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On Differential Equations

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ABSTRACT:

This paper explains the introduction on Differential equations and it explains basic concepts on these.It gives a few examples to illustrate how practical problems are modelled mathematically and how differential equations arise in them. It is important for engineers to be able to model physical problems using mathematical equations,and then solve these equations.This article attempts the concepts of LCR circuits in electrical circuits,free vibration of mass using Differential Equations.

Key words: differential equations ,LCR circuits,free vibration of mass

1.INTRODUCTION:

Many phenomena in engineering,physics and broad areas of applied mathematics involve entities which change as a function of one or more variables.

The movement of a car along a road,the propagation of sound and waves,the amount of charge in a capacitor in an electrical circuit,the way a cake taken out from a hot oven and left in room temperature cools down and many such processes can be described as Differential Equations.

Differential equations have wide applications in various engineering and science disciplenes.In general, variations of a physical quantity,such as temperatuure,pressure,displacement,velocity,stress,strain with the change of time t would lead to differential equations.

It is important for engineers to be able to model physical problems using mathematical equations,and then solve these equations.

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2. PRELIMINARIES AND BASICS:

2.1 Definition: A differential equation is an equation that relates an unknown function and one or more of its derivatives of with respect to one or more independent variables. For instance, the equation $\frac{dy}{dx} = -5x$

relates the first derivative of y with respect to x, with x. Here x is the independent variable and y is the unknown function (dependent variable).

If $\frac{d^n y}{dx^n}$ is the n-th derivative of y with respect to x. One can also use the notation y (n) to denote $\frac{d^n y}{dx^n}$. It is further convenient to write y ′ = y (1) and y ′′ = y (2). In physics, the notation involving dots is also common, such that y˙ denotes the first-order derivative.

Different classifications of differential equations are possible and such classifications make the analysis of the equations more systematic.

2.2 Classification of Differential Equations

There are various classifications for differential equations. Such classifications provide remarkably simple ways of finding the solutions (if they exist) for a differential equation. Differential equations can be of the following classes.

2.2.1 Ordinary Differential Equations

If the unknown function depends only on a single independent variable, such a differential equation is ordinary. The following is an ordinary differential equation:

$L\frac{d^2}{dt^2}Q(t) + R\frac{d}{dt^2}Q(t) + \frac{1}{C}Q(t) = E(t),$, which is an equation which arises in electrical circuits. Here, the independent variable is t.

2.2.2 Partial Differential Equations

If the unknown function depends on more than one independent variables, such a differential equation is said to be partial. Heat equation is an example for partial differential equations:

$\alpha^2\frac{\partial^2}{\partial x^2}f(x,t) = \frac{\partial}{\partial t}f(x,t)$

Here x, t are independent variables.

2.2.3 Homogeneous Differential Equations

If a differential equation involves terms all of which contain the unknown function itself, or the derivatives of the unknown function, such an equation is homogeneous. Otherwise, it is non-homogeneous.

2.2.4 N-th order Differential Equations

The order of an ordinary differential equation is the order of the highest derivative that appears in the equation. For example 2y (2) + 3y (1) = 5, is a second-order equation.

2.2.5 Linear Differential Equations

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A very important class of differential equations are linear differential equations. A differential equation F(y, y(1), y(2), . . . , y(n))(x) = g(x) is said to be linear if F is a linear function of the variables y, y(1), y(2), . . . , y(n)

2.3 Note:If a differential equation is not linear, it is said to be non-linear.

2.4 Solutions of Differential equations

2.4.1 Definition

To say that y = g(x) is an explicit solution of a differential equation $F(x, y, dy/dx, d^{n}y/dx^{n}) = 0$ on an interval I ⊂ R, means that F(x, g(x), dg(x) dx , . . . , d ng(x) dxn) = 0, for every choice of x in the interval I.

2.4.2Definition

We say that the relation G(x, y) = 0 is an implicit solution of a differential equation $F(x, y, dy/dx, d^{n}y/dx^{n}) = 0$ if for all y = g(x) such that G(x, g(x)) = 0, g(x) is an explicit solution to the differential equation on I.

