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STRUCTURE OF D-RINGS AND D* - RINGS

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ABSTRACT

In this paper we present some properties of D-ring and D*-ring. Using these properties, it is shown that if R is a D-ring satisfying $xy = (xy)^2p(x,y)$ for all $x, y \in R$ and $p(x,y)$ is a polynomial in two non-commuting indeterminates$x$ and y, then R is either a zero ring or a periodic field. Also we prove that any normal D*-ring is either periodic or D-ring.

Keywords:D-rings, D*-rings, zero divisors, nilpotent elements, idempotent elementsand periodic rings.

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I.INTRODUCTION

In 2001, Abu-khuzamet.al., [2] introduced the structure of certain classes of D-rings and periodic D-rings. In 2006, Abu-khuzam and Yaqub [1] extended the concept to the D*-rings and periodic D*-rings. In this paper, we discuss some results on D-rings and D*-rings and we prove that if R is a D-ring satisfying $xy = (xy)^2p(x,y)$ for all $x, y \in R$ and $p(x,y)$ is a polynomial in two non-commuting indeterminates$x$ and y, then R is either a zero ring or a periodic field. Also we prove that any normal D*-ring is either periodic or D-ring.

II. PRELIMINARIES

A ring R is a D-ring, if its every zero divisor is nilpotent. A ring R is a D*-ring, if every zero divisor x in R can be written as $x = a + b$, where $a\in N, b\in P$ and $ab = ba$. Nil ring, every domain of R and the ring of integers (modp^k), p prime are examples of D-rings. Clearly every D-ring is a D*-ring, but converse is not true. A Boolean ring is a D*-ring but not a D-ring.

Throughout this paper, R is an associative ring; C denotes the center of R, N denotes the set of nilpotents of R, P denotes the set of potents of R,$p(x,y)$ is a polynomial in two non-commuting indeterminates$x$ and y. [4] discussed about a method, End-to-end inference to diagnose and repair the data-forwarding failures, our optimization goal to minimize the faults at minimum expected cost of correcting all faulty nodes that cannot properly deliver data. First checking the nodes that has the least checking cost does not minimize the expected costin fault localization. We construct a potential function for identifying the candidate nodes, one of which should be first checked by an optimal strategy. We proposes efficient inferring approach to the node to be checked in large-scale networks.

We start with the following properties of D-rings.

Lemma 1 : Let R be a D-ring. Then aR is a nil right ideal for all $a\in N$.

Proof: Since $a^k = 0$, $a^{k-1}\neq 0$ implies $a^{k-1}(ax) = 0$, and thus $ax \in N$.
Hence aR is a nil right ideal of R. □

Lemma 2: Let R be a D-ring. If e is an idempotent element of R, then $e = 0$ or $e = 1$.

Proof: Suppose $e^2 = e\neq 0$, and $x\in R$.
Then $e(ex-x) = 0$ and hence $ex - x = 0$.
Otherwise, e will be nilpotent, since R is a D-ring. Thus $e = 0$.
Similarly, $xe - x = 0$ for all x in R, and thus $e = 1$. □

Theorem 3: Let R be a D-ring such that N is an ideal of R. Then, either $R = N$ or R/N is a domain.

Proof: Suppose that $R\neq N$.

Let $\overline{x} = x + N$ and $\overline{y} = y + N$ be two elements in *R/N* such that $xy = 0$.

Then $xy \in N$.
This implies that $(xy)^m = 0$ and $(xy)^{m-1} \neq 0$ for some positive integer *m*.
Hence $(xy)^{m-1}(xy) = 0$.
This implies that *y* is a zero divisor or $(xy)^{m-1} x = 0$.
Therefore, *y* is a zero divisor or *x* is a zero divisor, since $(xy)^{m-1} \neq 0$.

Hence $y \in N$ or $x \in N$, so $\overline{x} = \overline{0}$ or $\overline{y} = \overline{0}$.

Thus *R/N* is a domain. □

Corollary 4: Let *R* be a D-ring with *N* commutative. Then either $R = N$ or *N* is an ideal and *R/N* is a domain.

Proof: If *N* is commutative, *N* is an additive subgroup of *R*, hence an ideal by Lemma 1.
This corollary follows from the above theorem. □

Theorem 5: If *R* is a periodic D-ring, then *R* is either nil or local. Further, if *R* has an identity element, then *N* is an ideal and *R/N* is a field.

