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Equations of Motion in IntegralForm

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ABSTRACT

A fluid is composed of different particles for which the properties may change with respect to time and space. This description of fluid motion is somewhat different in comparison to solid body motion where the body can be tacked as it moves. Here, the fluid molecules are not identified as distinct one, rather a reasonably small chunk of fluid molecules are considered as particle for which the continuum assumption is valid. Then, the motion of this chunk is generally described by its velocity. Hence, the fluid velocity at a point is nothing but the velocity of fluid particle present at that point at that particular instant. Many a times, these chunks of molecules move randomly with different velocities. In such cases, the bulk of this chunk is often considered as of interest. So, the velocity can be thought of as mass averaged velocity of the system of molecules. Once, it is clear about what needs to be measured particle or bulk velocity, the entire domain of flow of this quantity (i.e., velocity) is described by two ways.In the first method, the individual fluid particle is studied as a function of time (Lagrangian approach). In the other case, the bulk motion is prescribed as the functions of space and time (Eulerian approach).

Key words: Small control volumes, Large control volumes, Steady and unsteady flow, Dimensionality of a flow field

Introduction:

Here, we consider the basic principles governing fluid motion: conservation of mass, momentum, and energy, as applied to large control volumes. This leads to the derivation of the general equations of motion expressed in integral form. Before doing so, we introduce the concepts of fluid particles and control volumes.

In Lagrangian description, any single particle of fluid from the flowis selected and its flow characteristics such as velocity, acceleration, pressure etc.are closely monitored and noted during the entire course of the flow through space. The position of particle at any instant of

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time becomes a function of its identity and time. In other words, a moving coordinate system is attached to the particle under study, it is equivalent to an observer sitting on a moving train and studying its motion.

The Eulerian approach deals with any fixed point in the space occupied by the fluid. The observations are made on the changes in flow characteristics that take place at that point.So, the coordinate system fixed to the point in space is selected and the attention is focussed on the fixed point as the fluid particles pass over it. It is similar to a situation where an observer standing on the ground watches the motion of a moving train.

Here, we consider the basic principles governing fluid motion: conservation of mass, momentum, and energy, as applied to large control volumes. This leads to the derivation ofthe general equations of motion expressed in integral form. Before doing so, we introducethe concepts of fluid particles and control volumes. [3] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. Utilizing the versatile back venturing method, every one of the six DOF are effectively followed utilizing just four control inputs while within the sight of un demonstrated flow and limited unsettling influences. Elements and speed vectors were thought to be inaccessible, along these lines a NN eyewitness was intended to recoup the limitless states. At that point, a novel NN virtual control structure which permitted the craved translational speeds to be controlled utilizing the pitch and the move of the UAV. At long last, a NN was used in the figuring of the real control inputs for the UAV dynamic framework. Utilizing Lyapunov systems, it was demonstrated that the estimation blunders of each NN, the spectator, Virtual controller, and the position, introduction, and speed following mistakes were all SGUUB while unwinding the partition Principle.

Fluid Particles and Control VolumesImagine what happens when a glass of water spills on the ground. The water spreads out inall directions, and parts of the water that were originally close together can end up in verydifferent places. In other words, there exist relative motions among different parts of thefluid. How are we to describe the displacement, velocity and acceleration of all these parts?We can either use a fluid particle, or a control volume approach. In the fluid particle, orLagrangian approach, we identify and follow small, fixed masses of fluid, as illustrated inFigure 1(a). The fluid making up the particle is always the

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same, no matter how muchthe particle deforms. As the particle moves through the duct, forces act on it and it mayexperience an acceleration so that its velocity changes. In the control volume, or Eulerianapproach, we do not follow individual fluid particles. Instead, we draw an imaginary boxaround a fluid system. The box can be large or small, and it can be stationary or moving ata constant velocity. Generally, there will be flow in and out of the control volume throughits surfaces, and the fluid inside the control volume is changing all the time, even if theboundary conditions are constant in time. [Figure 1(b)].