2.4.3 Definition

An n-th parameter family of functions defined on some interval I by the relation $h(x, y, c_1, \ldots, c_n) = 0$, is called a general solution of the differential equation if any explicit solution is a member of the family. Each element of the general solution is a particular solution.

2.4.4 Definition

A particular solution is imposed by supplementary conditions that accompany the differential equations. If all supplementary conditions relate to a single point, then the condition is called an initial condition. If the conditions are to be satisfied by two or more points, they are called boundary conditions. Recall that in class we used the falling object example to see that without a characterization of the initial condition (initial velocity of the falling object), there exist infinitely many solutions. Hence, the initial condition leads to a particular solution, whereas the absence of an initial condition leads to a general solution.

2.4.5Definition

A differential equation together with an initial condition (boundary conditions) is called an initial value problem (boundary value problem).

2.5 Basic definitions

- Order: Highest Derivative present in the differential equation
- Degree: Power of the higher derivative after free from radicals and fractions
- Formation of Differential Equation: 1. Consider the given equation f(x,y,a,b,c…d)=0

  2. Observe the no. of arbitrary constants if it is 'n'
  3. Based on Arbitrary constants differentiate the given Equation n times
  4. Using the n+1 equations including the givenequation We get a differential equation of nth order.

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• The standard form of First order linear differential equation is of the form

•Homogeneous Function: A function f(x,y) which satisfies the condition that

•A differential equation is said to be Homogeneous if the f(x, y) is homogeneous of degree zero

•For Homogeneous use substitution based on the equation

•For Non-Homogeneous in the case of use ax+by=t as substitution and in the case of take as substitution

•The necessary and sufficient condition for the differential equation M(x,y)dx+N(x,y)dy=o to be exact is
•The solution of the exact differential equation is ,under the first integral y constant and under the second integral terms do not contain x
•The equation is not an exact differential equation if and it can be solved by applying various conditions like homogeneous, inspection, functions in xy, function of x alone, function of y alone and powers of x,y

•The standard form of First order linear differential equation is of the form

• The linear differential equation of first order and first degree of Leibnitz is of the form

•The solution of the Leibnitz linear differential equation is

•The Bernoulli's differential equation is of the form
•To find the solution of Bernoulli's use the substitution as 3.Series RLC Circuit

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Series RLC Circuit

A circuit consisting of a resistor R, an inductor L, a capacitor C, and a voltage source $V(t)$ connected in series, shown in Figure 5.18, is called the series RLC circuit. Applying Kirchhoff's Voltage Law, one has

$$-V(t) + Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{C}\int_{-\infty}^{t} i\,\mathrm{d}t = 0.$$

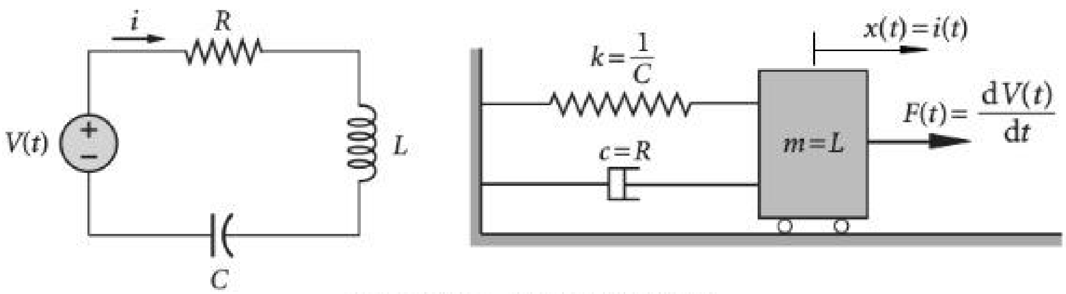

Figure 5.18 Series RLC circuit.

Differentiating with respect to t yields

$$L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + R\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{C}i = \frac{\mathrm{d}V(t)}{\mathrm{d}t},$$

or, in the standard form,

$$\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + 2\zeta\omega_0\frac{\mathrm{d}i}{\mathrm{d}t} + \omega_0^2 i = \frac{1}{L}\frac{\mathrm{d}V(t)}{\mathrm{d}t}, \quad \omega_0^2 = \frac{1}{LC}, \quad \zeta\omega_0 = \frac{R}{2L}.$$

The series RLC circuit is equivalent to a mass-damper-spring system as shown.

