

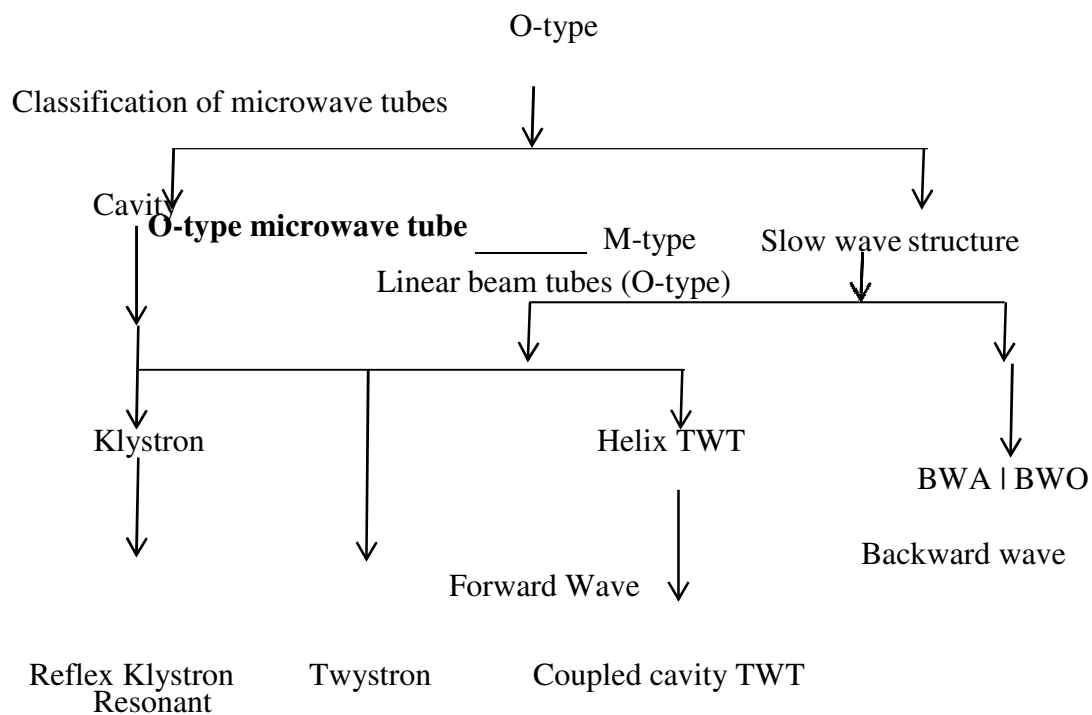
Monograph On Microwave Tubes And Measurements

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5.1 Microwave Tubes

→ Microwave tubes are constructed to overcome the limitations of conventional electronic vacuum tubes such as diodes, triodes, tetrodes and pentodes.



BWA → Backward Wave Amplifier;

BWO → Backward Wave Oscillator.

Christo Ananth et al.[1] discussed about E-plane and H-plane patterns which forms the basis of Microwave Engineering principles.

→ It is a vacuum tube that can be used either as a generator or an amplifier of power at microwave frequencies operated by the principles of velocity and current modulation (resonant periodic structure).

- | | | |
|----------|---|---|
| TWT | → | Non-resonant periodic structure for electron beam interactions. |
| Twystron | → | Hybrid amplifier that uses the combinations of klystron and TWT components. |

The diagram illustrates the internal structure and operation of a reflex klystron. At the bottom, an **Electron gun** provides electrons, which pass through a **Cathode R** and an **Accelerating grid G** (labeled 'c' with an upward arrow). The electrons are then accelerated into a **Cavity anode A**, represented by two horizontal plates. A **Repeller R** is positioned above the cavity, creating a retarding field that causes the electron beam to reflect. The distance between the anode and the repeller is labeled L . The gap between the anode plates is labeled d . An **RF o/p** (radio frequency output) is shown on the right. The diagram also shows the **VR** (variable resistor) and **V₀** (DC voltage) connected to the accelerating grid. A **V₁** (bias voltage) is connected to the cathode. The electron beam's path is shown as a dashed line, and the RF signal is shown as a solid line. Key points on the beam path are labeled a , b , c , and $t = 0$. The time taken for the beam to travel from the cathode to the anode is labeled $t = t_b$. The time taken for the beam to travel from the anode to the repeller is labeled t_0 .

- The reflex klystron is an oscillator with a built to feedback mechanism.
- The repeller electrode is a negative potential and sends the bunched electron beam back to the resonator cavity. This provides a positive feedback mechanism which support oscillations.
- Due to dc voltage (V_0) in the cavity circuit RF noise is generated in the cavity. This electromagnetic noise field in the cavity act as cavity resonant frequency.
- When the oscillation frequency is varied, the resonant frequency of cavity and the feedback path phase shift must be readjusted for a positive feedback.

Applegate diagram

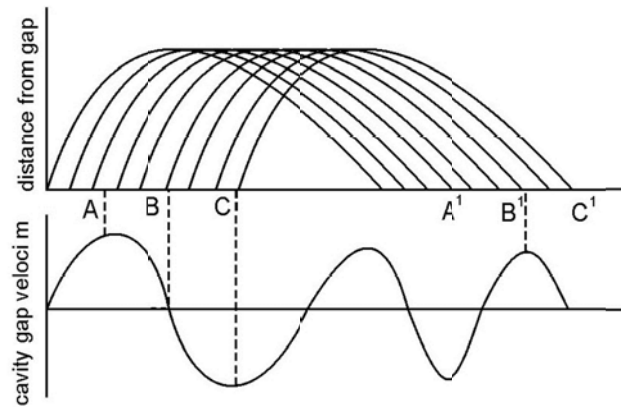


Fig. 5.2. : Applegate diagram with gap voltage for a Reflex Klystron

- When the gap voltage is as positive peak, electron passing at this moment is called early electron. This electron is accelerated towards repeller and travels a distance which is large comparatively.
- The electron at natural zero of gap voltage is called reference electron. When the gap voltage is at positive peak the corresponding electron is called late electron.

Modes of oscillation

The condition for oscillation $t_0 = (n + \frac{3}{4}) T = NT$

Where $N = n + \frac{3}{4}$ and mode of oscillation $= 0, 1, 2, 3, \dots$ T is the time period at the resonant frequency and t_0 is the time taken by the reference electron to travel in the repeller space.

Velocity Modulation

- The electron enters into the cavity gap from the cathode at $z = 0$ and time t_0 is assumed to have uniform velocity.

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \quad \text{----- (1)}$$

The same electron leaves the cavity gap at $z = d$ at time t_1 with velocity

$$v(t_1) = v_0 \left[1 + \frac{b_1 V_1}{2 V_0} \sin \left(\omega t_1 - \frac{\theta_g}{2} \right) \right] \quad \text{---- (2)}$$

The same electron is forced back to the cavity $z = d$ and time t_2 by the retarding electric field E .

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t)}{L} \quad \text{----- (3)}$$

The retarding field E is assumed to be constant in the z direction. The force equation on the repeller region is

$$E = \frac{V_r + V_0}{L} \quad \text{where } (V_1 \sin \omega t \ll (V_r + V_0)) \quad \text{----- (4)}$$

$$\text{Force of electron} = -e E = -e \left[\frac{V_r + V_0}{L} \right] \quad \text{----- (5)}$$

$$\text{Force of electron} = \text{mass} \times \text{acceleration} = m \frac{d^2 z}{dt^2} \quad (\text{Z distance})$$

$$-e E = m \frac{d^2 z}{dt^2} \quad V_r \rightarrow \text{magnitude of repeller voltage.}$$

$E \rightarrow -A V$ is used in z direction only

$$\frac{d^2 z}{dt^2} = -e \frac{(V_r + V_0)}{mL} \quad \text{----- (6)}$$

Integrating equation (6) with respect to 't' and 't₁'

$$\frac{dz}{dt} = -e \frac{(V_r + V_0)}{mL} \int_{t_1}^t dt = -e \frac{(V_r + V_0)}{mL} (t - t_1) + K_1 \quad \text{----- (7)}$$

If $t = t_1$, $\frac{dz}{dt} = v(t_1) = K_1$ then

$$\begin{aligned} z &= -e \left[\frac{V_r + V_0}{mL} \right] \int_{t_1}^t (t - t_1) dt + v(t_1) \int_{t_1}^t dt \\ &= -e \left[\frac{V_r + V_0}{mL} \right] \left[\frac{(t - t_1)^2}{2} + v(t_1)(t - t_1) + K_2 \right] \\ &= -e \left[\frac{V_r + V_0}{2mL} \right] (t - t_1)^2 + v(t_1)(t - t_1) + K_2 \quad \text{---- (8)} \end{aligned}$$

At $t = t_1$, $z = d = K_2$ then

$$z = -e \left[\frac{V_r + V_0}{2mL} \right] (t - t_1)^2 + v(t_1)(t - t_1) + d \quad \text{---- (9)}$$

→ The electron leaves the cavity gap at $z = d$ and time t_1 with a velocity of $v(t_1)$ and due to repeller negative potential returns to the gap $z = d$ and time t_2 then at $t = t_2$, $z = d$.

$$0 = -e \frac{(V_r + V_0)}{2 mL} (t_2 - t_1)^2 + v(t_1)(t_2 - t_1) \quad \text{----- (10)}$$

Transit time

The round trip transit time in the repeller region is given by

$$T' = \frac{2 (\text{velocity})}{\text{acceleration}} \quad \text{----- (11)}$$

The factor 2 in the numerator arises because of the t_0 and from journey of electrons.

$$T' = \frac{2v(t_1)}{\frac{d^2 Z}{dt^2}} = (t_2 - t_1) = \frac{2 mL}{e(V_r + V_0)} \cdot v(t_1) \quad \text{----- (12)}$$

Now the negative sign is not taken as electron bunch travels in the reverse direction.

Substitute equation (2) in (12)

$$T' = T_0^r \left[1 + \frac{b_1 V_1}{2 V_0} \sin \left(\omega t_1 - \frac{\theta_0}{2} \right) \right] \quad \text{----- (13)}$$

The round trip transit time of the center of the bunch electron

$$T_0^r = \frac{2 mL v_0}{e(V_r + V_0)} \quad \text{----- (14)}$$

Multiply the equation (13) by a radian frequency.

$$\omega(t_2 - t_1) = \omega T_0^r + \omega T_0^r \frac{b_1 V_1}{2 V_0} \sin \left(\omega t_1 - \frac{\theta_0}{2} \right)$$

$$\omega T_0^r = \theta_0^r + X' \sin \left(\omega t_1 - \frac{\theta_0}{2} \right) \quad \text{---- (15)}$$

The round trip at transit angle of the center of the bunch electron

$$\theta_0^r = \omega T_0^r \quad \text{---- (16)}$$

The bunching parameter of the reflex klystron oscillator

$$X' = \frac{b_1 V_1}{2 V_0} \theta_0^r \quad \text{----- (17)}$$

Output power

→ The maximum amount of kinetic energy can be transferred from the returning electrons to in the cavity walls.

→ For a maximum energy transfer the round trip transit angle is given by

$$\omega(t_2 - t_1) = \omega T_0^r = \left(n - \frac{1}{4}\right) 2\pi \quad \text{----- (18)}$$

$$= N 2\pi = 2\pi n - \frac{\pi}{2} \quad \text{----- (19)}$$

where $V_1 \ll V_0$ is assumed

n = any positive integer for cycle no and

$N = n - \frac{1}{4}$ is the no of modes.

The beam current injected into the cavity gap from the repeller region flows in negative z direction.

The beam current of a reflex klystron oscillator can be written as

$$i_{2t} = -I_0 - \sum_{n=1}^{\infty} 2 I_0 J_n(nX') \cos [n(\omega t_2 - \theta_0^r - \theta_g)] \quad \text{----- (20)}$$

$I_0 \rightarrow$ dc beam current.

