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EDGE BASED BASIS FUNCTIONS IN THE BEM SOLUTION OF HEAT CONDUCTION PROBLEM

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ABSTRACT

Boundary element solution is developed for a heat conduction problem in this work. The basis functions are defined usually on the nodes that are created on the mesh. This method is followed in finite element method and boundary element methods traditionally. In this work, in contrast to the traditional finite element method and boundary element methods, the basis functions are defined on the edges rather than on nodes. By defining the basis functions on the edges, the solution improves since the number of edges on the surface triangular mesh is always two times higher than that of nodes approximately for a closed body. The boundary element method is based on defining the basis functions on the edges that are resulted from triangular patch modeling. The numerical solution is tested with the exact solution. The exact solution is derived from the laws of heat conduction. The numerical results obtained are in well agreement with the exact solution for the case of sphere. The sphere is defined with a fixed temperature on the surface.

Keywords-- Boundary Integral Equations, Thermal Conduction, Edge Based Basis Functions, Error Analysis.

I INTRODUCTION

In this work, a three dimensional het conduction problem is solved using the BEM solution using the edge based basis functions. The heat conductions problems are usually formulated using differential equations and the solutions are based on the FDM or FEM. Of late, there is a great interest in the development of solutions using integral equation formulation among the research community. The integral equation formulation are solved numerically using BEM solutions. In BEM solutions only the boundary of the geometry are discretized in contrast to the discretization of the entire volume of interest.

BEM is usually implemented for the structural analysis problem and is popularly known as computational mechanics [1, 2]. The solution in the BEM involves defining the unknown functions on the nodes of the discretization. This is similar to the FEM and they are known as shape functions. However, in the acoustic scattering problems, the unknown functions in the numerical solution are defined distinctly; [1, 2] and they are

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referred to as basis function in this work. The numerical solution is referred to as the method of moment's solution in ref [1, 2]. In method of moment's solution procedure, the basis functions can be defined anywhere in the geometry like on the faces, edges or nodes. The BEM or FEM so special cases of the method of moment's solution procedure. If the method of moment's solution procedure is aimed at solving the integral equations, it can be treated as BEM. In ref [3, 4], the basis functions are defined on the faces. In this approach, authors have not used the concept of defining the basis or shape functions on the nodes. The basis functions are defined only on the faces. However, the problem solved in [3, 4] is acoustic scattering problem. Authors used the face based basis functions to solve a three dimensional heat conduction problem using BEM. In [6], the basis functions are defined on the edges in contrast to defining it on the faces to solve an acoustic scattering problem yet again. In this work, an endeavor is made to solve a heat conducting problem with the edge based basis functions similar to that defined in ref [6]. Other numerical methods that are available [7-15] to address the problem of heat conduction. [5] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. Utilizing the versatile back venturing method, every one of the six DOF are effectively followed utilizing just four control inputs while within the sight of un demonstrated flow and limited unsettling influences. Elements and speed vectors were thought to be inaccessible, along these lines a NN eyewitness was intended to recoup the limitless states. At that point, a novel NN virtual control structure which permitted the craved translational speeds to be controlled utilizing the pitch and the move of the UAV. At long last, a NN was used in the figuring of the real control inputs for the UAV dynamic framework. Utilizing Lyapunov systems, it was demonstrated that the estimation blunders of each NN, the spectator, Virtual controller, and the position, introduction, and speed following mistakes were all SGUUB while unwinding the partition Principle.

II. GENERAL PROCEDURE FOR BEM

For an equation resulting from governing equations, which is in the form of

$$\boldsymbol{L}\,\boldsymbol{f} = \boldsymbol{g} \tag{1}$$

Where

$\boldsymbol{L}$: Linear operator,

$\boldsymbol{g}$: Known function resulting from the forcing function

$\boldsymbol{f}$: Unknown function to be determined,

The solution can be derived as follows:

Let the unknown function $\boldsymbol{f}$ be approximated by a set of known functions $\boldsymbol{f}_j$, $j = 1,2,...,N$

$$\boldsymbol{f} = \sum_{n=1}^{N} \beta_j \boldsymbol{f}_j \tag{2}$$

Where

$\boldsymbol{f}_j$: Basis functions in the domain of $\boldsymbol{L}$

β_j are scalar coefficients to be determined.

