^{-}(t_{k,i-1}))$;

Step: 4 $t_{k,i+1} = t_{k,0} + (i+1)h_k$

Step: 5 Let i = i + 1

Step: 6 If $i \le N_k$, go to Step 3.

Step:7 The algorithm ends, and $[\underline{w_r^{+}}(t_{k+1}), \overline{w_r^{+}}(t_{k+1})]$ approximates the value of $[\underline{Y_r^{+}}(t_{k+1}), \overline{Y_r^{+}}(t_{k+1})]$ and $[\underline{w_r^{-}}(t_{k+1}), \overline{w_r^{-}}(t_{k+1})]$ approximates the value of $[\underline{Y_r^{-}}(t_{k+1}), \overline{Y_r^{-}}(t_{k+1})]$

Case:2 (1,2)-Differentiability

Step: 1 Let $h_k = \frac{t_{k+1} - t_k}{N_k}$,$\underline{w_r^{+}}(t_{k,0}) = a_0$,

$\overline{w_r^{+}}(t_{k,0}) = a_1$, $\underline{w_r^{-}}(t_{k,0}) = b_0$ and $\overline{w_r^{-}}(t_{k,0}) = b_1$

Step: 2 Let i=1.

Step:3 Let

$\underline{w_r^{+}}(t_{k,i}) = \underline{w_r^{+}}(t_{k,i-1}) + hF_r(t_{k,i-1}, w_r^{+}(t_{k,i-1}))$;

$\overline{w_r^{+}}(t_{k,i}) = \overline{w_r^{+}}(t_{k,i-1}) + hG_r(t_{k,i-1}, w_r^{+}(t_{k,i-1}))$;

$\underline{w_r^{-}}(t_{k,i}) = \underline{w_r^{-}}(t_{k,i-1}) + hI_r(t_{k,i-1}, w_r^{-}(t_{k,i-1}))$;

$\overline{w_r^{-}}(t_{k,i}) = \overline{w_r^{-}}(t_{k,i-1}) + hH_r(t_{k,i-1}, w_r^{-}(t_{k,i-1}))$;

Step: 4 $t_{k,i+1} = t_{k,0} + (i+1)h_k$

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

Step: 5 Let $i = i + 1$

Step: 6 If $i \le N_k$, go to Step 3.

Step:7 The algorithm ends, and $[\underline{w_r^+}(t_{k+1}), \overline{w_r^+}(t_{k+1})]$ approximates the value of $[\underline{Y_r^+}(t_{k+1}), \overline{Y_r^+}(t_{k+1})]$ and $[\underline{w_r^-}(t_{k+1}), \overline{w_r^-}(t_{k+1})]$ approximates the value of $[\underline{Y_r^-}(t_{k+1}), \overline{Y_r^-}(t_{k+1})]$

Case:3 (2,1)-differentiability

Step: 1 Let $h_k = \frac{t_{k+1}-t_k}{N_k}$, $\underline{w_r^+}(t_{k,0}) = a_0$,

$$\overline{w_r^+}(t_{k,0}) = a_1, \underline{w_r^-}(t_{k,0}) = b_0 \text{ and } \overline{w_r^-}(t_{k,0}) = b_1$$

Step: 2 Let i=1.

Step:3 Let

$$\underline{w_r^+}(t_{k,i}) = \underline{w_r^+}(t_{k,i-1}) + hG_r(t_{k,i-1}, w_r^+(t_{k,i-1}));$$
$$\overline{w_r^+}(t_{k,i}) = \overline{w_r^+}(t_{k,i-1}) + hF_r(t_{k,i-1}, w_r^+(t_{k,i-1}));$$
$$\underline{w_r^-}(t_{k,i}) = \underline{w_r^-}(t_{k,i-1}) + hH_r(t_{k,i-1}, w_r^-(t_{k,i-1}));$$
$$\overline{w_r^-}(t_{k,i}) = \overline{w_r^-}(t_{k,i-1}) + hI_r(t_{k,i-1}, w_r^-(t_{k,i-1}));$$

Step: 4 $t_{k,i+1} = t_{k,0} + (i+1)h_k$

Step: 5 Let $i = i + 1$

Step: 6 If $i \le N_k$, go to Step 3.

Step:7 The algorithm ends, and $[\underline{w_r^+}(t_{k+1}), \overline{w_r^+}(t_{k+1})]$ approximates the value of $[\underline{Y_r^+}(t_{k+1}), \overline{Y_r^+}(t_{k+1})]$ and $[\underline{w_r^-}(t_{k+1}), \overline{w_r^-}(t_{k+1})]$ approximates the value of $[\underline{Y_r^-}(t_{k+1}), \overline{Y_r^-}(t_{k+1})]$.