The fundamental component of the current induced in the cavity by the modulated e^{-n} beam is given by ($\theta_g \ll \theta_0$)

$$\begin{aligned} i_2 &= -\beta_i I_2 \\ &= 2 I_0 J_1(X') \cos(\omega t_2 - \theta_0^r). \beta_i \end{aligned} \quad \text{----- (21)}$$

The magnitude of fundamental component

$$I_2 = 2 I_0 \beta_i J_1(X') \quad \text{----- (22)}$$

The dc power supplied by the beam voltage V_0

$$P_{dc} = V_0 I_0 \quad \text{----- (23)}$$

The ac power delivered to the load

$$P_{ac} = \frac{V_1 I_2}{2} = V_1 I_0 \beta_i J_1(X') \quad \text{----- (24)}$$

X' – bunching parameter of reflex klystron.

The bunching parameter

$$X' = \beta_i V_i \frac{\omega_0}{2 V_0}$$

where $\theta_0^r = \omega T_0^r = 2\pi n - \frac{\pi}{2}$

$$2 V_0 X' = \beta_i V_1 \left(2\pi n - \frac{n}{2} \right)$$

$$\frac{V_1}{V_0} = \frac{2Xr}{\beta_i \left(2\pi n - \frac{n}{2} \right)}$$

$$V_1 = \frac{2X^F V_0}{\beta_i \left(2\pi n - \frac{n}{2} \right)} \quad \text{----- (25)}$$

Substitute equation (25) in equation (24)

$$P_{ac} = \frac{2X^F V_0 I_0 \beta_i J_1(X^F)}{\beta_i \left(2\pi n - \frac{n}{2} \right)} = \frac{2X^F V_0 I_0 J_1(X^F)}{2\pi n - \frac{n}{2}} \quad \text{----- (26)}$$

Efficiency

$$\eta = \frac{P_{ac}}{P_{dc}} = \frac{2X^F V_0 I_0 J_1(X^F)}{2\pi n - \frac{n}{2}} \times \frac{1}{V_0 I_0}$$

$$= \frac{2X^F J_1(X^F)}{2\pi n - \frac{n}{2}} \quad \text{----- (27)}$$

Maximum efficiency

→ The factor $X' J_1(X')$ reaches a maximum value of 1.25 at $X' = 2.408$ and $J_1(X') = 0.52$

The maximum efficiency is obtained when $n = 2$ or $1\frac{3}{4}$ mode

Maximum theoretical efficiency is

$$\eta_{\max} = \frac{2(2.408) J_1(2.408)}{2\pi(2) - \frac{2}{2}} = 22.78\% \quad \text{----- (28)}$$

Theoretical efficiency of Reflex Klystron ranges from 20 to 30%.

Output power in terms of repeller voltage V_R

→ For a given beam voltage V_0 , the relationship between the repeller voltage and cycle number of n required for oscillation is given by

$$\frac{V_0}{(V_r + V_0)^2} = \frac{\left(2\pi n - \frac{n}{2} \right)^2}{8 m^2 L^2} \frac{e}{m} \quad \text{----- (29)}$$

The output power can be expressed in terms of V_R

$$P_{ac} = \frac{V_0 I_0 X^F J_1(X^F) (V_r + V_0)}{mL} \mathbf{J} \frac{e}{2\pi m V_0} \quad \text{----- (30)}$$

Equivalent circuit of Reflex Klystron

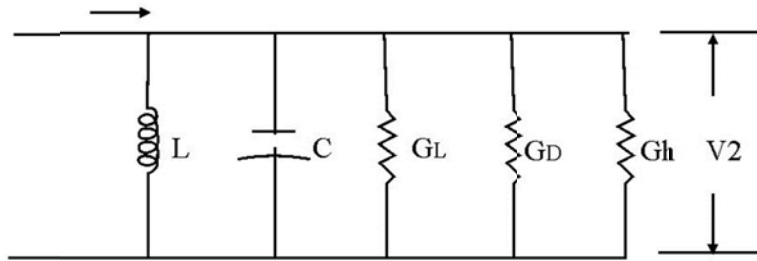


Fig. 5.3 : Equivalent circuit of Reflex Klystron

In the circuit L and C are energy storage elements, of the cavity

- G_0 copper losses of the cavity
- G_B Beam loading conductance &
- G_L Load conductance

5.3 Two cavity Klystron Amplifier

- A two cavity klystron Amplifier is a velocity modulated tube in which the velocity modulation process produces density modulation process of electrons. It consists of two cavity bunches (input) & catcher (output) cavity. The separation between bunches & catcher grids is called drift space.

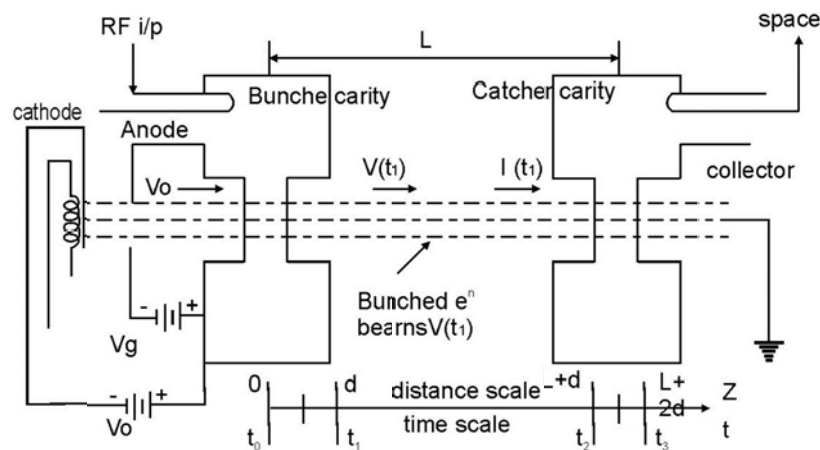


Fig. 5.4 : Two cavity klystron Amplifier

- Cathode emits the electron beam. This electron beam first reaches the anode. The accelerating anode produces a high velocity electron beam.

- Input RF signal to be amplified excites the buncher cavity with a coupling loop.
- The electron beam passing the buncher cavity gap at zeros of the gap voltage passes through with unchanged velocity.

Velocity Modulation

The variation in electron velocity in the drift space is known as velocity modulation.

Catcher cavity

The output cavity catches energy from bunched electron beam. It is called as catcher cavity.

- The electron beam passing the positive half cycle of the gap voltage under in velocity. In negative half cycle the gap voltage undergo decrease in velocity. As the electron gradually lunch together so they travel down the drift space.
- The first cavity acts as the buncher and velocity modulates the beam. Thus the electron beam is velocity modulated to form bunches or undergoes thereby modulation in accordance with the I/P RF single cycle.
- The ac current on the beam is such that the level of excitation of the second cavity is much greater than the buncher cavity.
- If desired a portion of the amplified output can be fed back to the regenerative manner to obtain self-sustained oscillation.
- The maximum bunching cavity occur between the second cavity grids during its retarding phase, thus the kinetic energy is transferred from the electron to the field of the second cavity.

Velocity modulation Process

- When electrons are first accelerated by the high dc beam voltage V_0 before entering the buncher grids, their velocity v_0 is uniform.

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \text{ m/s} \quad \text{----- (1)}$$

Assume that electrons leave the cathode with zero velocity.

- When a microwave signal applied to the input terminal of the buncher cavity the gap voltage between the buncher grids can be written as

$$V_S = V_1 \sin \omega t$$

$$V_1 \rightarrow \text{amplitude of signal and assume } (V_1 \ll V_0) \quad \text{----- (2)}$$

Average transit time through the buncher cavity grids gap distance d is

$$\tau = \frac{d}{V_0} = t_1 - t_0 \quad \text{----- (3)}$$

The average gap transit angle

$$\theta_g = \omega \tau = \omega (t_1 - t_0) = \frac{\omega d}{V_0} \quad \text{----- (4)}$$

The average microwave voltage in the buncher gap can be written as

$$V_S = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin (\omega t) dt$$

$$\text{Equation (4)} = \frac{-V_1}{\omega \tau} [\cos (\omega t_1) - \cos (\omega t_0)] \quad \text{----- (5)}$$

$$\omega(t_1 - t_0) = \frac{\omega d}{V_0} \quad ; \quad \omega t_1 = \frac{\omega d}{V_0} + \omega t_0 \quad \text{----- (6)}$$

Substitute equation (6) in (5)

$$= \frac{V_1}{\omega \tau} [\cos (\omega t_0) - \cos (\omega t_0 + \frac{\omega d}{V_0})]$$

$$\text{Let } \omega t_0 + \frac{\omega d}{2 V_0} = \omega t_0 + \frac{\theta_g}{2} = A \text{ \& } B = \frac{\omega d}{2 V_0} = \frac{\theta_g}{2}$$

$$A - B = \omega t_0$$

$$A + B = \omega t_0 + \theta_g = \omega t_0 + \frac{\omega d}{V_0}$$

By using trigonometric relation

$$\cos (A - B) - \cos (A + B) = 2 \sin A \sin B$$

$$V_S = \frac{V_1}{\omega \tau} 2 \sin \left[\frac{\theta_g}{2} \right] \sin \left(\omega t_0 + \frac{\omega d}{2 V_0} \right)$$

$$\text{Substitute } \tau = \frac{d}{V_0} = \frac{V_1 \sin \left(\frac{\theta_g}{2} \right)}{\frac{\omega d}{2 V_0}} \sin \left(\omega t_0 + \frac{\omega d}{2 V_0} \right)$$

$$V_S = V_1 \frac{\sin \left(\frac{\theta_g}{2} \right)}{\frac{\theta_g}{2}} \sin \left(\omega t_0 + \frac{\theta_g}{2} \right) \quad \text{----- (7)}$$

$$V_s = V_1 \beta_i \sin \left(\omega t_0 + \frac{\theta_g}{2} \right) \quad \beta_1 = \frac{\sin \left(\frac{\theta_g}{2} \right)}{\frac{\theta_g}{2}}$$

where β_1 – buncher cavity beam coupling co-efficient of the input cavity gap.

- Increasing the θ_g decreases the coupling between the electrons beam & buncher cavity, (i.e.) the velocity modulation of the beam for a given microwave signal is decreased.
- After velocity modulation the exit velocity from the buncher gap.

$$v(t_1) = \sqrt{\frac{2e}{m} \left[V_0 + \beta_i V_1 \sin \left(\omega t_0 + \frac{\theta_g}{2} \right) \right]}$$

$$v(t_1) = \sqrt{\frac{2eV_0}{m} \left[1 + \frac{\beta_i V_1}{2V_0} \sin \left(\omega t_0 + \frac{\theta_g}{2} \right) \right]} \quad \text{----- (8)}$$

Thus the electrons in the beam are velocity modulated by the input RF signal with depth of velocity modulation (m) = $\frac{\beta_i V_1}{V_0}$. Since $\beta_i V_1 \ll V_0$, the binormal expansion of equation (8).

$$v(t_1) = V_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin \left(\omega t_0 + \frac{\theta_g}{2} \right) \right] \quad \text{----- (9)}$$

It is equation of velocity modulation.

Alternatively the equation of velocity modulation can be given by

$$v(t_1) = V_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin \left(\omega t_1 + \frac{\theta_g}{2} \right) \right] \quad \text{----- (10)}$$

Bunching process

- The effect of bunching process produces bunching of electron beam (or) current modulation.
- The electron that pass the buncher cavity during the positive half cycles of microwave input voltage V_s travel faster than the electrons that passed the gap when $V_s = 0$.
- During the negative half cycle, V_s travel slower than the electrons that passed the gap when $V_s = 0$.