Parallel RLC Circuit

A circuit consisting of a resistor R, an inductor L, a capacitor C, and a current source $I(t)$ connected in parallel, as shown in Figure 5.19, is called the parallel RLC circuit. Applying Kirchhoff's Current Law at node 1, one has

$$I(t) = C\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{1}{L}\int_{-\infty}^{t} v\,\mathrm{d}t + \frac{v}{R}.$$

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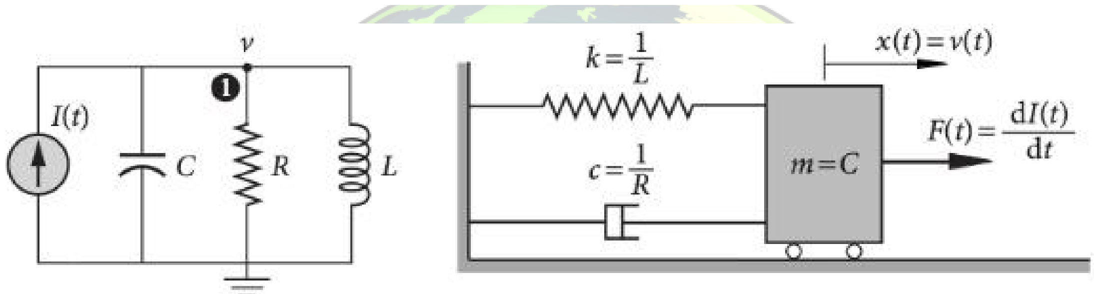
Differentiating with respect to t yields

$$C\frac{d^2v}{dt^2}+\frac{1}{R}\frac{dv}{dt}+\frac{1}{L}v=\frac{dI(t)}{dt},$$

or, in the standard form,

$$\frac{d^2v}{dt^2}+2\zeta\omega_0\frac{dv}{dt}+\omega_0^2v=\frac{1}{C}\frac{dI(t)}{dt},\quad \omega_0^2=\frac{1}{LC},\quad \zeta\omega_0=\frac{1}{2RC}.$$

The parallel RLC circuit is equivalent to a mass-damper-spring system as shown.



4.Free vibration:

free vibration takes place when a system oscillates under the action of forces inherent in the system itself due to initial disturbance and when the externally applied forces are absent.the system will oscillates about one of its static quilibrium positions.Basically there are two types of systems.they are the discrete and continuous systems.In the case of discrete systems,the physical properties are discrete quantites and the system behavior is described by ordinary differential equations.the system has finete number of degrees of freedom where as in the case of cotinuous system,the physical properties are function of spatial co-ordinates and it is described by partial differential equations. and has infinite number of degrres of freedom.

In other words, a system can be considered as discrete in which the whole mass of the system is lumped at some points and in case of continuous system the mass is distributed over the entire length of the system. An n-degrees-of-freedom system is governed by n coupled differential equations and has n natural frequencies. So the discrete system has finite number of natural frequencies and the continuous system has infinite number of natural frequencies. The system under free vibration will vibrate at one or more of its natural frequencies, which are properties of the dynamical system, established by its mass and stiffness distribution.

Frequency: The number of oscillations completed per unit time is known as frequency of the system.

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Natural Frequency: The frequency of free vibration of a system is called Natural Frequency of that particular system.

Damping: The resistance to the motion of a vibrating body is called Damping. In actual practice there is always some damping (e.g., the internal molecular friction, viscous damping, aero dynamical damping, etc.) present in the system which causes the gradual dissipation of vibration energy and results in gradual decay of amplitude of the free vibration. Damping has very little effect on natural frequency of the system, and hence, the calculations for natural frequencies are generally made on the basis of no damping. Damping is of great importance in limiting the amplitude of oscillation at resonance.

5.Differential Equation for a spring-mass system

Let us consider a spring-mass system as shown in Fig. 1.1. The system is constrained to move in the vertical direction only along the axis of the spring. Let k and m be the stiffness of the spring and the mass of the block, respectively.

Let x be the position of the mass at any instant from the equilibrium position of the mass and it is assumed that x is positive in the downward direction and negative in the upward direction. In the spring-mass system only one coordinate is enough to describe the position of the mass at any time, and hence, it is single degree-of-freedom system. Here the coordinate is x.