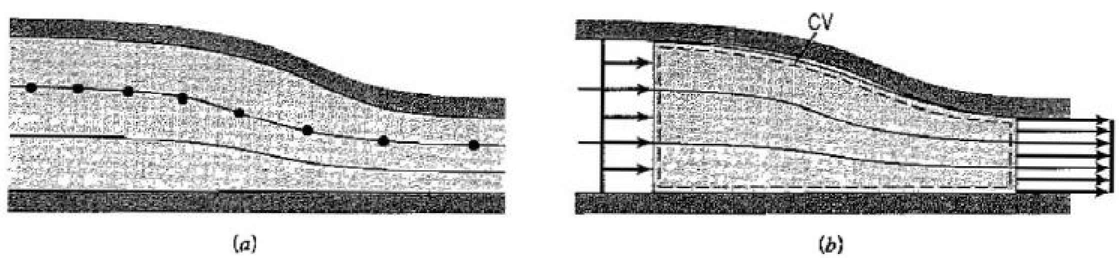

Figure 1:(a) Following a fluid particle in time; (b) Using a fixed control volume.

Lagrangian system:

In the Lagrangian system we use fluid particles, which are small parts of the fluid of fixedmass. They are called particles in analogy with the dynamics of solid bodies. We follow anindividual fluid particle as it moves through the flow, and the particle is identified by itsposition at some initial time and the time elapsed since that instant. Consider a velocityfield described byV = ui + vj + $w$kwhere i, j and k are the unit vectors in a Cartesian [x, y, z] coordinate system, and u, v,and w are the corresponding velocity components. In Lagrangian terms, then, the velocityof a fluid particle that was located at point [x_0, y_0, z_0] at time $t = t_0$ is given by V = dx/dt,and its acceleration is given byd$v$/dt.This particle description is the one normally used in describing the dynamics of rigidbodies because the particles tend to be few in number and easily identified. To describea fluid flow, however, we need to follow many fluid particles, and to resolve the smallestdetails of the flow we may need to follow a very large number of them. The motion ofeach particle can be described by a separate ordinary differential equation (ODE), such asNewton's second law, and each equation is coupled to all the others (that is, the solutionof one equation depends on the solution of all others), because the motion of each particlewill depend on the motion of all its neighbouring particles. The

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solutions of these coupledODE's are usually difficult to find because of their large number. The Lagrangian approach,therefore, is not widely used in fluid mechanics, except in some problems such as in trackingthe dispersion of pollutants.Eulerian systemIn the Eulerian system we try to find a description which gives the details of the entireflow field at any position and time. Instead of describing the fluid motion in terms of themovement of individual particles, we look for a “field" description. In other words, we searchfor a description that gives the velocity and acceleration of any fluid particle at any point[x; y; z] at any time t. For example, if we were given a Cartesian velocity field described by$V = 2x^2i - 3tj + 4xyk$, we would know its velocity at every point in the flow, at any time.At first sight, this approach appears to be very straightforward.

However, we are nolonger explicitly following fluid particles of fixed mass; at a given point in the flow, newparticles are arriving all the time. This makes it difficult to apply Newton's second lawsince it applies only to particles of fixed mass. We therefore need a relationship that givesthe acceleration of a fluid particle in terms of the Eulerian system, and this proves tobe somewhat complicated, as we shall see. Nevertheless, the Eulerian system is generallypreferred for solving problems in fluid mechanics.

Concepts of System and Control Volume:

The concepts of system and control volume are introduced here to tackle the mathematical model of the basic laws in fluid flows. In fluid mechanics, a system is defined as the chunk of fluid particles whose identity does not change during the course of flow. Here, the identity means that the chunk is composed of same fluid particles as it flows. The natural consequence of this definition is that the mass of the system is invariable since it is composed of the same fluid particles. The shaded oval shown in Fig. 2(a), is considered as the system which moves towards left as indicated by an arrow. Although, the particles inside the oval do not change as it moves, but the shape and size of this oval may change during the course of the flow because different particles have different velocities, Moreover, Lagrangian approach will be more appropriate for this method of description. A control volume is a volume or region in space whose identity is not same as fluid can enter and leave through the control surface which encloses this volume (Fig. 2-b).