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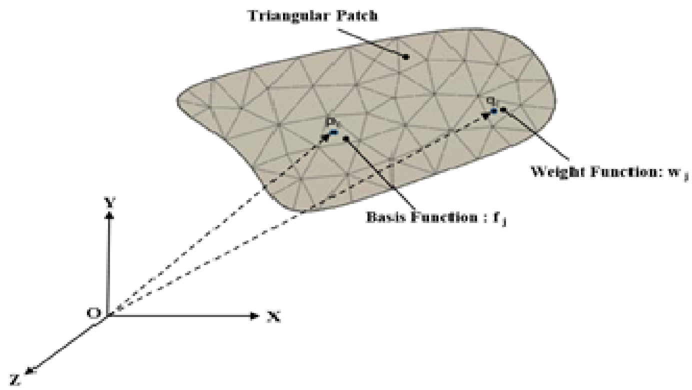

Fig.1: Edge Based Basis Functions

Substituting Eq. 2 into Eq. 1

$$\sum_{n=1}^{N} \beta_j \boldsymbol{L} \boldsymbol{f}_j = \boldsymbol{g} \tag{3}$$

Where the equality is usually approximate.

Let $\boldsymbol{w}_i$ testing functions in the range of $\boldsymbol{L}$. Now, taking the inner product of Eq. 3 with each w_i and using the linearity of inner product defined as $\langle \boldsymbol{f}, \boldsymbol{g} \rangle = \int_s \boldsymbol{f} \bullet \boldsymbol{g} \; ds$, we obtain a set of linear equations, given by

$$\sum_{n=1}^{N} \beta_j \langle \boldsymbol{w}_i, \boldsymbol{L} \boldsymbol{f}_j \rangle = \langle w_i, \boldsymbol{g} \rangle \quad i = 1,2,\ldots, N \tag{4}$$

The set of equations in Eq. 4 may be written in the matrix form as

$$\boldsymbol{ZX} = \boldsymbol{Y} \tag{5}$$

Which can be solved for $\boldsymbol{Z}$ using any standard linear equation solution methodologies. The simplicity, accuracy and efficiency of the BEM lies in choosing proper set of basis/testing functions and applying to the problem at hand. In this work, we propose a special set of basis functions and a novel testing scheme to obtain accurate results.

III.MATHEMATICAL FORMULATION

Let T is the scalar thermal potential satisfying the Helmholtz differential equation $\nabla^2 T + k^2 T = 0$ for the time harmonic waves present in the region exterior to the surface S of the body. Another condition that thethermal potential must satisfy is the appropriate boundary conditions on the surface S of the body.

Using the potential theory and the free space Green's function, the scattered thermal potential T^s may be defined as

$$T^s = \int_s \sigma(\boldsymbol{p}) G(\boldsymbol{p}, \boldsymbol{q}) ds' \tag{6}$$

In the above three equation,

σ Is the source density function dependent of $p$over the surface of the body,

P is the position vector of source points, with respect to a global co-ordinate system O.

q is the position vector of observation points, with respect to a global co-ordinate system O.

$G(\boldsymbol{p}, \boldsymbol{q})$ Is the free space Green's function.

$$G(\boldsymbol{p}, \boldsymbol{q}) = \frac{e^{k|p-q|} - 1}{|p-q|} \tag{7}$$

For a body, that has the fixed temperature defined on the surface of the body, the total thermal potential is zero, i.e.

$$\Phi^i + \Phi^s = 0 \tag{8}$$

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Hence,

$$\int_s \sigma(\boldsymbol{p}) G(\boldsymbol{p},\boldsymbol{q})\, ds' = -T^i \tag{9}$$

IV.NUMERICAL SOLUTION PROCEDURE

The surface the body is discretized into triangular patches. For a closed body, each edge of the triangular patch modeling, has two adjacent triangular patches. Let each triangle patch associated with the edge n is denoted as T_n^+ and T_n^-. The basis function is covered over the two adjacent triangles on the defined as. The area S_n is formed by connecting the centroids of the triangles T_n^+ and T_n^- with the nodes of the edge *n*. The edge based basis function may be defined as

$$f_j = \begin{cases} 1 & p_j \in S_n \\ 0 & otherwise \end{cases} \tag{10}$$

And the source distribution function may be defined as,

$$\sigma(p) = \sum_{n=1}^{N_e} \beta_j \boldsymbol{f}_j \tag{11}$$

Similarly, weight function may be defined as $w_j = \begin{cases} 1 & q_j \in S_n \\ 0 & otherwise \end{cases}$ (12)

The numerical solution procedure to solve the Eq. 9.Using the basis functions is explained below.
Testing Eq. 9 with a testing function w_m , results in

$$\left\langle w_q, \int_s \sigma(\boldsymbol{p})\, G(\boldsymbol{p},\boldsymbol{q}) ds' \right\rangle = \left\langle w_q, T^i \right\rangle \tag{13}$$

Using the inner product definition, 3, Eq. 4 can be written as

$$\int_s w_q \int_s \sigma(\boldsymbol{p})\, G(\boldsymbol{p},\boldsymbol{q}) ds'\, ds = \int_s w_q T^i ds \tag{14}$$