Case:4 (2,2)-differentiability

Step: 1 Let $h_k = \frac{t_{k+1}-t_k}{N_k}$, $\underline{w_r^+}(t_{k,0}) = a_0$,

$$\overline{w_r^+}(t_{k,0}) = a_1, \underline{w_r^-}(t_{k,0}) = b_0 \text{ and } \overline{w_r^-}(t_{k,0}) = b_1$$

Step: 2 Let i=1.

Step:3 Let

$$\underline{w_r^+}(t_{k,i}) = \underline{w_r^+}(t_{k,i-1}) + hG_r(t_{k,i-1}, w_r^+(t_{k,i-1}));$$
$$\overline{w_r^+}(t_{k,i}) = \overline{w_r^+}(t_{k,i-1}) + hF_r(t_{k,i-1}, w_r^+(t_{k,i-1}));$$
$$\underline{w_r^-}(t_{k,i}) = \underline{w_r^-}(t_{k,i-1}) + hI_r(t_{k,i-1}, w_r^-(t_{k,i-1}));$$
$$\overline{w_r^-}(t_{k,i}) = \overline{w_r^-}(t_{k,i-1}) + hH_r(t_{k,i-1}, w_r^-(t_{k,i-1}));$$

Step: 4 $t_{k,i+1} = t_{k,0} + (i+1)h_k$

Step: 5 Let $i = i + 1$

Step: 6 If $i \le N_k$, go to Step 3.

Step:7 The algorithm ends, and $[\underline{w_r^+}(t_{k+1}), \overline{w_r^+}(t_{k+1})]$ approximates the value of $[\underline{Y_r^+}(t_{k+1}), \overline{Y_r^+}(t_{k+1})]$ and $[\underline{w_r^-}(t_{k+1}), \overline{w_r^-}(t_{k+1})]$ approximates the value of $[\underline{Y_r^-}(t_{k+1}), \overline{Y_r^-}(t_{k+1})]$.

VI. NUMERICAL EXAMPLES

Let us consider the Weight-Loss problem:

Over-weight people on diet and treatment gradually reduce their weight W.The time rate of decrease of weight W is assumed to be proportional to the weight W itself.In the model, uncertainty is introduced if we have uncertain information on the initial weight W_0 of over-weight people. However, in some situations, there may be hesitation about over-weight. In order to take into account the uncertainty and hesitation, we consider initial value weight W_0 as a triangular intuitionistic fuzzy number (120,130,135:115,130,140) lb. Let us find the weight after 30 days? (The constant of proportionality is $k = \frac{1}{3500}$ lb/cal)

Solution:

$$\frac{dW}{dt} = -kW \qquad (6.1)$$

with the initial condition

$W(t_0) = (120,130,135{:}\,115,130,140)$,

an intuitionistic fuzzy number.

The (α,β)-cut of $W(t_0)$ is given by

$W(t_0,\alpha,\beta) = W_0(\alpha,\beta) =$

$\{[\underline{W_0^+}(\alpha), \overline{W_0^+}(\alpha)], [\underline{W_0^-}(\beta), \overline{W_0^-}(\beta)]\}$,

(i.e) $W(t_0,\alpha) = W_0(\alpha)$

$= \{[120 + 10\alpha, 135 - 5\alpha], [115 + 15\beta, 140 - 10\beta]\}$,

$0 \le \alpha + \beta \le 1$ and $\alpha, \beta \in [0,1]$.

Case[1] (1,1)-Differentiability:

The exact solution of Eq(5.1) under (1,1)-differentiability is given by

$$\underline{W^+}(t,\alpha) = (-7.5 + 7.5\alpha)e^{kt} + (127.5 + 2.5\alpha)e^{-kt},$$
$$\overline{W^+}(t,\alpha) = -(-7.5 + 7.5\alpha)e^{kt} + (127.5 + 2.5\alpha)e^{-kt},$$
$$\underline{W^-}(t,\beta) = (-12.5 + 12.5\beta)e^{kt} + (127.5 + 2.5\beta)e^{-kt},$$
$$\overline{W^-}(t,\beta) = -(-12.5 + 12.5\beta)e^{kt} + (127.5 + 2.5\beta)e^{-kt}.$$

Comparison of exact and approximate solutions of Eq(6.1) at t =30 with h=0.1 is given Fig.6.1

