



A New Approach on Finding Fuzzy Shortest Path Using Probability Measure

R.Sathya

Department of Mathematics,
Sona Group of Institution,
Salem - 636005,
E-mail: satmepri@gmail.com

Abstract

In this Paper Fuzzy Shortest Path Problem is determined based on mean value of Probability Measure. An acyclic network is consider for finding Fuzzy Shortest Path and Fuzzy Shortest Path Length from source node to destination node where as discrete Fuzzy Number is assigned for each arc length in the given network. An illustrative example also included to demonstrate our proposed algorithm.

Key words : Discrete fuzzy number
Probability Measure Fuzzy
Shortest Path Fuzzy Shortest
Path Length

1. INTRODUCTION

The Shortest path Problem was one of the first network problem and is widely applied in transportation, communication and computer network and it has been studied extensively in the fields of Computer Science, Operations Research and Transportation Engineering and so on.

In 1965, Zadeh [11] introduced the concept of fuzzy set theory which offers a powerful tool to deal with this case. In 1978, Dubois and Prade [4] defined any of the fuzzy numbers as a fuzzy subset of the real line. They itself introduced fuzzy shortest path problem [5] in the year 1980. Yager [10] studied a path problem in term of possibility production system in the year 1986. In 1997, Heilpern [6] proposed three definitions of the distance method is generated by expect values of fuzzy numbers.

Ranking fuzzy number by sign distance and universal approximation of fuzzy functions on a discrete set of points were developed by S. Abbasbandy and M. Amirfakhrian [1,2] in 2006. Lee and Li [7] developed the comparison of fuzzy numbers based on probability measure of fuzzy

events. Here, the mean of probability measure is used for finding fuzzy shortest path and fuzzy shortest path length with discrete fuzzy number.

2. PRELIMINARIES

2.1 Fuzzy Set:

X is a collection of objects denoted generally by x, then a fuzzy set A in X is defined as a set of ordered pairs $\tilde{A} = \{(x, \mu_A(x)) / x \in X\}$ where $\mu_A(x)$ is called the membership function maps each element of X to a membership grade (or membership value) between 0 and 1.

2.2 Discrete Fuzzy Number:

A fuzzy subset u of R with membership mapping $u : R \rightarrow \dots x_n$ such that $\text{supp}(u) = \{x_1, x_2, \dots, x_n\}$, and there are natural numbers s, t with $1 \leq s \leq t \leq n$ such that:

1. $u(x_i) = 1$ for any natural number and i with $s \leq i \leq t$ (core)
2. $u(x_i) \leq u(x_j)$ for each natural number i, j with $1 \leq i \leq j \leq s$.
3. $u(x_i) \geq u(x_j)$ for each natural number i, j with $t \leq i \leq j \leq n$.
- 4.

2.3 Addition of two discrete fuzzy numbers:

The addition of fuzzy sets A and B is denoted by $A + B$ and the membership function of $A + B$ is given by

$$(A + B)(z) = \sup_{z=x+y} \min \{A(x), B(y)\} \quad \forall z \in R$$

2.4 Mean of a Probability Measure:

The mean of a fuzzy event A relative to a probability measure P defined as follows:

$$M_p(A) = \frac{\int_A x \mu_A^2(x) dx}{\int_A \mu_A^2(x) dx}$$



For discrete universe of discourse \int is replaced by \sum .

3. ALGORITHM

Step 1 : Formation of Possible Paths

Form the possible paths from source node to destination node on given network.

Step 2 : Computation of Paths Length

compute the path lengths, L_i , $i = 1, 2, \dots, n$ for possible n paths.

Step 3 : Computation of Mean of Probability Measure . Compute the Mean value for all possible paths length

$$M(L_i) = \frac{\sum_{j=1}^n x_j \mu_A^2(x_j)}{\sum_{j=1}^n \mu_A^2(x_j)}$$

Step 4 : Fuzzy Shortest Path and Length

Decide the shortest path with minimum mean value and that corresponding path length is the shortest path length.

4. NUMERICAL EXAMPLE

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 10 edges with discrete fuzzy number.

Solution:

The edge Lengths are

$P = \{ 0.4/2, 0.3/3 \}$

$Q = \{ 0.3/1, 0.7/3 \}$

$R = \{ 0.6/2, 0.2/3, 0.1/4 \}$

$S = \{ 0.7/2, 0.5/4 \}$

$T = \{ 0.6/3, 0.4/4 \}$

$U = \{ 0.5/4, 0.6/5, 0.5/6 \}$

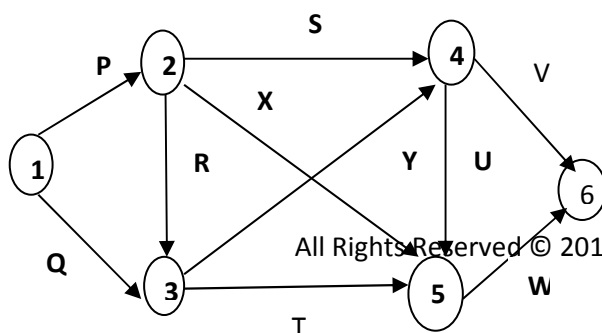
$V = \{ 0.8/4, 0.6/5 \}$

$W = \{ 0.7/3, 0.3/5 \}$

$X = \{ 0.2/1, 0.6/3 \}$

$Y = \{ 0.6/3, 0.5/4 \}$

Fig 4.1



Step 1 : Formation of Possible Paths

Form the possible paths from source node to destination node on given network

$P_1 = 1 - 2 - 4 - 5 - 6$

$P_2 = 1 - 2 - 3 - 5 - 6$

$P_3 = 1 - 3 - 5 - 6$

$P_4 = 1 - 2 - 4 - 6$

$P_5 = 1 - 2 - 5 - 6$



$$P_6 = 1 - 3 - 4 - 6$$

$$P_7 = 1 - 2 - 3 - 4 - 6$$

$$P_8 = 1 - 2 - 3 - 4 - 5 - 6$$

$$P_9 = 1 - 3 - 4 - 5 - 6.$$

Step 2 : Computation of Paths Length

compute the path lengths, L_i , $i =$

1,2,...,n for possible n paths.