Let
Approximating the Eq. 14 over the source triangular patch,

$$\int_s w_q \int_s \sum_{j=1}^{N_e} \beta_j \boldsymbol{f}_j G(\boldsymbol{p_c},\boldsymbol{q})\, ds'\, ds = \int_s w_q T^i ds \tag{15}$$

Approximating the integration over the field triangular patch at the centroids, Eq. 15 becomes

$$\int_s w_i \int_s \sum_{j=1}^{N_e} \beta_j \boldsymbol{f}_j G(\boldsymbol{p_c},\boldsymbol{q}_c) ds'\, ds = \int_s w_i T^i ds \tag{16}$$

$$A_i \sum_{j=1}^{N} \beta_j \boldsymbol{f}_j\, G(\boldsymbol{p_c},\boldsymbol{q_c}) ds' = A_i\, T^i \tag{17}$$

Where A_i is the area of field triangular patch. This approximation is justified because the domains are sufficiently small, which is a necessary condition to obtain accurate solution using BEM. For a pulse function defined on the source triangular patch, it results in a system of linear equations, which can be represented in the matrix form as

$$\boldsymbol{Z}\ \boldsymbol{X} = \boldsymbol{Y} \tag{18}$$

Where $\boldsymbol{Z}$ is the impedance matrix of the single layer formulation of size $N_f\ X\ N_f$, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are the column vectors of size N_f .The elements of $\boldsymbol{Z}$, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are given below.

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$$\boldsymbol{Z}_{m,n} = \frac{A_m^+}{3}\int_{s+} G\left(p_c^+, q_c^+\right)ds' + \frac{A_m^-}{3}\int_{s-} G\left(p_c^-, q_c^-\right)ds'$$
$$\frac{A_m^+}{3}\int_{s-} G\left(p_c^-, q_c^+\right)ds' + \frac{A_m^-}{3}\int_{s+} G\left(p_c^+, q_c^-\right)ds' \qquad (19)$$

And

$$\boldsymbol{Y_m} = \left[\frac{A_m^+}{3} + \frac{A_m^-}{3}\right]T^i \qquad (20)$$

Where q_c is the position vector to the centroids of the triangular patches associated with field edge, p_c is the position vector to the centroids of triangular patches associated with source edge. Once the matrix Z is determined and vector Y is calculated, vector X can be calculated using any standard linear equation solvers. Analytical Solution of heat conduction from a sphere into a stagnant fluid like air is given as follows:

Consider a sphere of radius R is placed in a stagnant fluid of thermal conductivity k. Let the temperature of the surface of the sphere is TR and the temperature at infinity is TS. Let r be the radius in the space of stagnant fluid from origin O. Heat is conducted in the direction of r only. For energy balance, consider a small shell of thickness Δr.

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$$4\pi r^2 q\Big|_r - 4\pi r^2 q\Big|_{r+\Delta r} = 0$$

$$\frac{4\pi r^2 q\Big|_r - 4\pi r^2 q\Big|_{r+\Delta r}}{\Delta r} = 0$$

$$\lim_{\Delta r \to 0} \frac{4\pi r^2 q\Big|_{r+\Delta r} - 4\pi r^2 q\Big|_r}{\Delta r} = 0$$

$$\frac{d}{dr}\left(r^2 q\right) = 0$$

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

Integrating the above equation twice,

$$\left(r^2 \frac{dT}{dr}\right) = A \quad \text{And} \quad T = -Ar + B$$

Where A and B are constants. Using the boundary conditions,

At r = R; $T = T_R$

And

At r = ∞; $T = T_S$

The above equation becomes

$$\frac{T - T_S}{T_R - T_S} = \frac{R}{r}$$

Neglecting the far field temperature T_S since only near field temperatures considered in this work,

$$T = \frac{R}{r} T_R$$

V. NUMERICAL RESULTS

In this section, numerical results are plotted for the edge based BEM solution for the cases of Sphere. The sphere of radius 1m, is discretized into four mesh sizes. The sphere is divided into 10 equal parts along the polar and azimuthal directions resulting in 270 edges. Similarly the mesh size is decreased by dividing the sphere into 12, 15, and 20 equal parts resulting in 396, 630, and 1140 edges respectively.

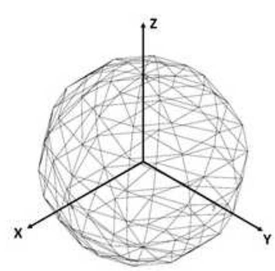

Fig. 1. Triangular Patch Model of a Sphere

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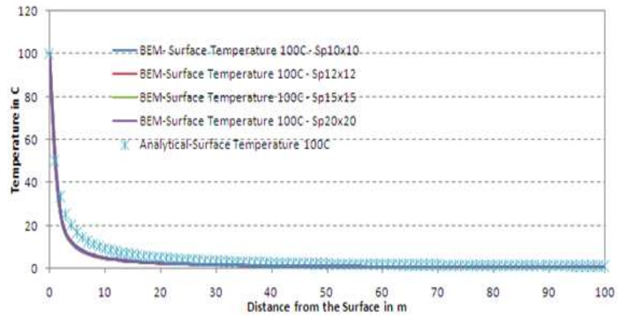

Fig.2: Temperature Distribution outside a Sphere of Radius 1m, Maintained at a Temperature of 100^{0}C upto a Distance of 100m.