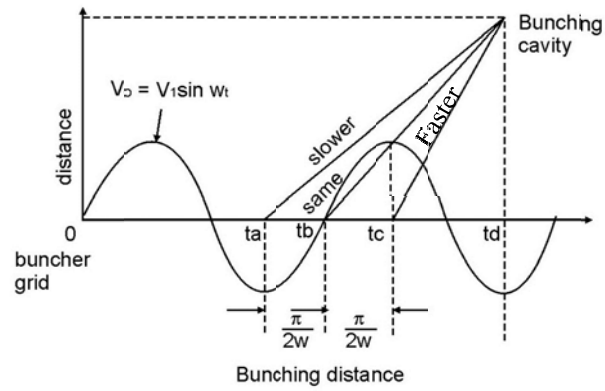


Fig. 5.5 : Bunching distance

Bunching distance from the buncher grid to the location of dense electron bunching for the electron at t_b is

$$\Delta L = v_0 (t_d - t_b) \quad \text{----- (11)}$$

where $t_c = t_b + \frac{u}{\omega}$; $t_b = t_a + \frac{u}{2\omega}$; $t_a = t_b - \frac{u}{2\omega}$

The distance for the electrons at t_a and t_c are

$$\Delta L = v_{\min} (t_d - t_a) = v_{\min} (t_d - t_b + \frac{u}{2\omega}) \quad \text{----- (12)}$$

$$\Delta L = v_{\max} (t_d - t_c) = v_{\max} (t_d - t_b - \frac{u}{2\omega}) \quad \text{----- (13)}$$

From equation (9) (or) equation 10 the maximum & minimum velocities are

$$v_{\min} = V_0 \left(1 - \frac{b_1 V_1}{2 V_0} \right) \quad \text{----- (14)}$$

$$v_{\max} = V_0 \left(1 + \frac{b_1 V_1}{2 V_0} \right) \quad \text{----- (15)}$$

Substitute equation (14) in equation (12)

$$\begin{aligned} \Delta L &= v_{\min} \left((t_d - t_b) + \frac{\pi}{2\omega} \right) \\ &= v_0 \left(1 - \frac{b_1 V_1}{2 V_0} \right) \left((t_d - t_b) + \frac{\pi}{2\omega} \right) \\ &= v_0 (t_d - t_b) + v_0 \frac{u}{2\omega} - \frac{V_0 b_1 V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_1 V_1}{2 V_0} \frac{u}{2\omega} \end{aligned} \quad \text{----- (16)}$$

Substitute equation (15) in equation (13)

$$\begin{aligned}\Delta L &= V_{\max} \left((t_d - t_b) - \frac{\pi}{2m} \right) \\ &= v_0 (t_d - t_b) + r - \frac{V_0 u}{2\omega} + \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_i V_1}{2 V_0} \frac{u}{2\omega} \quad \text{----- (17)}\end{aligned}$$

Applegate diagram

→ It represents the internal operation of two cavity klystron by distance time plot. It include velocity modulation process, bunching & energy transfer etc.

From equation (16) & equation (17), the necessary condition for those electrons at t_a , t_b and t_c to meet at the same distance ΔL is

$$\frac{V_0 u}{2\omega} + \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_i V_1}{2 V_0} \frac{u}{2\omega} = 0 \quad \text{----- (18)}$$

$$- \frac{V_0 \pi}{2m} + \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_i V_1}{2 V_0} \frac{u}{2\omega} = 0 \quad \text{----- (19)}$$

Equating equation (18) & equation (19)

$$\begin{aligned}\frac{V_0 u}{2\omega} + \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_i V_1}{2 V_0} \frac{u}{2m} \\ - \frac{V_0 u}{2\omega} + \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_i V_1}{2 V_0} \frac{u}{2m}\end{aligned}$$

$$\frac{V_0 u}{2\omega} + \frac{V_0 u}{2\omega} - \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b) - \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b)$$

$$\frac{V_0 u}{\omega} = \frac{V_0 b_i V_1}{2 V_0} (t_d - t_b)$$

$$(t_d - t_b) = \frac{u V_0}{\omega b_i V_1} \quad \text{----- (20)}$$

Substitute equation (20) in equation (11) we get the expression for min distance at which maximum bunching occur

$$\Delta L = V_0 (t_d - t_b)$$

$$\Delta L = \frac{V_0 u V_0}{m b_i V_1} \quad \text{----- (21)}$$

Maximum bunching

→ Now, the spacing between the buncher & catcher cavities in order to achieve the maximum degree of bunching.

→ The transit time for an electron to travel at distance of L

$$T = t_2 - t_1 = \frac{L}{V(t)} \quad \text{----- (22)}$$

Substitute equation (10) for V(t) in equation (22) and use the binomial expansion.

$$\begin{aligned} (1+x)^{-1} &= 1-x \quad \text{for } |x| \ll 1. \\ &= \frac{L}{V_0 \left[1 + \frac{b_1 V_1}{2 V_0} \sin \left(\omega t_1 - \frac{\omega g}{2} \right) \right]} \\ T &= T_0 \left[1 + \frac{b_1 V_1}{2 V_0} \sin \left(\omega t_1 - \frac{\omega g}{2} \right) \right] \quad \text{----- (23)} \end{aligned}$$

where [@ $T_0 = \frac{L}{V_0}$] is the dc transit time.

→ Bunching parameter & DC Transit angle

In terms of radians, the equation (23) becomes

$$\begin{aligned} \omega T &= \omega t_2 - \omega t_1 = \omega T_0 - \frac{\omega T_0 b_1 V_1}{2 V_0} \sin \left(\omega t_1 - \frac{\omega g}{2} \right) \\ &= \theta_0 - X \sin \left(\omega t_1 - \frac{\omega g}{2} \right) \quad \text{----- (24)} \end{aligned}$$

$$\text{dc transit angle between cavities } \theta_0 = \frac{\omega L}{V_0} = 2\pi N \quad \text{----- (25)}$$

where, N is the number of electron transit cycle in the drift space.

The bunching parameter of a klystron.

$$X = \frac{b_1 V_1}{2 V_0} \theta_0 \quad \text{----- (26)}$$

Current modulation

- **Beam current in catcher cavity**

→ The bunched beam current at the catcher cavity is a periodic wave form of period $\frac{2\pi}{\omega}$ about dc current.

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2 I_0 J_n(nX) \cos [\omega n (t_2 - \tau - T_0)] \quad \text{----- (27)}$$

$I_0 \rightarrow$ dc beam current in buncher cavity.

The klystron is generally tuned in fundamental ac component of current given by

$$I_f = 2 I_0 J_1(X) \cos(\omega t_2 - \tau - T_0) \quad \text{----- (28)}$$

The fundamental ac component of the beam current at the catcher cavity has a magnitude

$$I_f = 2 I_0 J_1(X) \quad \text{----- (29)}$$

This fundamental ac component I_f can be maximum when $J_1(X) = 0.582$ at $X = 1.841$ by adjusting the beam voltage V_0 . So the optimum distance L at which the maximum fundamental ac component of current occurs.

$$\text{we know that } X = \frac{b_1 V_1}{2 V_0} \quad \theta_0 = \frac{b_1 V_1}{2 V_0} \frac{\omega L}{v_0}$$

$$L = \frac{2 X V_0 v_0}{\omega b_1 V_1} \quad \text{----- (30)}$$

In equation (30) $L = L_{\text{opt}}$ when $X = 1.841$

$$I_{\text{opt}} = \frac{2 X V_0 r_0}{\omega b_1 V_1} = \frac{2 \times 1.841 \times V_0 \times r_0}{\omega b_1 V_1}$$

$$I_{\text{opt}} = \frac{3.682 V_0 v_0}{\omega b_1 V_1} \quad \text{----- (31)}$$

Output Power

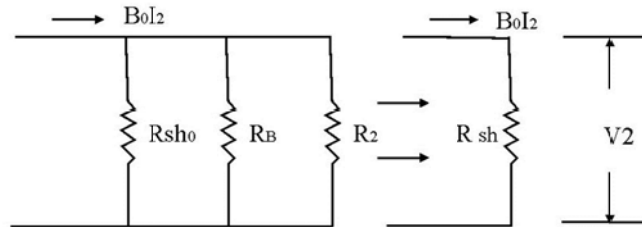


Fig. 5.6 : Equivalent circuit of output power

$R_{Sho} \rightarrow$ Wall resistance of catcher cavity

$R_B \rightarrow$ Beam loading resistance

$R_L \rightarrow$ External load R

$R_{Sh} \rightarrow$ Total equivalent shunt R of the catcher cavity including the load.

Christo Ananth et al. [2] discussed about Improved Particle Swarm Optimization. The fuzzy filter based on particle swarm optimization is used to remove the high density image impulse noise, data acquisition and processing.

- **Induced current (i_{2ind}) in the catcher cavity**

→ The fundamental component of RF beam current passing through the catcher cavity induces a current in the catcher cavity.

$$I_{2ind} = \beta_0 i_2 = \beta_0 2 I_0 J_1(X) \cos(\omega(t_2 - \tau - T_0)) \quad \text{----- (32)}$$

$$= I_2 \beta_0 \cos(\omega(t_2 - \tau - T_0)) :$$

Where $I_2 = 2 I_0 J_1(X)$

The corresponding RF Voltage $V_2 = V_2 \cos(\omega(t_2 - \tau - T_0)) = \beta_2 I_2 R_{sh}$

$\beta_0 \rightarrow$ Beam coupling co-efficient

$\beta_0 = \beta_i$ when both buncher & catcher cavities are identical.

The magnitude of the induced current in the cavity is given by

$$I_2 = \beta_0 2 I_0 J_1(X) \quad \text{----- (33)}$$

The output power delivered to the catcher cavity and the load is given as

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} \quad \text{---- (34)}$$

$$\text{where, } R_{sh} = \frac{V_2}{\beta_0 I_2}$$

$V_2 \rightarrow$ fundamental component of catcher gap voltage

$$P_{out} = \frac{\beta_0^2 I_2^2}{2} \times \frac{V_2}{\beta_0 I_2} = \frac{\beta_0 I_2 V_2}{2} \quad \text{----- (35)}$$

- **Efficiency**

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{ac}}{P_{dc}} \quad \text{----- (36)}$$

The dc power supplied by beam voltage $P_{in} = V_0 I_0$ ----- (37)

$$\eta = \frac{\beta_0 I_2 V_2}{2} \times \frac{1}{V_0 I_0} = \frac{\beta_0 I_2 V_2}{2 V_0 I_0} \quad \text{----- (38)}$$

- **Maximum efficiency**

$$\eta = \frac{\beta_0 I_2 V_2}{2 V_0 I_0}$$

substitute equation (33) in above equation

$$= \frac{\beta_0 2 I_2 J_1(X) V_2}{2 I_0 V_0} = \frac{\beta_0 J_1(X) V_2}{V_0}$$

The efficiency becomes maximum when $J_1(X) = 0.582$ in $X = 1.841$ output voltage is V_0 ($V_2 = V_0$)

$$\begin{aligned}\eta_{\max} &= 0.582 \frac{\beta_0 V_2}{V_0} \\ &= 0.582\%\end{aligned}$$

If the coupling is perfect $\beta_0 = 1$ then

$$\eta_{\max} = 58.2\% \quad \text{----- (39)}$$

ii) Voltage Gain

The input voltage V_1 is the bunching parameter X

$$V_1 = \frac{2 V_0}{\beta_0 Q_0} X$$

$$\text{we know } R_{sh} = \frac{V_2}{\beta_0 I_2}$$

$$V_2 = \beta_0 I_2 R_{sh}$$

$$\begin{aligned}A_v &= \left| \frac{V_2}{V_1} \right| = \frac{\beta_0 I_2 R_{sh}}{V_1} = \frac{\beta_0 I_2 R_{sh}}{2 V_0 X} \beta_0 \theta_0 \\ &= \frac{\beta_0^2 \theta_0 R_{sh} I_2}{2 V_0 X} = \frac{\beta_0^2 \theta_0 R_{sh} \cdot 2 I_0 J_1(X)}{2 V_0 X} \\ &= \frac{\beta_0^2 \theta_0 R_{sh} \cdot J_1(X)}{V_0} = J_1 \\ &= \frac{\beta_0^2 \theta_0}{R_0} \cdot \frac{J_1(X)}{X} \cdot R_{sh} \quad (R_0 = \frac{V_0}{I_0} \text{ is the dc beam resistance})\end{aligned}$$

$$A_v = G_m \cdot R_{sh}, \text{ (or) } \frac{G_m}{G_{sh}}.$$

Multi-cavity Klystron Amplifiers (or) Four cavity Klystron Amplifier

- The typical power gain of a two-cavity klystron is about 30 dB.
- In order to achieve higher overall gain, connect several two-cavity tubes in cascade, feeding the output of each of the tubes to the input of the following one.
- Multi cavity klystron is to serve the high gain requirement.