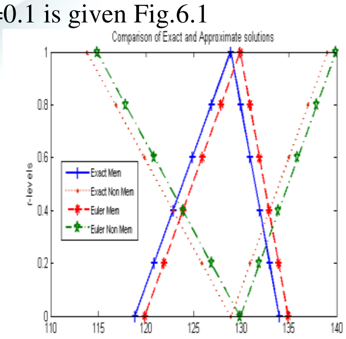

Fig.6.1

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Case[2] (1,2)-Differentiability

The exact solution of Eq (6.1) under (1,2)-differentiability is given by

$\underline{W^+}(t,\alpha) = (-7.5 + 7.5\alpha)e^{kt} + (127.5 + 2.5\alpha)e^{-kt}$,

$\overline{W^+}(t,\alpha) = -(-7.5 + 7.5\alpha)e^{kt} + (127.5 + 2.5\alpha)e^{-kt}$,

$\underline{W^-}(t,\beta) = (115 + 15\beta)e^{-kt}$,$\overline{W^+}(t,\beta) = (140 - 10\beta)e^{-kt}$.

Absolute errors between exact and approximate solutions of membership and non membership functions Eq (6.1) at t =30 with h=0.1 is given Table.6.1.

Table.6.1

R	Error Mem	Error Non Mem
0	1.957892599	1.957892599
0.2	1.96557061	1.96557061
0.4	1.97324862	1.97324862
0.6	1.98092663	1.98092663
0.8	1.98860464	1.98860464
1	1.996282652	1.996282652

Case[3] (2,1)-Differentiability

The exact solution of Eq (6.1) under (2,1)-differentiability is given by

$\underline{W^+}(t,\alpha) = (120 + 10\alpha)e^{-kt}$,$\overline{W^+}(t,\alpha) = (135 - 5\alpha)e^{-kt}$,

$\underline{W^-}(t,\beta) = (-12.5 + 12.5\beta)e^{kt} + (127.5 + 2.5\beta)e^{-kt}$,

$\overline{W^-}(t,\beta) = -(-12.5 + 12.5\beta)e^{kt} + (127.5 + 2.5\beta)e^{-kt}$.

Absolute errors between exact and approximate solutions of membership and non membership functions Eq (6.1) at t =30 with h=0.1 is given Table.6.2.

Table.6.2

R	Error Mem	Error Non Mem
0	1.957892599	1.957892599
0.2	1.965570609	1.96557061
0.4	1.97324862	1.97324862
0.6	1.98092663	1.98092663
0.8	1.98860464	1.98860464
1	1.996282652	1.996282652

Case [4] (2,2)-Differentiability

The exact solution of Eq(6.1) under (2,2)-differentiability is given by

$\underline{W^+}(t,\alpha) = (120 + 10\alpha)e^{-kt}$,$\overline{W^+}(t,\alpha) = (135 - 5\alpha)e^{-kt}$,

$\underline{W^-}(t,\beta) = (115 + 15\beta)e^{-kt}$,$\overline{W^+}(t,\beta) = (140 - 10\beta)e^{-kt}$.

Comparison of exact and approximate solutions at t =30 with h=0.1 is given Fig.6.2

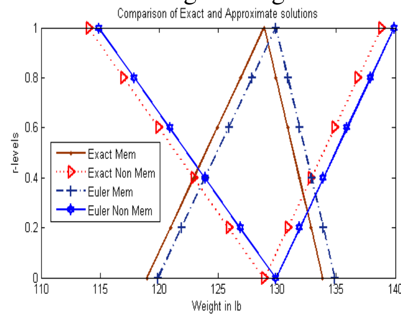

Fig.6.2

The solutions of Eq(6.1) in cases(1,1),(1,2) and (2,1) have an increasing length of its support, which leads us to the conclusion that there is a possibility that, the weight of the person on diet and treatment increases as days goes on and even a non-zero possibility that it is negative. Fortunately, the real situation is different, and the weight always decreases with time and it cannot be negative. Therefore, (2,2)-differentiability is suitable for this type of problems.

In all the cases, the numerical solutions of Eq(6.1) by Euler method is closer to the exact solutions. However, the errors can be minimised by taking smaller step size h .

VII. Conclusion

We propose a general numerical procedure for treating intuitionistic fuzzy Cauchy problems in the (α,β)-cut representation. The original initial value problem is replaced by four set of parametric ordinary differential equations, where each set consists of four simultaneous equations which are then solved numerically using classical algorithms. In this work the standard Euler approximation method is used. The method's applicability is illustrated by solving a first order intuitionistic fuzzy differential equation. In future, higher order methods will be used to study numerical solution of intuitionistic fuzzy differential equations.

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