ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 4, Special Issue 21, August 2017

PSEUDO SYMMETRIC IDEALS IN SEMIGROUPS

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ABSTRACT:

In this paper some basic notions,definitions and results related to semigroups,ideals in semigroups are introduced. Thenotions of pseudo symmetric ideals and pseudo symmetric semigroups with some examples are presented. It is proved that every left(right) duo semigroup S is a pseudo symmetric semigroup. And every idempotent semigroup S is a pseudo symmetric semigroup.It is attempted to characterize pseudo symmetric ideals. Properties of pseudo symmetric ideal in a semigroup S are discussed.

KEY WORDS:

Semi groups,ideals in semigroups, pseudo symmetric ideals, pseudo symmetric semi groups,duo semigroup, pseudo commutative semigroup, quasi commutative semigroup, generalized commutative semigroup, normal semigroup, idempotent semigroup, mid unit, completely semi prime ideal, prime ideal, completely prime ideal.

1.Introduction:

CLIFFORD, PETRICH and LYAPIN studied the algebraic theory of semigroups. ANJANEYULU developed the ideal theory in general semigroups. The ideal theory in commutative semigroups was developed by BOURNE,HARBANS LAL.In this paper we introduced pseudo symmetric ideals in semigroups.

2.PRELIMINARIES:

Definition 2.1: A semigroup is a system S=(S,.) where s is a nonempty set and . is an associative binary operation on S.

Definition 2.2:A semigroup S is said to be

1.Commutative if ab = ba for all a,b ∈ S.

2.quasi commutative if there exists a natural number n such that $ab = b^n a$ for any a,b ∈ S.

3.normal if as = sa for all a∈ S.

4.left(right) pseudo commutative provided abc = bac(abc = acb) for all a,b,c ∈ S.

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Definition 2.3:A nonempty subset A of a semigroup S is said to be

1.a subsemigroup of S if a,b$\in$ A implies ab$\in$ A.

2. a left(right) ideal provided SA$\subseteq$A(AS$\subseteq$A)

3.a two sided ideal or simply an ideal provided it is both a left and a right ideal of S.

Definition 2.4: Let S be any semigroup.Then

1.a nonempty set A is said to generate S if every element of S is a finite product of elements of A.

2.The intersection of all (left,right) ideals of S containing a nonempty set A is called the (left,right) ideal generated by A.

3.The intersection of all ideals in S,if nonempty,is said to be the kernel of S and is denoted by K.

Definition 2.5: An ideal A of a semigroup S is called a

1. principal ideal if A is an ideal generated by single element set.
2. finitely generated ideal if it is a union of finite number of principal ideals.
3. proper ideal if A $\neq$S.
4. trivial ideal if S\A is singleton.
5. maximal idealifA is a proper ideal of S and is not properly contained in any proper ideal of S.
6. minimal ideal ifd it does not contain any ideal of S properly.
7. prime ideal if XY$\subseteq$ A; X,Y are ideals of S,then either X $\subseteq$A or Y $\subseteq$ A.
8. completely prime ideal provided xy$\in$ A; x,y$\in$ S implies either x$\in$A or y$\in$A
9. semiprime ideal if xsx $\subseteq$ A; x$\in$ S implies x $\in$ A
10. completely semiprime ideal if $x^n$$\in$A; x$\in$ S for some natural number n implies x$\in$ A.
11. globally idempotent ideal if A^2 = A.

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Definition 2.6: An element 'a' of a semigroup S is said to be

1.an idempotent if $a^2 = a$.

2. a mid unit provided $xay = xy$ for any x,y ∈ S.

3.an r-element provided as = sa for all s ∈ S and if x,y ∈ S,we have $axy = byx$ for some b∈ S.

Definition 2.7:A semigroup S is said to be

1. a group provided S has no left and right ideals.
2. a simple semigroup provided S has no proper ideals.
3. a left(right) duo semigroup provided every left(right) ideals of S is two sided ideal of S.
4. a duo semigroup provided it is both a left and a right duo semigroup.

We introduce the notion of pseudo symmetric ideals and pseudo symmetric semigroups. We characterize pseudo symmetric ideals in a semigroup and give some examples and some classes of pseudo symmetric semigroups.

Definition 2.8: An ideal A in a semigroup S is said to be pseudo symmetric provided

xy ∈ A;x,y ∈ S implies xsy ∈ A for all s∈ S.

Definition 2.9: A semigroup is said to be pseudo symmetric provided every ideal is pseudo symmetric ideal.

Every commutative semigroup is a pseudo symmetricsemigroup and the converse need not true.

Example 2.10: Let S = {a,b,c} and '.' And ' *' be two binary operations in S defined as

*	a	b	c
a	a	b	a

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b	a	b	a
c	a	b	c

.	a	b	c
a	a	a	a
b	a	a	a
c	a	b	c

Here(S,.) and (S,*) are not commutative semigroups but pseudo symmetric semigroups.

Theorem 2.11: The following statements are equivalent for an ideal A in a semigroup S.

1.A is a pseudo symmetric ideal.

2.$A_r(a) = \{x \in S: ax \in A\}$ is an ideal of s for all $a \in S$.