- The cavities can all be tuned to the same frequency for narrow band operation.
- The gain increases exponentially with the number of cavities employed. The last cavity in the chain used as the output cavity.
- Power gains of 50 to 60 dB can be achieved with multi cavity klystrons.

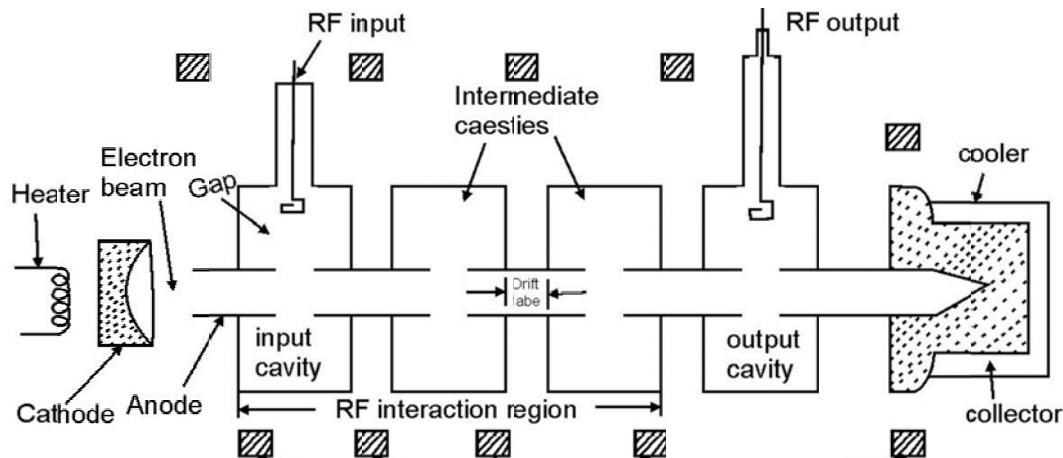


Fig. 5.7 : Schematic diagram of a four (or) multi cavity klystron amplifier

Beam-current Density

The space – charge forces within electron bunches vary with the size and shape of an electron beam.

Plasma frequency

The electron plasma frequency is the frequency at which the electrons will

$$\text{Charge density } \rho = B \cos(\beta_e z) \cos(\omega_q t + \theta) \quad \text{----- (1)}$$

$$\text{Velocity perturbation } \gamma = -C \sin(\beta_e z) \sin(\omega_q t + \theta) \quad \text{----- (2)}$$

Where B = Constant of charge – density perturbation

C = Constant of velocity perturbation

Christo Ananth et al.[3] presented a short overview on two port RF networks. They widely used microwave and RF applications and the denomination of frequency bands. The monograph start outs with an illustrative case on wave propagation which will introduce fundamental aspects of high frequency technology.

Reduced Plasma frequency

In practical, beams of finite diameter are characterized by plasma frequency i.e. less than ω_P . It is designated ω_q .

The total charge density and electron velocity are given by,

$$\rho_{\text{tot}} = -\rho_0 + \rho \quad \text{----- (3)}$$

$$\gamma_{\text{tot}} = \gamma_0 + \gamma \quad \text{----- (4)}$$

$\rho_0 \rightarrow$ dc electron charge density

$\rho \rightarrow$ Instantaneous RF charge density

$\gamma_0 \rightarrow$ dc electron charge density

$\gamma \rightarrow$ Instantaneous electron velocity perturbation

The total electron beam current density can be written as

$$J_{\text{tot}} = -J_0 + J \quad \text{----- (5)}$$

$J_0 \rightarrow$ dc beam current density

$J \rightarrow$ Instantaneous RF beam-current perturbation

The instantaneous convection beam-current density at any point in the beam is expressed.

$$\begin{aligned} J_{\text{tot}} &= \rho_{\text{tot}} \gamma_{\text{tot}} = (-\rho_0 + \rho) (\gamma_0 + \gamma) \\ &= -\rho_0 \gamma_0 - \rho_0 \gamma + \rho \gamma_0 + \rho \gamma \\ J_{\text{tot}} &= -J_0 + J \quad \text{---- (6)} \end{aligned}$$

$$\text{where, } J = \rho \gamma_0 - \rho_0 \gamma$$

$$J_0 = \rho_0 \gamma_0$$

$\rho \gamma \rightarrow$ Very small is ignored.

In accordance with the law of conservation of electric charge, the continuity equation can be written as

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

where, $J = \rho \gamma_0 - \rho_0 \gamma$ is in positive z direction only

For practical microwave tubes the beam-current density and the modulated velocity are expressed as,

$$J = \gamma_0 B \cos(\beta z - \omega t) \cos(\omega t_1 + \theta) + \frac{m_q}{m} \gamma_0 B \sin(\beta z - \omega t) \sin(\omega t_1 + \theta) \quad \text{----- (7)}$$

$$\gamma = -\gamma_0 \frac{b_i V_1}{2 V_0} \cos(\beta_q z) \sin(\beta_c z - \omega t) \quad \text{----- (8)}$$

The electrons leaving the input gap of a klystron amplifier have a velocity at the exit grid as

$$\gamma(t_1) = \gamma_0 \left[1 + \frac{b_i V_1}{2 V_0} \sin(\omega \tau) \right] \quad \text{---- (9)}$$

$V_1 \rightarrow$ magnitude of the input signal voltage.

$$\tau = \frac{d}{v_0} = t_1 - t_0 \rightarrow \text{transit time}$$

$d \rightarrow$ gap distance

The velocity at a later time t is given by

$$\gamma_{\text{tot}} = \gamma_0 \left[1 + \frac{b_i V_1}{2 V_0} \sin(\omega \tau) \cos(\omega_p t - \omega_p \tau) \right] \quad \text{----- (10)}$$

$\omega_p \rightarrow$ plasma frequency

$$\text{The current density } J = \frac{1}{2} \frac{J_0 m}{V_0 m_q} \beta_i V_1 \sin(\beta_q z) \cos(\beta_c z - \omega t) \quad \text{----- (11)}$$

where, $\beta_q = \frac{m_q}{P_0} \rightarrow$ plasma phase constant.

Output current and output power of Two-cavity Klystron

If the two cavities of a two-cavity klystron amplifier are identical, the magnetic of RF convection current at the output cavity for a two cavity klystron can be written as

$$|i_2| = \frac{1}{2} \frac{I_0 m}{V_0 m_q} \beta_i |V_1| \quad \text{----- (12)}$$

where $V_1 \rightarrow$ magnitude of the input signal voltage.

The magnitudes of the induced current & voltage in the output cavity are equal.

$$|I_2| = \beta_0 |i_2| = \frac{1}{2} \frac{I_0 m}{V_0 m_q} \beta_0^2 |V_1| \quad \text{----- (13)}$$

$$\text{and } |V_2| = |I_2| R_{shl} = \frac{1}{2} \frac{I_0 m}{V_0 m_q} \beta_0^2 |V_1| R_{shl} \quad \text{----- (14)}$$

$\beta_0 = \beta_i \rightarrow$ beam coupling coefficient.

$R_{shl} \rightarrow$ Total shunt resistance of the output cavity in a two cavity klystron amplifier including the external load.

The output power delivered to the load in a two cavity klystron amplifier is given by

$$P_{out} = |I_2|^2 R_{shl} = \frac{1}{4} \left(\frac{I_0 m}{V_0 m_q} \right)^2 \beta_0^4 |V_1|^2 R_{shl} \quad \text{----- (15)}$$

The power gain of a two cavity klystron amplifier is then expressed by

$$\begin{aligned} \text{Power gain} &= \frac{P_{out}}{P_{in}} = \frac{P_{out}}{\frac{|V_1|^2}{R_{sh}}} \\ &= \frac{1}{4} \left(\frac{I_0 m}{V_0 m_q} \right)^2 \beta_0^4 \frac{R_{sh}}{R_{shl}} \end{aligned} \quad \text{----- (16)}$$

The electronic efficiency of a two-cavity klystron amplifier is

$$\begin{aligned} \eta &= \frac{P_{out}}{P_{in}} = \frac{P_{out}}{I_0 V_0} \\ \eta &= \frac{1}{4} \left(\frac{I_0}{V_0} \right) \left(\frac{|V_1| m}{V_0 m_q} \right)^2 \beta_0^4 \frac{R_{sh}}{R_{shl}} \end{aligned} \quad \text{----- (17)}$$

Output power of four-cavity Klystron

High power may be obtained by adding additional intermediate cavities in a two cavity klystron.

In the four cavity klystron amplifier, the four activities are assumed to be identical and they have same unloaded Q and beam coupling coefficient.

The RF convection current

$$|i_3| = \frac{1}{2} \frac{I_0 m}{V_0 m_q} \beta_0 |V_2| \quad \text{----- (18)}$$

Substitute equation (14) in equation (18)

$$|i_3| = \frac{1}{4} \left(\frac{I_0 m}{V_0 m_q} \right)^2 \beta_0^3 |V_1| R_{sh} \quad \text{----- (19)}$$

The output voltage is given by

$$|V_3| = \beta_0 |i_3| R_{sh} = \frac{1}{4} \left(\frac{I_0 m}{V_0 m_q} \right)^2 \beta_0^4 |V_1| R_{sh}^2 \quad \text{----- (20)}$$

Velocity modulation is convert into an RF convection current the output cavity for four cavity klystron as

$$|i_4| = \frac{1}{2} \frac{I_0 m}{V_0 m_q} \beta_i |V_3| = \frac{1}{8} \left(\frac{I_0 m}{V_0 m_q} \right)^3 \beta_0^5 |V_1| R_{sh}^2 \quad \text{----- (21)}$$

and $|I_4| = \beta_0 |i_4|$

$$= \frac{1}{8} \left(\frac{I_0 m}{V_0 m_q} \right)^3 \beta_0^6 |V_1| R_{sh}^2 \quad \text{---- (22)}$$

The output voltage is then $|V_4| = |I_4| R_{shl}$

$$= \frac{1}{8} \left(\frac{I_0 m}{V_0 m_q} \right)^3 \beta_0^6 |V_1| R_{sh}^2 R_{shl} \quad \text{---- (23)}$$

The output power from the o/p cavity in a 4-cavity klystron amplifier can be expressed as

$$P_{out} = |I_4|^2 R_{shl}$$

$$P_{out} = \frac{1}{64} \left(\frac{I_0 m}{V_0 m_q} \right)^6 \beta_0^{12} |V_1|^2 R_{sh}^4 R_{shl}^2 \quad \text{----- (24)}$$

where, R_{shl} = Total shunt resistance of the output cavity including the external load.

The high – power klystron amplifiers are available with a power gain of 40 to 50 dB.