$$L_1 = \{ 0.4/11, 0.4/12, 0.4/13, 0.4/14, 0.4/15, 0.3/16, 0.3/17, 0.3/18 \}$$

$$L_2 = \{ 0.4/10, 0.4/11, 0.3/12, 0.3/13, 0.3/14, 0.2/15, 0.1/16 \}$$

$$L_3 = \{ 0.3/7, 0.3/8, 0.6/9, 0.4/10, 0.3/11, 0.3/12 \}$$

$$L_4 = \{ 0.4/8, 0.4/9, 0.4/10, 0.4/11, 0.3/12 \}$$

$$L_5 = \{ 0.2/6, 0.2/7, 0.4/8, 0.4/9, 0.3/10 \}$$

$$L_6 = \{ 0.3/8, 0.3/9, 0.6/10, 0.6/11, 0.5/12 \}$$

$$L_7 = \{ 0.4/11, 0.4/12, 0.4/13, 0.3/14, 0.2/15, 0.1/16 \}$$

$$L_8 = \{ 0.4/14, 0.4/15, 0.3/16, 0.3/17, 0.2/18, 0.3/19, 0.3/20, 0.2/21, 0.1/11 \}$$

$$L_9 = \{ 0.3/11, 0.3/12, 0.5/13, 0.6/14, 0.5/15, 0.5/16, 0.3/17, 0.3/18 \}$$

Step 3 : Computation of Mean of Probability Measure

Compute the Mean value for all possible paths length

$$M(L_i) = \frac{\sum_{j=1}^n x_j \mu_A^2(x_j)}{\sum_{j=1}^n \mu_A^2(x_j)}$$

$$M(L_1) = \frac{1.76 + 1.92 + 2.08 + 2.24 + 2.4 + 1.44 + 1.53 + 1.62}{0.16 + 0.16 + 0.16 + 0.16 + 0.16 + 0.09 + 0.09 + 0.09}$$

$$M(L_1) = 14.009$$

$$M(L_2) = 11.922$$

$$M(L_3) = 9.386$$

$$M(L_4) = 9.8082$$

$$M(L_5) = 8.4489$$

$$M(L_6) = 10.513$$

$$M(L_7) = 12.548$$

$$M(L_8) = 16.7532$$

$$M(L_9) = 14.4625$$

Step 4 : Fuzzy Shortest Path and Length

Decide the shortest path with minimum mean value and that corresponding path length is the shortest path length. Here $M(L_5)$ is having minimum Mean value. Hence $P_5 = 1 - 2 - 5 - 6$ is the shortest path and that corresponding path length is the shortest path length

$$L_5 = \{ 0.2/6, 0.2/7, 0.4/8, 0.4/9, 0.3/10 \}$$

5. CONCLUSION



This paper defines a solution for a fuzzy shortest path problem with discrete fuzzy number. The introduced algorithm is based on probability measure for finding fuzzy shortest path is effective in ranking fuzzy number and with minimum number of processing steps. It provides better output for more number of vertices and edges in a network.

REFERENCES

1. S. Abbasbandy & M. Amirfakhrian, A new approach to universal approximation of fuzzy functions on a discrete set of points, *Applied Mathematical Modelling*, 30: 2006, pp.1525 – 1534.
2. S. Abbasbandy & B. Asady, Ranking of fuzzy numbers by sign distance, *Information Sciences*, 176: 2006, pp. 2405 – 2416.
3. V. Anusuya and R. Sathya, “Distance based similarity measure for discrete fuzzy shortest path”, *Proceedings of the International conference on Mathematics – A Global Scenario*, 2012, pp. 55 – 58.
4. D. Dubois and H. Prade, Operations on fuzzy numbers, *J. Systems Sci.*, vol.9, 1978, pp. 613 – 626.
5. D. Dubois and H. Prade, *Fuzzy sets and systems, Theory and Applications*, Academic Press, New York, 1980.
6. S. Heilpern, Representation and Application of fuzzy numbers, *Fuzzy Sets and systems*, Vol.91, No.2, 1997, pp. 259 – 268.
7. E.S. Lee and R.J. Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, *Comp. Math. Applic.*, Vol.15, No.10, 1988 pp. 887 – 896.
8. Nagoorgani, A. and A. Mumtaj Begam, A new approach on shortest path in Fuzzy environment, *ICTACT Journal on soft computing*, Oct. 2010, Issue 02.
9. Xiaoyu Ji, Kakuzo Iwamura, Zhen shao, New models for shortest path problem with fuzzy arc lengths, *Applied mathematical modeling*, 31, 2007, pp. 259 – 269.
10. R. Yager, Paths of least resistance on Possibilistic production systems, *Fuzzy Set. Syst.*, 19 1986, pp. 121 – 132.
11. L. A. Zadeh, *Fuzzy sets, Information and control* 8, 1965, pp. 338-356.