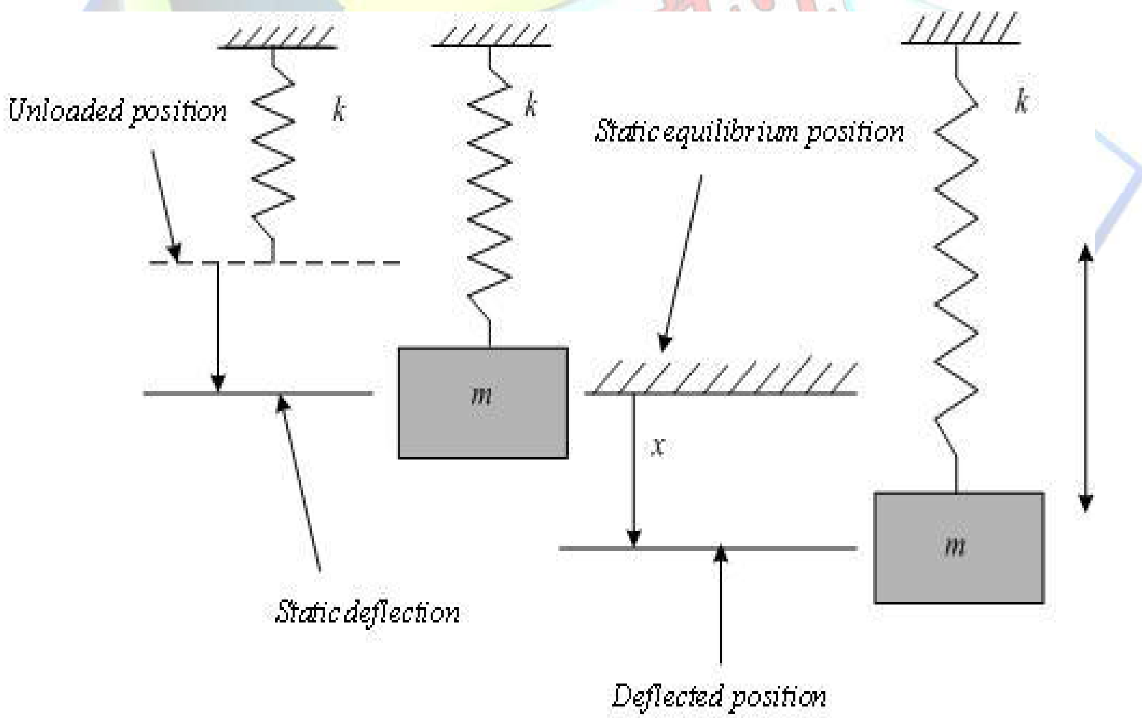

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Fig.1.1: Spring-mass system

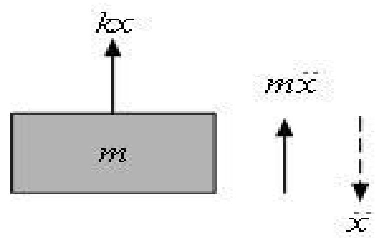

Fig. 1.2: Free Body diagram of the mass

free-body diagram of the mass is shown in Fig.2. Applying D'Alembert's principle, the equation of motion of the mass can be obtained as,

$$m\ddot{x} + kx = 0 \quad \textbf{(1.1)}$$

The natural frequency of the system, ω_n is,

$$\omega_n = \sqrt{\frac{k}{m}} \quad \textbf{(1.2)}$$

Let

$$x = A\sin\omega_n t + B\cos\omega_n t \quad \textbf{(1.3)}$$

be the solution for this differential equation (1.1).

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Initial conditions:

$$x = x(0) \text{ at } t = 0$$
$$\dot{x} = \dot{x}(0) \text{ at } t = 0 \quad \textbf{(1.4)}$$

Substitution of Eq.1.3 into Eq.1.1 and application of Eq.1.4 yields,

$$x = \left(\frac{\dot{x}(0)}{\omega_n}\right)\sin\omega_n t + x(0)\cos\omega_n t \quad \textbf{(1.5)}$$

Time period:

The time taken to complete one cycle, is,

$$\tau = 2\pi / \omega_n \quad \textbf{(1.6)}$$

It is important to note that, even though there is no specific damper attached to the system, there will always be the presence of damping of very small in amount. Because of this the response obtained from experiment always corresponds to a small amount of damping.