5.5 Traveling Wave Tube Amplifier (TWTA)

→ A TWTA circuit uses a helix slow wave non resonant microwave guiding structure and thus a broadband microwave amplifier. Christo Ananth et al. [4] discussed about RF Transistor Amplifier Design and Matching Networks, amplifier power relation, impedance, $T \pi$ and microstripline matching networks. Christo Ananth et al.[5] analyzed Microwave Passive Components, microwave waveguides such as microwave T junctions, circulators, attenuators and Isolators. Christo Ananth et al.[6] discussed about Microwave Semiconductor Devices such as Tunnel diode, Gunn diode and valanche transit time devices and analyzes Monolithic Microwave Integrated Circuits (MMIC)

Helix Traveling Wave Tube

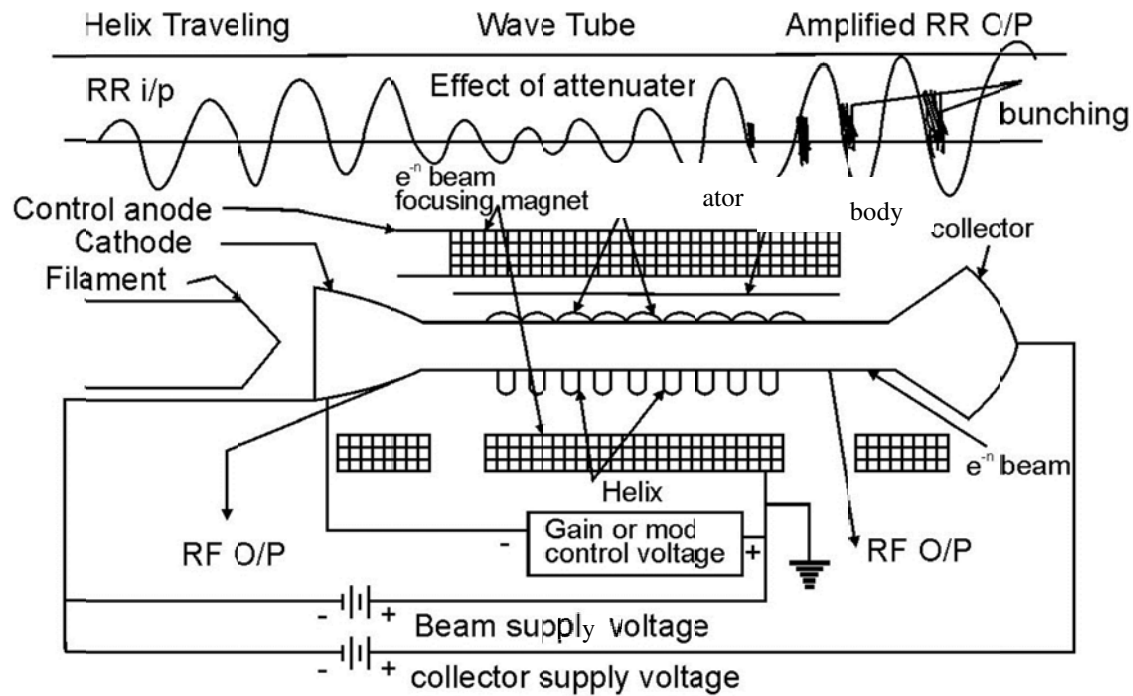


Fig. 5.8 : TWT amplifier tube

Operation

- The electrons beam is focused axially by a static magnetic field and collected in a collector circuit.
- The microwave input signal is injected on the helix slow wave circuit surrounding the electrons beam which produces an axial electric field of the signal at the center of the helix that can interact with the electrons beam.
- The dc beam voltage is adjusted so that the beam velocity is slightly greater than the axial component of field on the slow wave structure.
- During transit along the axis the electron beam transfers energy to the traveling signal wave and thus signal field increases.

Attenuator

- An attenuator is placed over a part of the helix near the out put end to attenuate any reflected waves due to impedance mismatch that can be fed back to the input to cause oscillations.

Magnet

- The magnet produces an axial magnetic field to prevent spreading of the electron beam as it travels down the tube.
- Need of slow – wave structure (helix tube).

Slow wave structures are special circuits that are used in microwave velocity in a certain direction. So that the electron beam and the signal wave can interact.

Magnetron oscillators

- Magnetron provide microwave oscillations at very high peak power. All magnetrons operated in a dc magnetic field normal to a dc electric field between the cathode and anode.

Classification

i) Split anode ii) cyclotron – frequency iii) Traveling wave

Split anode → This type of magnetron uses a static negative resistance between two anode segments but has low efficiency and is useful only at low frequencies.

Cyclotron – Frequency → It operates under the influence of synchronous between an alternating component of electric field and a periodic oscillation of electrons in a direction parallel to the field.

- It is useful only for frequencies greater than 100 MHz.

Traveling – Wave Magnetron → This type referred to simply as magnetrons. These magnetrons depend upon the interaction of electrons with a traveling electromagnetic field of linear velocity.

- Cylindrical magnetrons, linear magnetron, co-axial magnetron, voltage tunable magnetrons, inverted co-axial magnetrons & frequency agile magnetron output.

Power and Efficiency

- A magnetron can deliver a peak output power of up to 40 MW with the dc voltage 50 KV at 10 GHz.
- The average output power at 800 KW.
- It provides a very high η ranging from 40 to 70%.
- It is available for peak power output from 3 KW & higher.

5.6.1 Cylindrical Magnetron

- It is a high power microwave oscillator. It is also called as conventional magnetron. It has several reentrant cavities connected to gaps.
- The anode is a slow wave structure consisting of several re-entrant cavities equispaced around the circumference.
- The dc voltage V_0 is applied between the cathode and anode and dc magnetic flux density B_0 is maintained in the positive z direction by means of a permanent magnet or an electromagnet.
- The electrons emitted from the cathode try to travel to anode but with the influence of cross fields E&H in the space between anode and cathode the electrons take curved path.
- When the dc voltage and magnetic flux are adjusted, the electron will follow cycloidal paths in the cathode anode space under the combined force of both electric and magnetic field.

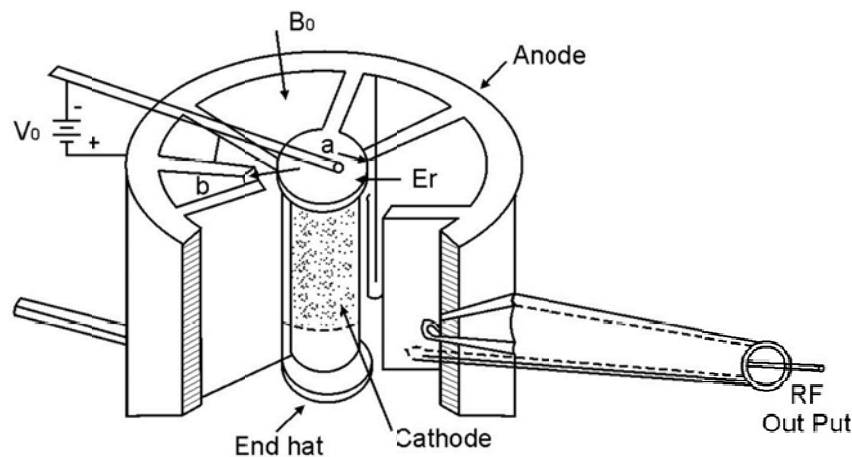


Fig. 5.9 : Schematic diagram of a Cylindrical Magnetron

Electron Motion (or) Hull Cut Off Voltage

- The equations of motion for electron in a cylindrical magnetron can be written as

$$\frac{d^2r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 = \frac{e}{m} E_r - \frac{e}{m} r B_0 \frac{d\theta}{dt} \quad \text{----- (1)}$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\theta}{dt} \right) = \frac{e}{m} B_0 \frac{dr}{dt} \quad \text{----- (2)}$$

Where $\frac{e}{m} \rightarrow$ charge to mass ratio of electron = 1.759×10^{11} C/kg.

@ $B_0 = B_z$ is assumed in the positive z direction

$$\begin{aligned} \text{Equation (2)} \Rightarrow \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) - \frac{e}{m} B_z \frac{r dr}{dt} \\ = \frac{1}{2} \omega_c \frac{d(r^2)}{dt} \end{aligned} \quad \text{----- (3)}$$

Where $\omega_c \rightarrow \frac{e}{m} B_z$ is cyclotron angular frequency.

Integrating equation (3), we get

$$r^2 \frac{d\phi}{dt} = \frac{1}{2} \omega_c r^2 + \text{constant} \quad \text{---- (4)}$$

At $r = a$ is the radius of the cathode cylinder & $\frac{d\phi}{dt} = 0$

$$0 = \frac{1}{2} \omega_c r^2 + \text{constant}$$

$$\text{@ constant} = -\frac{1}{2} \omega_c a^2 \quad \text{---- (5)}$$

Substitute equation (5) in (4)

$$\begin{aligned} r^2 \frac{d\phi}{dt} &= \frac{1}{2} \omega_c r^2 - \frac{1}{2} \omega_c a^2 \\ \frac{d\phi}{dt} &= \frac{m_e}{2} \left[1 - \frac{a^2}{r^2} \right] \end{aligned}$$

\rightarrow The angular velocity of the electrons is

$$\frac{d\phi}{dt} = \frac{m_e}{2} \left[1 - \frac{a^2}{r^2} \right] \quad \text{----- (6)}$$

The electron move in direction perpendicular to the mag field the kinetic energy of the electron is given by

$$\begin{aligned} \frac{1}{2} m v^2 &= eV \\ v^2 &= \frac{2 e V}{m} \end{aligned}$$

The electron velocity has r and ϕ components

$$v_r^2 + v_\phi^2 = \frac{2 e V}{m}$$

$$\left(\begin{aligned} \text{@} \frac{dr^n}{dt} &= n r^{n-1} \frac{dr}{dt} \\ \frac{dr^2}{dt} &= 2r \frac{dr}{dt} \\ \frac{r dr}{dt} &= \frac{1}{2} \frac{dr^2}{dt} \end{aligned} \right)$$

$$\left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 = \frac{2eV}{m} \quad \text{----- (7)}$$

At $r = b$, radius from the center of cathode to the edge of the anode $V = V_0$ and $\frac{dr}{dt} = 0$ for the electrons just graze the anode equation (6) & equation (7) becomes

$$\frac{d\phi}{dt} = \frac{m_e}{2} \left[1 - \frac{a^2}{b^2} \right] \quad \text{----- (8)}$$

$$\left(r \frac{d\phi}{dt}\right)^2 = \frac{2eV_0}{m} \quad \text{----- (9)}$$

Substitute equation (8) in equation (9)

$$\begin{aligned} \left(\frac{d\phi}{dt}\right)^2 b^2 &= \frac{2eV_0}{m} \\ \frac{2eV_0}{m} &= b^2 \left[\frac{\omega_c}{2} \left(1 - \frac{a^2}{b^2}\right) \right]^2 \\ \frac{2eV_0}{m} &= \frac{h^2 m_e^2}{4} \left(1 - \frac{a^2}{b^2}\right)^2 \\ \frac{2eV_0}{m} &= \frac{b^2 e^2 B_{0c}^2}{4 m^2} \left[1 - \frac{a^2}{b^2} \right] \\ \frac{V_0 e}{m} &= \frac{b^2 e B_{0c}^2}{4 m^2} \left(1 - \frac{a^2}{b^2}\right)^2 \end{aligned}$$

$B_{0c} \rightarrow$ cut off magnetic flux density

$$\begin{aligned} 8 V_0 &= \frac{b^2 e}{m} B_{0c}^2 \left(1 - \frac{a^2}{b^2}\right)^2 \\ B_{0c} &= \frac{8 V_0 \frac{m}{e}}{b^2 \left(1 - \frac{a^2}{b^2}\right)^2} \\ B_{0c} &= \frac{(8 V_0 \frac{m}{e})^{1/2}}{b \left(1 - \frac{a^2}{b^2}\right)} \quad \text{----- (10)} \end{aligned}$$

Hull cut-off magnetic equation

\rightarrow The electron will just graze the anode and return toward the cathode depends on relative magnitudes of V_0 and B_0

$$B_{0c} = \frac{(8 V_0 \frac{m}{e})^{1/2}}{b \left(1 - \frac{a^2}{b^2}\right)} \quad \text{----- (11)}$$

This equation is called as Hull cut off magnetic equation.

- The magnetic field required to return electron back to cathode just grazing of the anode is called as cut-off magnetic field (or) cut-off magnetic flux density.
- If $B_0 > B_{0C}$ for a given V_0 , the electron will not reach the anode.

Hull cut-off voltage equation

The cut-off voltage

$$V_{0C} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2 \quad \text{----- (12)}$$

This equation is called hull cut off voltage equation

If $V_0 < V_{0C}$, the given B_0 , the electron will not reach the anode.