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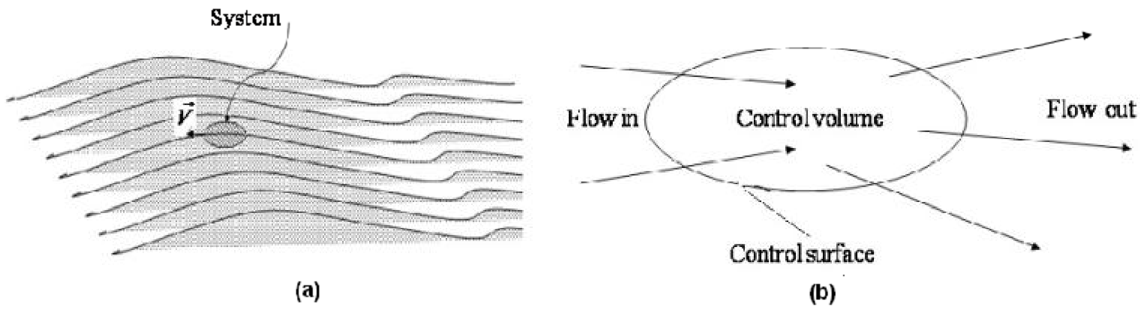

Fig. 2: Concept of system and control volume.

The shape and size of the control volume may be fixed or can change depending upon the choice of coordinate system used to analyze the flow situation. Here, the Eulerian variables are more suitable for analysis of flow field.

Basic Physical Laws

In the theory of fluid mechanics, the flow properties of fluid are generally predicted without actually measuring it. If the initial values of certain minimum number of quantities are known, then the values at some other locations can be obtained by using certain fundamental relationships. However, they are very much local in the sense that they cannot be used for different set of conditions. Such relationships are called as empirical laws/formulae and there are certain relationships which are broadly applicable in a general flow field, falling under the category of 'basic laws'. Pertaining to the theory of fluid mechanics, there are three most relevant basic laws namely;

- ♣Conservation of mass (continuity equation)
- ♣ Conservation of momentum (Newton's second law of motion)
- ♣ Conservation of energy (First law of thermodynamics
- ♣ Second law of thermodynamics

All these basic laws involve thermodynamic state relations (equation of state, fluid property relation etc.) for a particular fluid being studied.

Small control volumes: fluid elements

In deriving the equations of motion, we often choose fixed, infinitesimally small controlvolumes. Such fluid elements" are control volumes that are small enough so that thevariations in fluid properties over its volume, such as density, temperature, velocity andstress, are approximately linear.That is,we need to take only the first two terms in aTaylor

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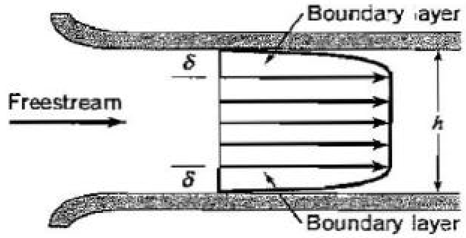
series expansion. Fluid elements are different fromfluid particles.A fluid element has a fixed volume and it is fixed in space. A fluid particle has a fixedmass and moves with the flow.

FLUID PARTICLES AND CONTROL VOLUMES



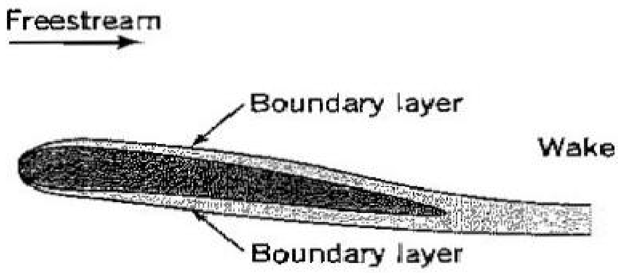


Fig3: Flow in a duct, showing boundarylayers near the walls.