3.$A_l(a) = \{x \in S: xa \in A\}$ is an ideal of s for all $a \in S$.

Corollary 2.12: Every left(right) duo semigroup S is a pseudo symmetric semigroup.

Proof: Let A be any ideal in S.Since for all $a \in S$,$A_l(a)$ is a left ideal and hence by the above theorem, A is a pseudo symmetric ideal. Therefore S is a pseudo symmetric semigroup. Similarly every right duo semigroup is a pseudo symmetric semigroup.

Here are the examples of pseudo symmetric semigroups.

1.Every left(right) pseudo commutative semigroup is a pseudo symmetric semigroup.

2.Every quasicommutative semigroup is a pseudo symmetric semigroup.

3.Every generalized commutative semigroup is a pseudo symmetric semigroup.

4.Every normal semigroup is a pseudo symmetric semigroup.

Theorem 2.13: Every idempotent semigroup S is apseudo symmetric semigroup.

Proof: Let A be any ideal in S and let $ab \in A$.Then $ba = baba \in A$ and hence

$asb = asbasb \in A$.Therefore A is a pseudo symmetric ideal.

Theorem 2.14: If S is a semigroup in which every element is a mid unit then S is a pseudo symmetric semigroup.

Proof: Let A be any ideal in S and let $ab \in A$.Now for any $s \in S$,$asb = ab \in A$.

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So A is a pseudo symmetric ideal.

Theorem 2.15: Every completely semiprime ideal A in a semigroup S is a pseudo symmetric ideal and the converse is not true.

Proof: Let $xy \in A$. Then $(yx)^2 = yxyx \in A$. Since A is a completely semiprime ideal, $yx \in A$.

Now $(xsy)^2 = xsyxsy \in A$ for all $s \in S$ and hence $xsy \in A$.Therefore Ais a pseudo symmetric ideal.

Theorem 2.16: Let A be any pseudo symmetric ideal in a semigroup S. Then $a_{1,}a_{2,} \ldots \ldots . a_n \in A$

if and only if $< a_1 >< a_2 > \cdots \ldots \ldots \ldots . . < a_n > \subseteq A$.

Proof : Clearly if $< a_1 >< a_2 > \cdots \ldots \ldots \ldots . . < a_n > \subseteq A$,then $a_{1,}a_{2,} \ldots \ldots . a_n \in A$

Conversely if $a_{1,}a_{2,} \ldots \ldots . a_n \in A$,then for any $t \in < a_1 >< a_2 > \cdots \ldots \ldots \ldots . . < a_n >$, we have $s_1 a_1 s_2 a_2 \ldots \ldots \ldots \ldots . . a_n s_{n+1,}$ where $s_i \in s'$. Since A is a pseudo symmetric ideal,we have $t \in A$.

Therefore ,$< a_1 >< a_2 > \cdots \ldots \ldots \ldots . . < a_n > \subseteq A$.

Theorem 2.17: If A is a pseudo symmetric ideal in a semigroup S,then for any natural number n, $a^n \in A$ implies $< a >^n \subseteq A$.

It can be proved by taking $a_1 = a_2 = \cdots \ldots . a_n = a$ in the above theorem.

Theorem 2.18: Every prime ideal P minimal relative to containing a pseudo symmetric ideal A in a semigroup S is completely prime.

Proof : Let T be the subsemigroup generated by s\ p.

First we show that $A \cap T = \emptyset$.If $A \cap T \neq \emptyset$,then there exists $x_{1,}x_{2,} \ldots \ldots . x_n \in$ s \ p such that

$x_{1,}x_{2,} \ldots \ldots . x_n \in A$.We have $< x_1 >< x_2 > \cdots \ldots . < x_n > \subseteq A \subseteq P$.

Since p is a prime ideal,we have $< x_1 > \subseteq P$ for some I, a contradiction. Thus $A \cap T = \varphi$.

Consider the set Σ ={B:B is an ideal s containing A such that $B \cap T = \emptyset$}.

Since A∈ Σ, Σ is not empty. Now Σ is a poset under set inclusion and satisfies the hypothesis of Zorn's lemma.Thus by Zorn's lemma Σ contains a maximal element,say M.

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Let X and Y be two ideals in S such that $XY \subseteq M$ .If $X \nsubseteq M$ and $Y \nsubseteq M$,then M∪X and M∪Y are ideals in S containing M properly and hence by the maximality of M,we have

$(MUX) \cap T \neq \emptyset$ and $(MUY) \cap T \neq \emptyset$

Since $M \cap T = \emptyset$, we have $X \cap T \neq \emptyset$ and $Y \cap T \neq \emptyset$.

so there exists$x \in X \cap T$ and $y \in Y \cap T$.

Now $xy \in XY \cap T \subseteq M \cap T = \emptyset$, a contradiction.

Therefore M is a prime ideal containing A.

Now A⊆ $M \subseteq S \setminus T \subseteq P$.

Since P is minimal prime ideal relative to containing A, we have M= $S \setminus T = P$

Therefore P is a completely prime ideal .

Hence we deduce that every prime ideal P minimal relative to containing a completely semiprime ideal A in a semigroup S is completely prime.

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