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A Note on Characteristic function of Near-semirings

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Abstract : Near-rings are algebraic systems with binary operations of addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. A natural example of a Near-ring is given by the set M(G) of all mappings of an additive group G (not necessarily abelian) into itself with addition and multiplication as composition of mappings.A semiring is an algebraic system which is closed and associative under two operations usual addition and multiplication and satisfies both distributive laws. Semirings abound in the mathematical world around us. Indeed, the first mathematical structure we encounter, the set of natural numbers with the usual operations of addition and multiplication of integers is an example of a semiring. In this paper,we consider the algebraic system near-semiring which is a generalization of both a near-ring and a semiring. More precisely, a near-semiring S is an algebraic system with two binary operations: usual addition and usual multiplication such that S forms a semigroup with respect to both the operations, and satisfies the right distributive law. A natural example of a near-semiring is obtained by considering the operations usual addition and composition of mappings on a set of all mappings of an additive semigroup S into itself. In this paper, Characteristic function of near-semirings is considered and some related results are proved.

Keywords : near-semiring, s-ideal, s-k-ideal, characteristic function

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1.1 INTRODUCTION:

The theory of fuzzy sets was first motivated by Zadeh [10]. The term Fuzzy means "imprecise", "unclear", "indistinct". Fuzzy set is a generalization of the notion classical set. To distinguish between Fuzzy sets and classical sets, we refer to the latter as 'crisp' sets. When A is a fuzzy set and x is a relevant object, the projection that "x is a member of A" is not necessarily either true or false, as required by two-values logic, but it may be true only to some degree, that is, the degree to which x is member of A.The Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics. [6] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. Utilizing the versatile back venturing method, every one of the six DOF are effectively followed utilizing just four control inputs while within the sight of un demonstrated flow and limited unsettling influences.

Definition:A subset I of a near-semiring S is a right (respectively, left) s-ideal if

(i) $x + y \in I$ for all $x, y \in I$.

(ii) $xr \in I$ (right s-ideal) and $rx \in I$ (left s-ideal) for all $x, y \in I$, and $r \in S$.

1.2 Definition:Let I be a subset of X. We define $\lambda_I: X \to [0, 1]$ as

$$\lambda_I(x) = \begin{cases} 1 & \text{if } x \in I \\ 0 & \text{otherwise} \end{cases}$$

is called the characteristic function on I.

1.4 Definition: A left and (or right) s-ideal I of S is called a left (right) s-k–ideal of S if $y \in I$ and $x \in S$, $x + y \in I$ implies that $x \in I$.

1.5 Definition: A non-empty fuzzy subset μ (that is, $\mu(x) \neq 0$ for some $x \in S$) of a near-semiring S is called a fuzzy s-ideal with thresholds $\alpha, \beta \in [0, 1]$, $\alpha < \beta$ if it satisfies

(i) $\alpha \vee \mu(x + y) \geq \beta \wedge (\mu(x) \wedge \mu(y))$

(ii) $\alpha \vee \mu(xy) \geq \beta \wedge (\mu(x) \vee \mu(y))$

This is also called generalized fuzzy s-ideal or (α, β) fuzzy s-ideal. In case if $\alpha = 0$ and $\beta = 1$, then this definition coincides with the usual fuzzy s-ideal.

In this note we consider the notion (α, β) fuzzy s-ideal where $\alpha, \beta \in [0, 1]$, $\alpha < \beta$.

1.6 Definition: A fuzzy s-ideal μ of S is called a fuzzy s-k- ideal of S if for all $x, y, z \in S$, $x + y = z$ implies that $\alpha \vee \mu(x) \geq \beta \wedge \{\mu(y) \wedge \mu(z)\}$

1.7 Definition: Let μ be a fuzzy subset of a near-semiring S. Then the set defined by

$$\mu_t = \{x \in S \mid \alpha \vee \mu(x) \geq \beta \wedge t,\ t \in [0,1]\},\ \beta \geq t.$$

is called the level subset of S with respect to μ.