Fig4: Flow over an airfoil, showing boundary layers near the surface and the formation of a wake downstream of the air foil

There are upper and lower limits on the size of the element. The characteristic dimensionof the element, `*l*(its height, width or length), should always be large compared to the meanfree path $l_m$so that the continuum approximation holds (`$l \geq l_m$).Atthe other end of the scale, `*l*should be small enough so that the changes in fluid velocityor pressure across the element can be found by linear interpolation. In other words, thefluid element needs to be small compared to the characteristic scale of the flow field. Forexample, if we have a duct of height h, then we should certainly make sure that `$l \leq$h. Inaddition, near solid walls, thin boundary layers can form. To be able to say something aboutthe flow inside the boundary layer, we need `$l \leq$ d, where d is a measure of the boundarylayer thickness.

Large control volumes

Instead of using a small fluid element as a control volume, we can choose a large control volume that encloses the entire flow [for instance, the flow inside the duct shown inFigure 1(b). Whatever their size, control volumes allow us to think in terms of overallbalances of mass, momentum and energy. For example, mass must be conserved in any fluidflow. That is, we need to account for all the mass of fluid entering and leaving the controlvolume, as well as the rate of change of the amount of mass contained inside the volume. Apiping system, for instance, will have a number of places where fluid enters, and a numberof other places where fluid exits. If the amount of mass entering over a given time

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exceedsthe amount of mass leaving during the same time, we know that mass must be accumulatingsomewhere inside the system.

We generally use fixed control volumes, large and small, although it issometimes useful to follow a fluid particle or use a moving control volume. The large controlvolume approach is one of the most basic tools for the study of fluid flow, in that it can givemany insights into fluid flow problems without solving all the detailed aspects of the fluidbehaviour inside the control volume.On the other hand, it is sometimes essential to know the details in the control volume.For instance, the aerodynamic performance of an airplane wing depends critically on itsshape, and using a large control volume that encloses the entire wing cannot give anyguidance as to its correct shape. To design the wing shape, we need to use sufficiently smallcontrol volumes or fluid elements, since this is the only way to obtain detailed informationon the pressure distribution and velocity field generated by the wing.Consider a large control volume such as that shown in Figure 1(b). Usually, there isfluid flowing in and out of the control volume through its surfaces. The collection of matterinside the control volume at any given time, therefore, is continually changing. However, allphysical laws are originally stated in terms of a system, which is a particular collection ofmatter. For example, the law of conservation of mass states that the mass of a system doesnot change. Newton's second law states that the force acting on a system is equal to thetime rate of change of momentum of the system. To apply these physical laws to a controlvolume, we need to consider the system that occupies the control volume at the instant ofour analysis. Because the system properties may be changing in time, and the system itselfis generally moving relative to the control volume, we need to relate the system propertiesto the control volume properties. For large control volumes, this relationship is called theReynolds transport theorem and for infinitesimally small control volumes,it is called the total derivative.

Steady and unsteady flow:

Another very important concept is the idea of a steady flow. If we have a large, fixed controlvolume, it is possible that the inflow and outflow conditions do not change with time. Ifthe fluid properties inside the control volume are also independent of time, we say thatthe flow is steady. However, if we followed an individual fluid particle passing through thecontrol volume, as shown in Figure 1(a), we see that its velocity can change as it moves,and therefore

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the particle experiences unsteady conditions. Whether the flow is steady orunsteady often depends on how we choose our point of view. In the case of the large controlvolume with boundary conditions that are independent of time, we may see variations ofvelocity, momentum, and energy in space, but no variation in time, whereas when we movewith a fluid particle we would see only a variation in time.It is sometimes possible to change an unsteady flow into a steady flow by changing thepoint of view of the observer. For instance, if you stand by the side of the road as a car isapproaching, you will feel no air flow at first, then a sudden rush as the car goes by, andthen, later, nothing again. Standing by the side of the road, you experience an unsteadyflow. However, if you were travelling with the car, and the car was moving at a constantvelocity, the flow relative to your new vantage point would not vary with time: it is steady.Steady flows are much more easily analyzed than unsteady flows, and it is always useful if a coordinate transformation can be found whereby an unsteady flow becomes steady.