Let *c* be the damping coefficient, then the equation of motion changes to,

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \textbf{(1.7)}$$

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Damping ratio (ζ):

It can be defined as the ration of the damping coefficient to the critical damping coefficient.

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It can be defined as the ration of the damping coefficient to the critical damping coefficient.

$$\zeta = \frac{c}{c_c} \quad \textbf{(1.8)}$$

where C_G is the Critical damping coefficient, given by:

$$c_c = 2m\omega_n \quad \textbf{(1.9)}$$

If $c > c_c$ or $\zeta > 1$ the system is said to be over-damped system.

If $c = c_c$ or $\zeta = 1$, the system is said to be critically damped system.

If $c < c_c$ or $\zeta < 1$, the system is said to be under-damped system

The motion governed by this solution is of oscillatory type whose amplitude decreases in an exponential manner with the increase in time as shown in Fig. 3.

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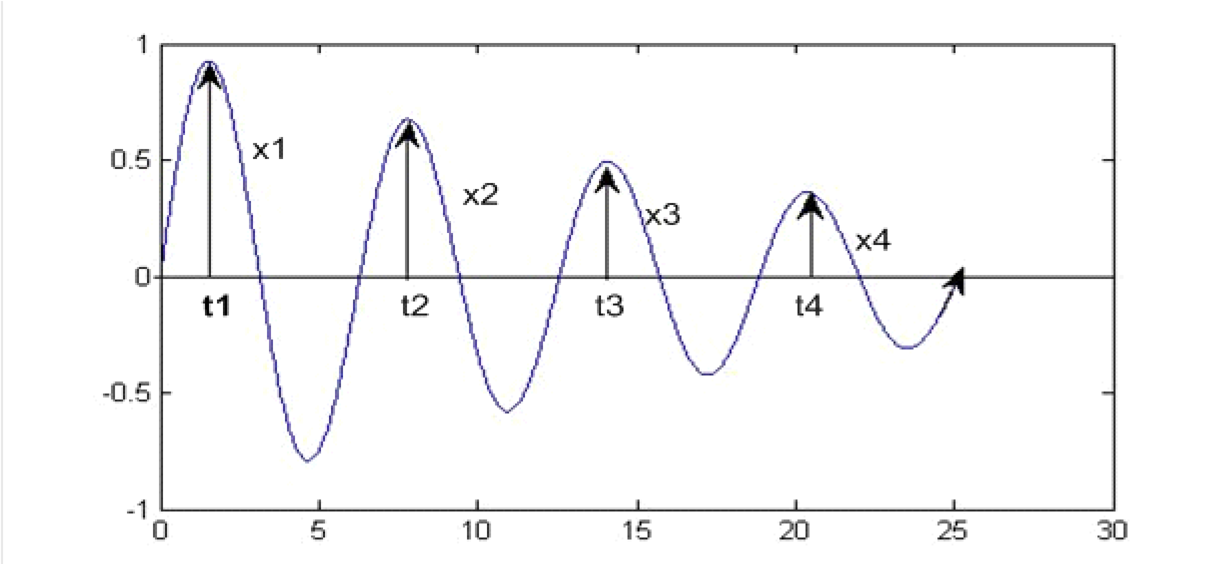

Fig. Displacement-time plot of an under-damped system with general initial conditions

The logarithmic decrement is given by**,**

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \zeta\omega_n\tau_d \quad \textbf{(1.14)}$$

Substituting Eq.1.13 into Eq.1.14,

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad \textbf{(1.15)}$$

Conclusion :

This paper explains the introduction on Differential equations and it explains basic concepts on these and a few examples to illustrate how practical problems are modelled mathematically and how differential equations arise in them explaines clearly.It is important for engineers to be able to model physical problems using mathematical equations,and then solve these equations.This article attempts the concepts of LCR circuits in electrical circuits,free vibration of mass using Differential Equations.

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