Cyclotron Angular Frequency

The magnetic field is normal to the motion of electrons that travel in a cycloidal path the outward centrifugal force is equal to the pulling force.

$$\frac{m v^2}{R} = e v B \quad \text{----- (13)} \quad \text{Where } R \rightarrow \text{Radius of cycloidal path}$$

$V \rightarrow$ Tangential velocity of electron

The cyclotron angular frequency of the circular motion of the electron

$$\omega_c = \frac{V}{R} = \frac{eB}{m} \quad \text{----- (14)}$$

The period of one complete revolution

$$T = \frac{2\pi}{\omega_c} = \frac{2\pi m}{eB} \quad \text{----- (15)}$$

Resonant modes in a magnetron

- For N resonant coupled cavities of the anode there exist N resonant frequencies or modes.

- If there are N reentrant cavities in the anode structure, the phase shift between two adjacent cavities can be expressed as $\phi_n = \frac{2\pi n}{N}$ ----- (16)

where mode of oscillation $n = 0, \pm 1, \pm 2, \dots, \pm N/2$

π mode of operation of an 8 cavity magnetron

→ Magnetron oscillators are operated in π mode where $n = \frac{N}{2}$

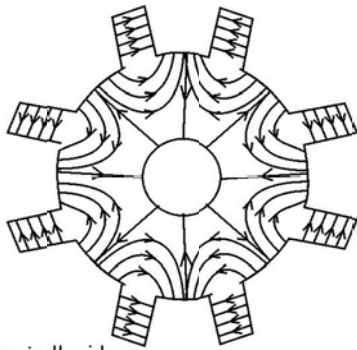
$$\phi_n = \pi \quad \text{----- (17)}$$

→ The successive cavities in anode have opposite phase excitation is maximum in the cavities.

→ If L is mean separation between cavities the phase constant of the fundamental mode field

$$\beta_0 = \frac{2\pi n}{NL} \quad \text{----- (18)}$$

The traveling field of the fundamental mode travels around the structure with angular velocity



$$\frac{d\phi}{dt} = \omega_c \quad \text{----- (19)}$$

→ When the cyclotron frequency is equal to the angular frequency the interaction between the field and electron occurs & energy is transferred.

$$\omega_c = \beta_0 \frac{d\phi}{dt} \quad \text{----- (20)}$$

Output power and Efficiency

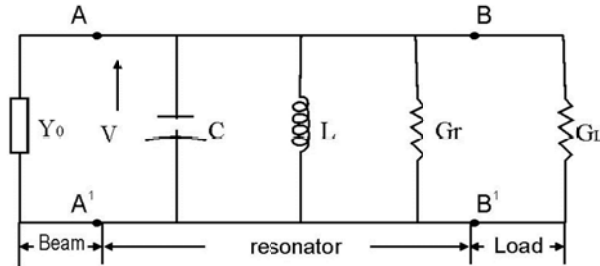


Fig. 5.10 : Equivalent circuit for one resonator of a magnetron

The efficiency and power output of a magnetron depend of the resonant structure and dc power supply.

$Y_e \rightarrow$ Electronic admittance

$C \rightarrow$ Capacitance of vane tips

$L \rightarrow$ Inductance

$G_r \rightarrow$ Conductance

$G_L \rightarrow$ load conductor of resonator

$V \rightarrow$ RF voltage across the vane tips

The unloaded quality factor of the resonator

$$Q_{un} = \frac{m_0 \epsilon}{G_r} \bigg| \text{ at } \omega_0 \quad \text{----- (1)}$$

where angular resonance frequency ($\omega_0 = 2 \pi f_0$)

The unloaded Q is a measure of the quality of the resonant circuit.

\rightarrow The external quality factor of load circuit.

$$Q_{ex} = \frac{m_0 \epsilon}{G_L} \bigg| \text{ at } \omega_0 \quad \text{----- (2)}$$

External Q is a measure of degree to which the resonant circuit is coupled to the external circuiting.

The loaded quality Q_L of the resonant circuit.

$$\frac{1}{Q_L} = \frac{1}{Q_{ex}} + \frac{1}{Q_{un}}$$
$$Q_L = \frac{m_0 \epsilon}{G_r + G_L} \bigg| \text{ at } \omega_0 \quad \text{----- (3)}$$

Circuit efficiency

$$\eta_c = \frac{G_L}{G_L + G_r} \cdot \frac{1}{1 + \frac{G_r}{G_L}} = \frac{1}{1 + \frac{Q_{ex}}{Q_{un}}} \quad \text{----- (4)}$$

The maximum circuit efficiency is obtained when the magnetron is heavily loaded for $G_L \gg G_r$.

Electronic Efficiency

$$\eta_e = \frac{P_{gen}}{P_{dc}} = \frac{V_0 I_0 - P_{lost}}{V_0 I_0} \quad \text{----- (5)}$$

where $P_{gen} \rightarrow$ RF power induced into anode current

$P_{dc} \rightarrow V_0 I_0$ power from the dc power supply

$V_0 \rightarrow$ Anode voltage

$I_0 \rightarrow$ Anode current

$P_{lost} \rightarrow$ Power lost in the anode current.

The RF power generated by the electrons

$$\begin{aligned} P_{gen} &= V_0 I_0 - P_{lost} \\ &= V_0 I_0 - I_0 \frac{m}{2e} \frac{m^2}{b^2} + \frac{E^2_{max}}{R_Z} \\ P_{gen} &= \frac{1}{2} N |V|^2 \frac{m \epsilon}{Q_l} \end{aligned} \quad \text{----- (6)}$$

$N \rightarrow$ Total number of resonator.

$V \rightarrow$ RF Voltage across resonator gap

$E_{max} = \frac{M_1 |V|}{L}$ is the maximum electric field.

$M_1 = \frac{\sin(\frac{\beta}{2})}{(\frac{\beta}{2})} = 1$ for small δ is the gap factor for the n mode operator.

$\beta \rightarrow$ Phase constant,

$\beta_z \rightarrow$ magnetic flux density

$L \rightarrow$ Center – to – center spacing of the vane tips.

The power generated by the electronic may be simplified to

$$P_{gen} = \frac{N L^2 m_0 \epsilon}{2 M^2 Q_l} F^2_{max} \quad \text{----- (7)}$$

The electronic efficiency may be rewritten as

$$\eta_e = \frac{P_{gen}}{V_0 I_0} = \frac{1 - \frac{m m \theta}{2 e V_0 b^2}}{1 + \frac{I_0 m M_1^2 Q_l}{B_z e N L^2 m_0 \epsilon}} \quad \text{----- (8)}$$

5.7 Microwave measurements

5.7.1 Wavelength & Frequency Measurement

→ The frequency can be computed from measured guide wavelength in a voltage standing wave pattern along a short circuited line by using a slotted line.

Slotted line Method

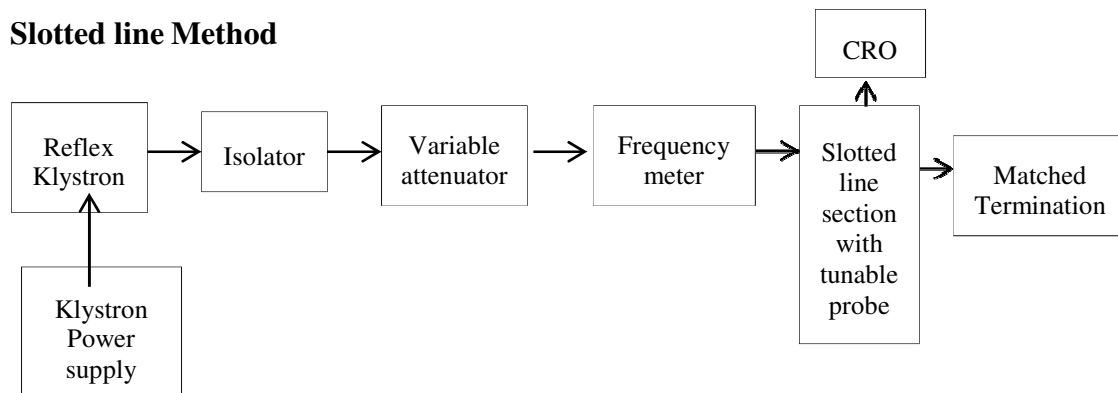


Fig. 5.11 : Wavelength Measurement

Wavelength Measurements

For dominant TE_{10} mode in rectangular waveguide λ_0 , λ_g , λ_c are related as below.

$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} \quad \text{---- (1) where } \lambda_0 \rightarrow \text{free space wavelength}$$

$\lambda_g \rightarrow$ guide wavelength

$\lambda_c \rightarrow$ Cut off wavelength

Guide Wavelength

By moving the probe between two successive minima a distance equal to $\frac{\lambda_g}{2}$ is found to determine the λ_g .

$$\lambda_g = 2 d_{\min} = 2 (d_1 - d_2) \quad \text{----- (2)}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} \quad \text{----- (3)}$$

For TE₁₀ mode:

Cut off wavelength $\lambda_c = 2a$

Free space wavelength $\lambda_0 = \frac{c}{f}$

Where, a – Broad dimension of waveguide,

C → velocity of light and f- frequency.

Frequency measurement

Using the equations (1) , (2) and (3) frequency of an unknown microwave

signal is $f = \frac{c}{\lambda} \sqrt{\left(\frac{1}{h_g}\right)^2 + \left(\frac{1}{h_e}\right)^2}$

5.7.2 Insertion loss and Attenuation Measurements

→ When a device (or) network is inserted in the transmission line, from the input power signal P_i , a part of input power P_r reflected from the input terminal & remaining part $P_i - P_r$ which actually enters the network is attenuated due to the non-zero loss of the network.

Insertion Loss

→ It is measure the loss of energy in transmission through a line (or) device compared to direct delivery of energy without the line (or) device.

→ Let P_i be the power received by the load when connected directly to source without the line (or) device, and P_0 the power received by the load when the line (or) the device is inserted between the source and the load, while the input power is held constant.

$$\begin{aligned} \text{Insertion loss (dB)} &= 10 \log \frac{\text{Power received by load without line (or) device}}{\text{Power received by load with line (or) device}} \\ &= 10 \log \frac{P_i}{P_0} \quad \text{----- (1)} \end{aligned}$$

The insertion loss is contributed by

- Mismatch loss at the input.
- Attenuation loss through the device.
- Mismatch loss at the output.

$$\frac{P_i}{P_0} = \frac{P_i - P_r}{P_0} + \frac{P_i}{P_i - P_r}$$

$$10 \log \frac{P_i}{P_0} = 10 \log \left[\frac{P_i - P_r}{P_0} \right] + 10 \log \left[\frac{P_i}{P_i - P_r} \right] \quad \text{----- (2)}$$

Insertion loss (dB) = Attenuation loss (dB) + Reflection loss (dB)

Attenuation loss

→ It is a measure of the power loss due to signal absorption in the device.

$$\text{Attenuation loss (dB)} = 10 \log \frac{\text{Input Energy} - \text{Reflected energy at the input}}{\text{Transmitted energy to the load}}$$

$$= 10 \log \frac{P_i - P_r}{P_0}$$

Reflection loss

→ It is a measure of power loss during transmission due to the reflection of the signal as a result of impedance mismatch.

$$\text{Reflection loss (dB)} = 10 \log \frac{\text{Input Energy}}{\text{Input Energy} - \text{Reflected Energy}}$$

$$= 10 \log \frac{P_i}{P_i - P_r} = 10 \log \left(1 - \frac{P_r}{P_i} \right)$$

$$= 10 \log \frac{1}{1 - |\Gamma|^2}$$

$$\text{Reflection loss (dB)} = 10 \log \frac{(S+1)^2}{4S} ; S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Return loss (dB)

→ It is a measure of the power reflected by a line (or) network (or) device.