Proof: Since *R* is periodic, for each $x \in R$, there exists a positive integer $k = k(x)$ such that x^k is idempotent [3]. Using Lemma 2, $x^k = 0$ or $x^k = 1$, and hence *x* is either nilpotent or invertible.
Therefore, *R* is nil or local.
If *R* has an identity element, then *R* is local and hence *N* is an ideal.
Thus *R/N* is a periodic division ring and hence *R/N* is a field. □

Theorem 6: If *R* is a periodic D-ring, then *C*(*R*) is nil.

Proof: If *R* is nil, there is nothing to prove.
Suppose $R \neq N$, and let $x \in R/N$.
Then $x^n = x^m$ for some integers $n > m \geq 1$.
It is easily verified that $x^{m(n-m)}$ is a nonzero idempotent, and hence by Lemma 2, $1 \in R$.
By theorem 5, *R* is local, *N* is an ideal, and *R/N* is a field.
Thus, *C*(*R*) is nil. □

Now we prove that a D-ring with a condition is either a zero ring or a periodic field.

Theorem 7: Let *R* be a D-ring such that for each $x, y \in R$ there exists a polynomial $p(x,y)$ in two noncommutingindeterminates, with integer coefficient, for which

$$xy = (xy)^2\, p(x,y). \qquad \text{…………………………} \quad (1)$$

Then *R* is either a zero ring or a periodic field.

Proof: Theorem 1 states that any ring *R* satisfying (1) is a direct sum of a J-ring and a zero ring. In view of theorem 12 [2], a D-ring with (1) must be either a J-ring or a zero ring. By Lemma 2, D-rings which are also J-rings must be periodic division rings; and J-rings are commutative by Jacobson's famous "$a^n = a$ Theorem". □

Now we present a result on a ring *R* in which every zero divisor is potent.

Theorem 8: Let *R* be a ring in which every zero divisor is potent. Then $N = \{0\}$ and *R* is normal. Moreover, if *R* is not a domain, then $J = \{0\}$.

Proof: If $a \in N$, then a is a zero divisor and hence potent by hypothesis.

So an = a for some positive integer n, and since $a \in N$, there exist a positive integer k such that $0 = a^{n^k} = a$.
So N = {0}.
Let e be any idempotent element of R and x is any element of R.
Then $ex - exe \in N$, and hence $ex - exe = 0$.
Similarly, xe = exe. So ex = xe and R is normal.
Let x be a nonzero divisor of zero.
Then xJ consists of zero divisors, which are potent.
Therefore xJ = {0}.
But then J consists of zero divisors, hence potent elements, and therefore J = {0}. □

Next we prove some results on D*-rings.
Theorem 9: A ring R is a D*-ring if and only if every zero divisor of R is periodic.
Proof: We assume R is a D*-ring and let x be any zero divisor. Then

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$x = a + b,\ a \in N,\ b \in P,\ ab = ba.$

So (x – a) = b = bn = (x – a)n.

This implies, since x commutes with a, that (x – a) = (x – a)n = xn + sum of pairwise commuting nilpotent elements.

Hence x – xn∈ N for every zero divisor x.

Since each such x is included in a subring of zero divisors, which is periodic by Chacron's theorem, x is periodic.

Suppose, conversely, that each zero divisor is periodic.

Then by the proof of Lemma 1 [5], R is a D*-ring. □

Theorem 10: If R is any normal D*-ring, then either R is periodic or R is a D-ring. Moreover, aR⊆N for each a ∈ N.

Proof: If R is a normal D*-ring which is not a D-ring, then R has a central idempotent zero divisor e.

Then R = eR⊕A(e), where eR and A(e) both consist of zero divisors of R, hence (in view of Theorem 10) are periodic. Therefore R is periodic.

Now consider a∈ N and x ∈ R.

Since ax is a zero divisor, hence a periodic element, (ax)j = e is a central idempotent for some j.

Thus (ax)j+1 = (ax)j ax = a2y for some y ∈ R.

By repeating this argument we see that for each positive integer k, there exists m such that $(ax)^m = a^{2^k} w$ for some w ∈ R.

Therefore aR⊆ N. □

Corollary 11: Let R be a D*-ring which is not a D-ring.

If N ⊆ C, then R is commutative.

Proof: Since N ⊆ C, R is normal. Therefore commutativity follows from theorem 10 and a theorem of Herstein. □

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