As a first case, the surface of the sphere that is maintained at a temperature of 100C is modeled and simulated. For the four mesh sizes, simulations are run. Results are plotted in Fig.2.Temperatures are plotted for a distance of upto a distance of 100m from the surface of the sphere. It can be observed that the numerical BEM results are very close to that of the analytical solution. Also, it can be observed that there is no much difference in the solutions of different mesh sizes, meaning that the largest mesh size that is having a sphere with 270 edges yields results as good as 1140 edges.

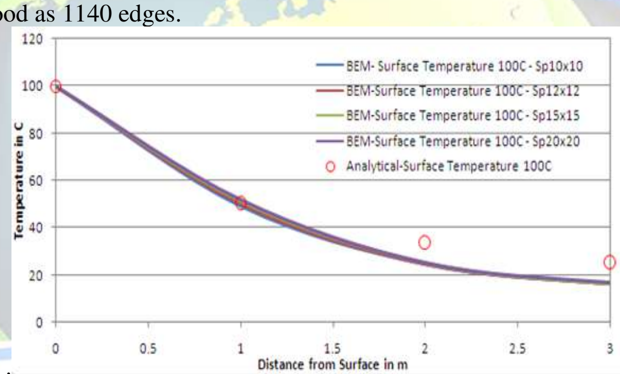

Fig.3: Temperature Distribution outside a Sphere of Radius 1m, Maintained at a Temperature of 100C up to a Distance of 3m.

As shown in Fig. 3, when the results plotted in Fig. 2 is magnified to capture the temperature distribution for a distance of 3m, the results show that there is still not much difference in the numerical results of the BEM solution based on the edge based basis functions. It again emphasizes the fact that a smaller number of edges in the mesh will yield good results which prove that the edge based basis functions is more efficient, and it requires less computational resources. All the numerical results for the four different mesh sizes are almost equi- distant in the Fig.3 from the analytical solution.

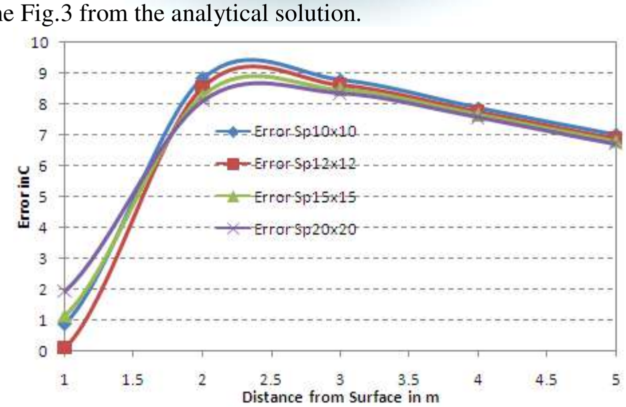

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Fig 4: Temperature Difference w.r.t Exact Solution outside a Sphere of Radius 1m, Maintained at a Temperature of 100^0C up to a Distance of 5m.

Fig. 4 shows the temperature difference outside a sphere of radius 1m, maintained at a temperature of 100^0C up to a distance of 5m. The error is less than 2C in all the cases of 4 mesh sizes at a distance of 1m from the surface of the sphere. The error grows to 9C at a distance of 2m. As the mesh size is decreased and the number of edges in the model increases, the error drops by almost 1C.

VII. CONCLUSIONS

In this research article, the numerical results of thermal conduction problem are presented. The numerical solution is based on the boundary element method. The basis functions are used in the numerical solution. The basis functions are defined on the edge of the mesh created on the surface of the object. The surface of the object is approximated with the triangular patch modeling. The triangular patch modeling creates the triangular mesh on the surface of the body. The triangular patches that are created on the surface, has faces, edges and nodes. The basis functions are defined on the edges in this work. The numerical solution is tested on the sphere of radius 1m. The surface of the sphere is divided into 270, 396, 630 and 1140 triangular patches. The temperature of the surface is defined to be 100^0C. The temperatures of the surrounding medium is computed at a distance up to 5m from the center of the sphere. The temperatures obtained using the numerical solution is compared with that of exact solution. There is a maximum error of 9.3^0C at a distance of 2.3m. The error falls gradually as the distance increases.

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