Dimensionality of a flow field:

The dimensionality of a flow field is governed by the number of space dimensions needed todefine the flow field completely. For example, in a one-dimensional flow the flow variablescan only vary in one direction. The direction might coincide with a coordinate axis such asx, or it may be directed along the flow direction, as in Figure 5(a). In a two-dimensionalflow, the flow variables vary along the flow direction as well as across it, as in Figure 5(b),and in a three-dimensional flow they vary in all three directions.The dimensionality of a flow field is equal to the number of space dimensions needed todefine the flow field.Perfectly one-dimensional flows are not often found in nature. For example, if the flowdiverges so that its cross-sectional area increases with distance, the flow will also diverge[Figure 5(a)], and it will no longer be strictly one-dimensional. Even if the streamwisevelocity is constant over the area of a duct, the other components need not be. For instance,in a symmetric diverging flow in the x-y plane, $v = 0$ on the centerline, but v will be positiveabove the centerline and negative below (Figure :5).

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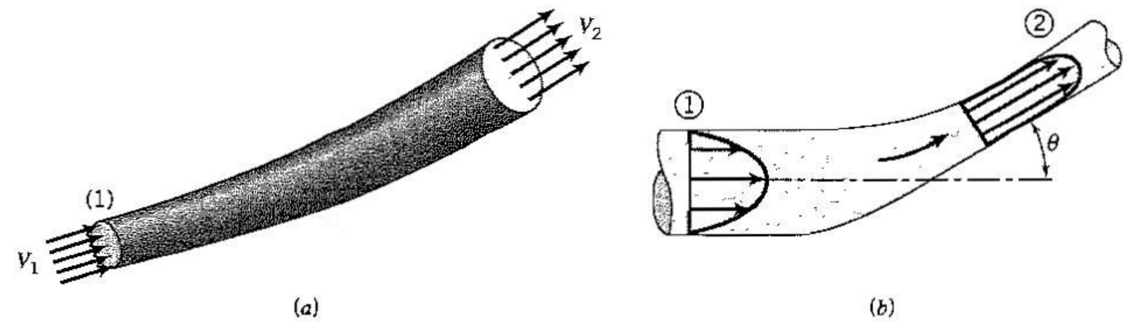

Figure 5(a): One-dimensional flow; Figure 5(b):Two-dimensional flow.

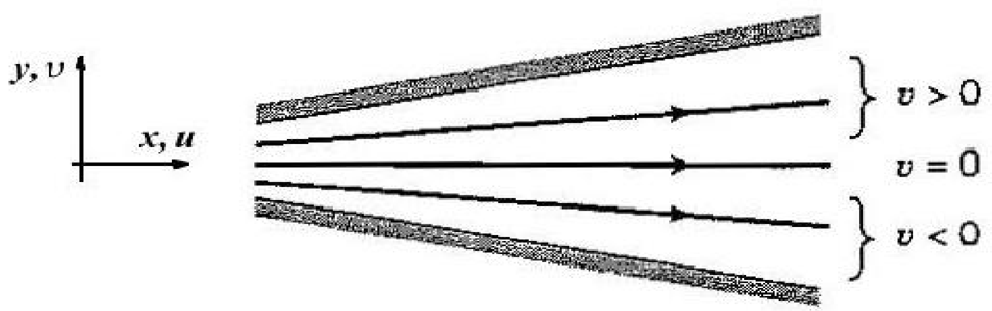

Figure 6: Flow in a symmetrically diverging duct.

We therefore define a class of "quasi" one-dimensional flows where the flow variables,including the streamwise velocity, are assumed to be constant across the flow area. Thiswill be a good approximation to one-dimensional flow as long as the other components ofvelocity (*v* and *w)* are small compared to the streamwise component, *u*.In addition, for the flow in a duct or pipe, the no-slip condition guarantees that the flowvelocity at the wall must be zero. So that there will alwaysbe a variation in velocity across the duct, even if the bulk of the flow has a relatively constantvelocity.Nevertheless, the concept of a one-dimensionalflow, is still a very useful approximation in many cases.

Conclusion:

In fluid mechanics, a system is defined as the chunk of fluid particles whose identity does not change during the course of flow. Here, the identity means that the chunk is composed

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A control volume is a volume or region in space whose identity is not same as fluid can enter and leave and through the control surface which encloses the volume. The shape and size of the control volume may be fixed or can change depending upon the choice of coordinate system used to analyse the flow field. Here, the Eulerian variables are more suitable for analysis of flow field.

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