2. Characteristic Function of Near-semirings:

2.1 Theorem: Let I be an s- ideal of a near-semiring S. Then the characteristic function λ_I defined by

$$\lambda_I(x) = \begin{cases} 1 & \text{if } x \in I \\ 0 & \text{otherwise} \end{cases}$$

is a fuzzy s-ideal of S.

Further, if I is an s-k-ideal, then λ_I is a fuzzy s-k-ideal of S.

Proof: Suppose I is an s-ideal of S.

First we show that λ_I is a fuzzy ideal of S.

Take x, y $\in$ S. If x, y $\in$ I, then x + y $\in$ I (since I is an s-ideal of S). Now

$$\alpha \vee \lambda_I(x+y) = 1 \geq \beta \wedge \min\{1, 1\}$$
$$= \beta \wedge \min\{\lambda_I(x), \lambda_I(y)\}.$$

Suppose x $\notin$ I and y $\in$ I.
If x + y $\in$ I then it is clear from above.

If x + y $\notin$ I, then $\alpha \vee \lambda_I(x+y) = 0 = \beta \wedge \min\{0,1\}$
$$= \beta \wedge \{\lambda_I(x), \lambda_I(y)\}$$

If x$\notin$ I, y $\notin$ I, then x + y $\notin$ I or x + y $\in$ I.
Now, from each of these cases we get

$$\alpha \vee \lambda_I(x+y) \geq \beta \wedge \min\{\lambda_I(x), \lambda_I(y)\}$$

Also, take x $\in$ I, y $\in$ S $\Rightarrow$xy$\in$ I. Now

$$\alpha \vee \lambda_I(xy) = 1$$
$$= \beta \wedge \max\{1, 0\}$$
$$= \beta \wedge \max\{\lambda_I(x), \lambda_I(y)\}$$

Suppose I is an s-k-ideal of S. We show that λ_I is a fuzzy s-k-ideal of S.
Take y $\in$ I and every x $\in$ S, x + y $\in$ I, since I is an s-k-ideal, we have x $\in$ I.
Further,

$$\alpha \vee \lambda_I(x) = 1$$
$$\geq \beta \wedge \min\{1, 1\}$$
$$= \beta \wedge \min\{\mu(y), \mu(x+y)\}$$

Therefore, λ_I is a fuzzy s-k- ideal of S.

2.2 Theorem: Let μ be a fuzzy s-ideal of a near-semiring S. Then the level set μ_t, $t \leq \mu(0)$ is an s-ideal of S.

Proof. Take x, y $\in \mu_t$
This implies $\alpha \vee \mu(x) \geq \beta \wedge t$, $\beta \geq t$, and $\mu(t) \geq t$.
Now

$$\alpha \vee \mu(x+y) \geq \beta \wedge \min\{\mu(x), \mu(y)\}$$
(since μ is a fuzzy s-ideal)
$$\geq \{t, t\}$$
$$= t.$$

This implies x + y $\in \mu_t$

For any s $\in$ S, we have

$$\alpha \vee \mu(sx) \geq \beta \wedge \mu(x)$$
(Since μ is a fuzzy s-ideal)
$$= t.$$

This implies that sx$\in \mu_t$. Similarly, $\alpha \vee \mu(xs) \geq \beta \wedge \mu(x) \geq t$. This implies that xs$\in \mu_t$.
Therefore, μ_t is an s-ideal of S.

2.3 Example: Take S = $\mathbb{Z}^+ \cup \{0\}$.
Take I = <{2, 3}> , an s-ideal of S.
Define μ : S$\rightarrow$ [0,1]

$$\text{by}\ \mu(x) = \begin{cases} 1 \text{ if } & x \in <\{2,3\}> \\ 0, & \text{otherwise} \end{cases}$$

By Theorem 2.2, μ is a fuzzy s-ideal of S.
Further if μ is an s-k fuzzy ideal whenever I is an s-k-ideal of S.
Now for t = 0,
μ_0 = { x $\in$ S I $\mu(x) = \mu(0)$}
= <{2, 3}> is not an s-k- ideal of S.

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