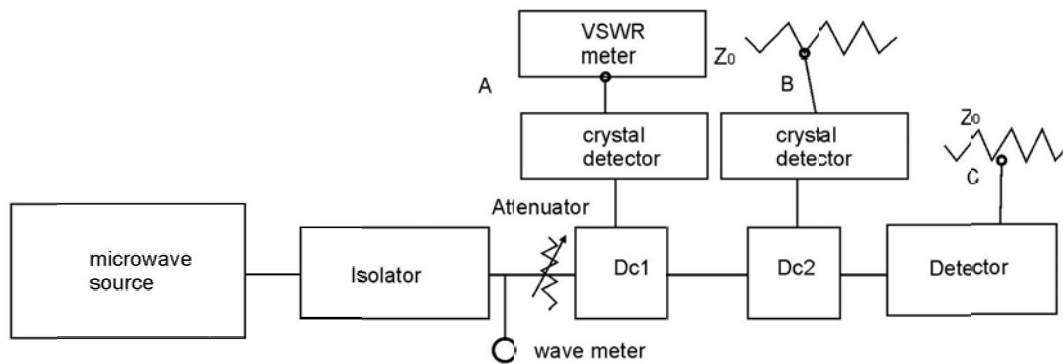


Fig.5.12 : Insertion loss & attenuation measurements

$$\text{Return loss (dB)} = 10 \log \frac{\text{Input Energy to the device}}{\text{Reflected Energy at the input of devices}}$$

$$\text{Return loss (dB)} = -20 \log |\Gamma|$$

5.7.3 Power measurements

→ It is defined as the quantity of energy dissipated or stored per unit time.

Categories

- i) Measurement of low power (less than 10 MW)
- ii) Measurement of medium power (from 10 MW to 10 W) &
- iii) Measurement of high power (> 10W)

Average power

$$P \text{ (dBm)} = 10 \log \frac{P \text{ (mw)}}{1 \text{ mw}}$$

→ The most convenient unit of power at microwave is dBm.

Power sensor

- The microwave power consists of power sensor, which converts the microwave power into heat energy.
- The sensors used for power measurements are the schottky barrier diode, bolometer and thermo couple sensor.

High frequency power

- It is measured by calorimeters, in which the temperature rise of the load provides a direct measure of power absorbed by the load.

- At high power, capability decreases due to over loading.
- It provides good impedance match, low loss, good isolation from thermal & physical shock and good shielding against energy leakage.

a) Schottky Barrier Diode Sensor (SBD)

- A zero – biased schottky barrier diode is used as a square law detector whose output is directly proportional to the input power.
- The diode resistance is a strong function of temperature, the circuit is designed such that the input matching is not affected by diode resistance.
- The SBD diode detectors can be used to measure power levels as low as 70 dBm.

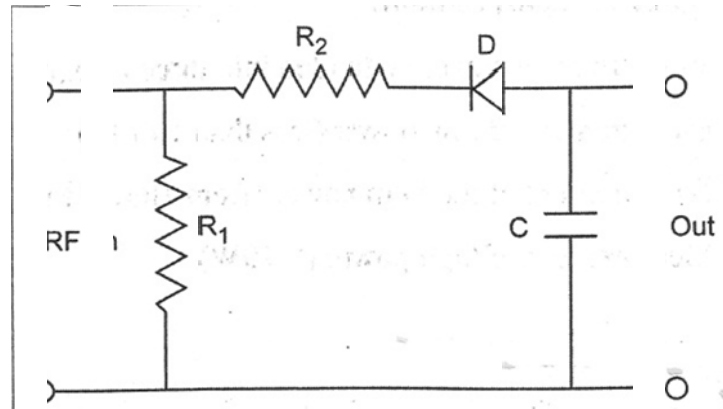


Fig. 5.13 : Schottky barrier diode sensor

b) Bolometer sensor

- A bolometer is a power sensor whose reactance changes with temperature as it absorbs microwave power.
- The two most common types of bolometer are , the barretter and the thermistor.

Barretter

- The barretter is a short thin metallic wire sensor which has a positive temperature coefficient of resistance . They are used only very low power

Thermistor

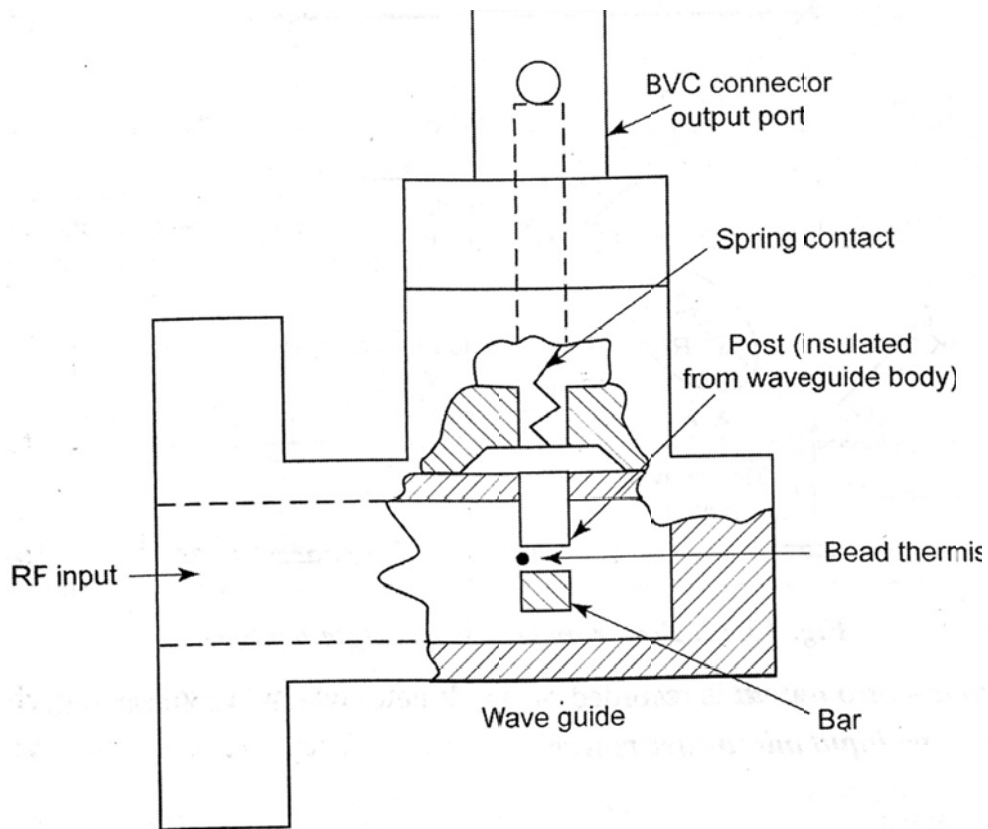
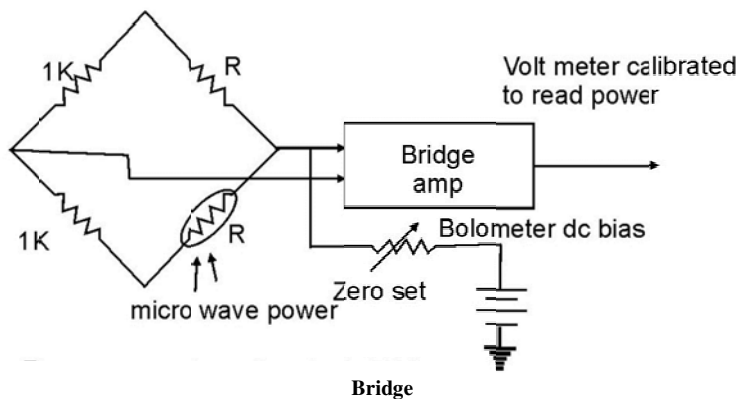


Fig. 5.14 : Thermistor mount

- It is a semiconductor sensor which has a negative temperature coefficient of resistance and can be easily mounted in microwave lines due to its smaller and more compact size.
- Medium and high power are measured with a low – power thermistor sensor, after precisely attenuating the signal.
- The thermistor is basically more sensitive than the barretter but it is also much more sensitive to the changes in the ambient temperature.
- The sensitivity of a thermistor is limited to 20dBm.
- The thermistor mount provides good impedance match, low loss , good isolation from thermal , physical shock and good shielding against energy leakage.

c) Power meter

i) Single bridge power meter



→ The microwave power applied to the arm will change the bolometer's resistance causing on an unbalance in the bridge from its initial balance condition under zero incident power.

→ Heating effect causes the bolometer's resistance to decrease & unbalances the bridge in proportion to the power applied.

→ The non-zero output is recorded on a voltmeter which is calibrated to read the level of the input microwave power.

ii) Double bridge power meter

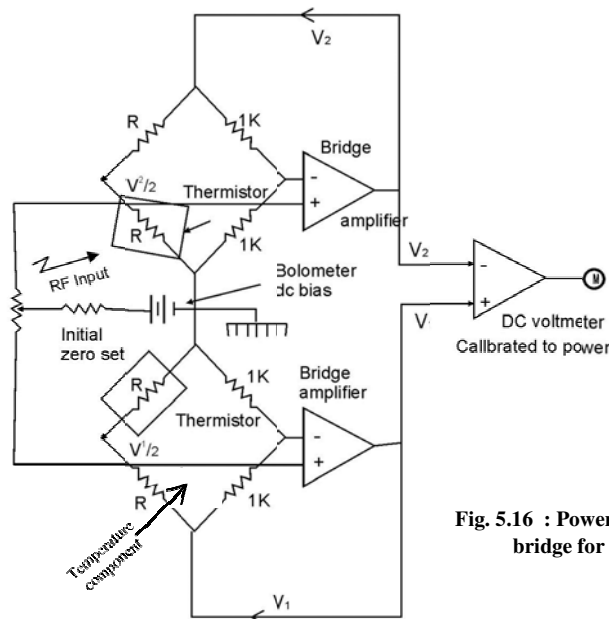


Fig. 5.16 : Power meter using double bridge for compensation

- The upper bridge circuit measures the microwave power and lower bridge circuit compensates the effect of ambient temperature variation ($V_1 = V_2$).
- Initial zero setting of the bridge is done by adjusting $V_2 = V_1 = V_0$. With an microwave input signal applied, where R is the resistance of thermistor at balance.
- Without & with microwave power the dc voltages across the sensor at balance $\frac{V_1}{2}$ and $\frac{V_2}{2}$
- The average input power P_{av} is equal to change in the power.

$$P_{av} = \frac{V_1^2}{4R} - \frac{V_2^2}{4R} = \frac{(V_1 - V_2)(V_1 + V_2)}{4R}$$

For any change in temperature if the voltage changes by Δ_v , the change in RF power is

$$\begin{aligned} P_{av} + \Delta P &= \frac{(V_1 + \Delta_v)^2}{4R} - \frac{(V_2 + \Delta_v)^2}{4R} \\ &= \frac{(V_1^2 + (\Delta_v)^2 + 2\Delta_v V_1)}{4R} - \frac{(V_2^2 + (\Delta_v)^2 + 2\Delta_v V_2)}{4R} \\ &= \frac{(V_1^2 - V_2^2) + 2\Delta_v(V_1 - V_2)}{4R} \\ P_{av} + \Delta P &= \frac{(V_1 - V_2)(V_1 + V_2 + 2\Delta_v)}{4R} \end{aligned}$$

Since $V_1 + V_2 \gg \Delta V$ in practice $\Delta P = 0$.

d) Thermocouple sensor

- It is a junction of two dissimilar metals or semiconductor. It generates an emf when two ends are heated up differently by absorption of microwaves in a thin film tantalum – nitride resistive load deposited on a S_1 substrate which forms one electrode to the thermocouple.
- The emf is proportional to the incident microwave power to be measured.

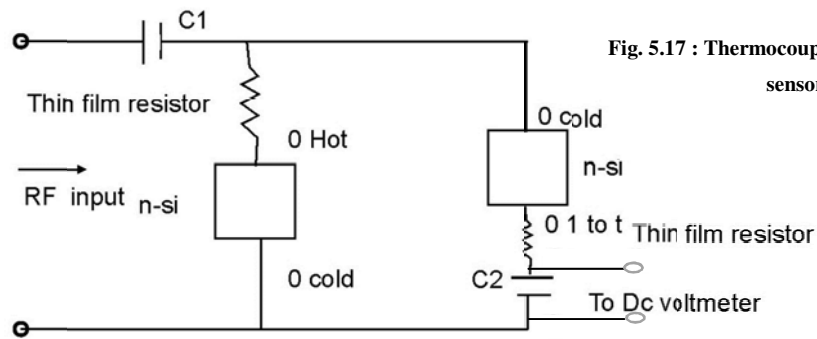


Fig. 5.17 : Thermocouple power sensor

→ Here capacitor C₂ is RF by-pass capacitor it is input coupling capacitor.

Peak Power

$$P_{\text{peak}} = \frac{P_{\text{av}} T}{\tau}$$

$T \rightarrow$ Time period
 $\tau \rightarrow$ Pulse width

e) High power measurements by the calorimetric method

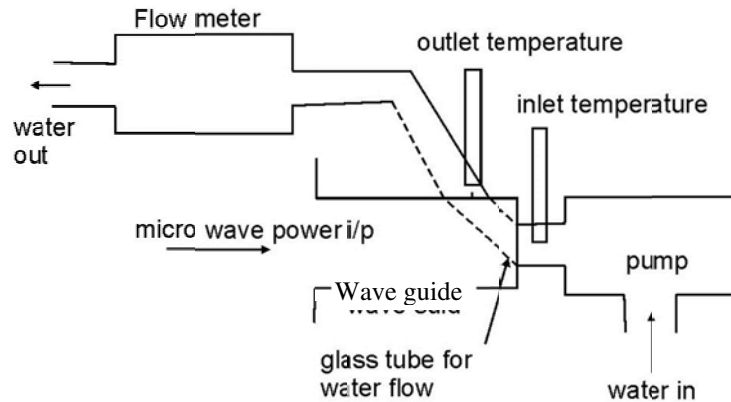


Fig. 5.18 : Microwave calorimeter

→ The microwave energy is converted into heat, absorbing this heat in a field and then measuring the temperature rise of the field.

Types

- i) **Direct heating method :** The rate of production of heat can be measured by observing one rise in the temperature.

- ii) **Indirect heating method** : Heating is transferred to another medium before measurement.

→ In both the methods static & circulating calorimeter are used.

Static calorimeters

→ It consist of 50 ohm coaxial cable which is filled by a dielectric load with a high hysteresis loss.

$$P = \frac{4.187 M C_p T}{t} \text{ Watts} \quad P \rightarrow \text{Average power}$$

$M \rightarrow$ Mass of thermometric medium, $t \rightarrow$ Time

$C_p \rightarrow$ Its specific heat in cal/gm. $T \rightarrow$ Temperature rise in C

Circulating calorimeters

→ The calorimeter field is constantly flowing through a water load. The heat introduced into the field makes exit temperature higher than input temperature.

$$P = 4.187 V \cdot d C_p T \text{ watts}$$

$V \rightarrow$ Rate of flow of calorimeter field in 'C

$d \rightarrow$ specific gravity of the field as in gm/oc.

5.7.4 VSWR Measurements

- VSWR & the magnitude of Voltage reflection coefficient Γ determine the degree of impedance matching and the measurement of load impedance by the slotted line method.

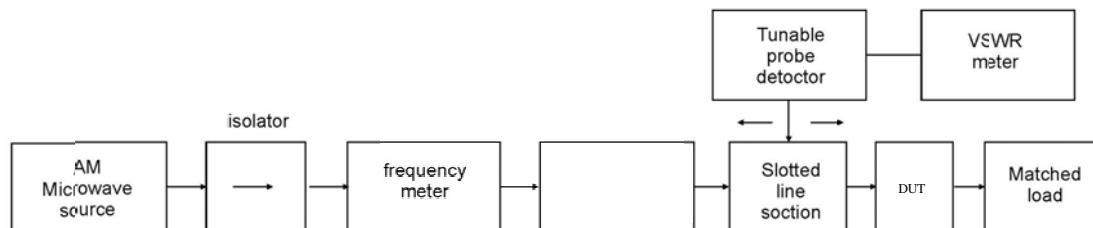


Fig. 5.19 : Basic experimental set-up for VSWR measurement

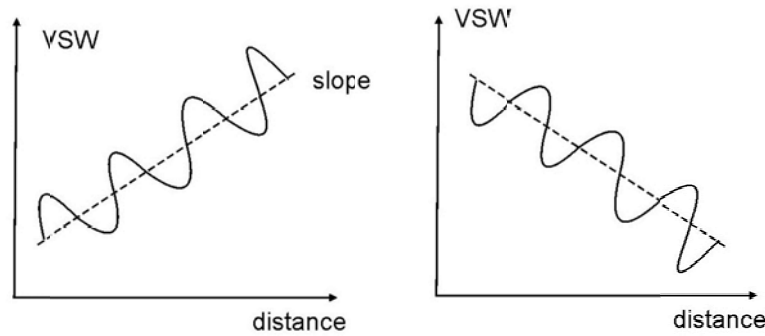


Fig. 5.20 : Slant patierial due to mechanical slope error

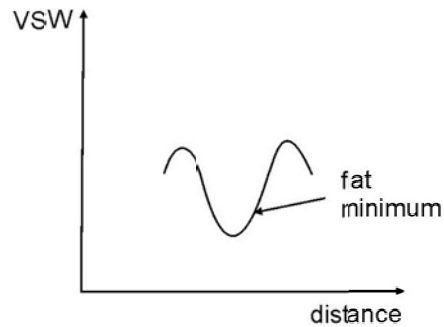


Fig. 5.21 : Flat minimum position due to imperfect square wave modulating S_{11}

- When a load $Z_L \neq Z_0$ is connected to a transmission line, standing waves are produced.
- By inserting a slotted line system in the line, standing waves can be traced by moving the carriage with a tunable probe detection along the line.
- VSWR can be measured by detecting V_{\max} & V_{\min} in the VSWR meter.

$$S = \frac{V_{\max}}{V_{\min}}$$

Low VSWR ($S < 20$)

Low values of VSWR can be measured **directly** from the VSWR meter.

- 1) Variable attenuation is adjusted to 10 dB.
1 KHZ modulation is adjusted for maximum reading on VSWR meter in a 30 dB scale.
- 2) The probe carriage is slid along the non-radiating slot from the load end until a peak reading is obtained in VSWR meter. The meter's gain control is adjusted to get the meter reading at 1.0 or 0 dB.

- 3) For VSWR between 3.2 and 10, a 10 dB lower range should be selected.
- 4) For VSWR between 10 and 40, a 20 dB range sensitivity increase is required.
- 5) For VSWR between 32 and 100, a 30 dB lower RANGE must be selected.

The possible sources of error in this measurement are

- 1) V_{\max} & V_{\min} may not be measured in the square law region of the crystal detector.
- 2) Depth of penetration should be kept as small as possible otherwise values of VSWR measured would be lower than actual.
- 3) When $VSWR < 1.05$, the associated VSWR of connector produces significant error in VSWR measurement.
- 4) If the modulating 1 KHZ signal is not a perfect square wave, the microwaves will be frequency modulated & at each frequency there will be a different set of standing waves.
- 5) Any harmonics & spurious signals from the source may be tuned by the probe to cause measurement error.
- 6) A residual VSWR of slotted line arises due to mismatch impedance between slotted line and the main line.

$\rho_L \rightarrow$ Actual load reflection co-efficient.

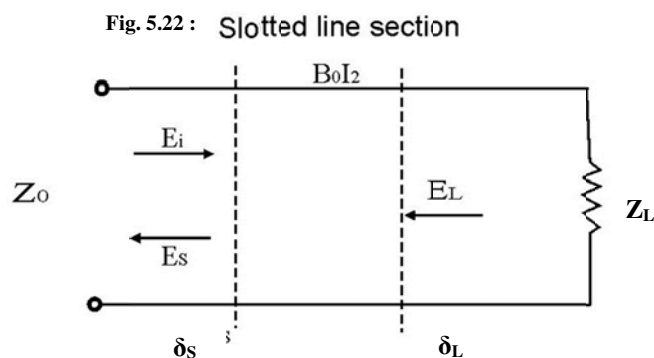
$\rho_S \rightarrow$ Slotted line reflection co-efficient on mainline.

$E_i \rightarrow$ Incident electric field at any point on the mainline.

$E_L \rightarrow$ Reflected electric field from the load.

$E_S \rightarrow$ Reflected electric field on the main line because of slotted line.

Then the total reflected field at a point = $|E_S + E_L|$



$$S_{\max} = \frac{E_i + (E_S + E_L)}{E_i - (E_S + E_L)}$$

$$S_{\min} = \frac{E_i + (E_S - E_L)}{E_i - (E_S - E_L)}$$

$$\rho_{\max} = \frac{S_{\max} - 1}{S_{\max} + 1} = |\rho_L| + |\rho_S|$$

$$\delta_{\min} = \frac{S_{\min} + 1}{S_{\min} - 1} = |\rho_L| - |\rho_S|$$

Then, the residual VSWR

$$S_S = \frac{1 + |\rho_S|}{1 - |\rho_S|}$$

High VSWR ($S > 20$)

- For high VSWR the difference of power a voltage maximum and voltage minimum is large.
- So it would be difficult to remain on the detector's square-large region at maximum positions when diode current may exceed $20 \mu A$.
- Here, VSWR meter calibrated on a square-law basis ($J = kV^2$) will be inaccurate.
- Hence, double minimum method is used where measurements are carried out at two positions around a Voltage minimum point.

Let the ratio of line voltage neat a minimum and the voltage at the minimum be

$$\gamma_n = \frac{|V(x)|}{|V_{\min}|}$$

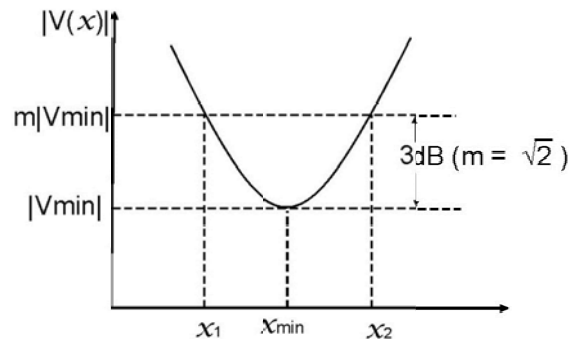


Fig. 5.23 : Double minima method

For $\Gamma = pe^{i\theta}$

$$|V(x)| = |V_{inc}| |1 + p e^{j(\phi - 2\beta x)}|$$

$$|V(x)| = |V_{inc}| [1 + 2p \cos(\phi - 2\beta x) + p^2]^{1/2}$$

The voltage minimum

$$|V_{min}| = |V_{inc}| (1 - P) \text{ at } x = x_{min}$$

If x_1 & x_2 are two points around x_{min}

where $|V(x_1)| = |V(x_2)| = m|V_{min}|$.

$$M = \frac{|V(s_1)|}{|V_{min}|} = \frac{[1 + 2p \cos(\phi - 2\beta s_1) + p^2]^{1/2}}{1 - p}$$

Substitute $p = \frac{(S-1)}{(S+1)}$

$$S = \frac{[m^2 - \cos^2(\frac{2u(x_1 - x_{min})}{\lambda_g})]^{1/2}}{\sin[\frac{2u(x_1 - x_{min})}{\lambda_g}]}$$

$$\text{where } \beta = \frac{2u}{\lambda_g}$$

$\lambda_g \rightarrow$ Guide wavelength

$$\Delta x = 2(x_1 - x_{min})$$

$$S = \frac{[m^2 - \cos^2(\frac{n\Delta x}{\lambda_g})]^{1/2}}{\sin^2(\frac{n\Delta x}{\lambda_g})}$$

$$\lambda_g = \left[\frac{m^2 - 1}{\sin^2(\frac{n\Delta x}{\lambda_g})} + 1 \right]^{1/2}$$

If $m = \sqrt{2}$

$$\begin{aligned} S &= \frac{[2 - 1}{\sin^2(\frac{n\Delta x}{\lambda_g})} + 1]^{1/2}}{\sin^2(\frac{n\Delta x}{\lambda_g})} \\ &= \frac{1 + \sec^2(\frac{n\Delta x}{\lambda_g})}{\sin^2(\frac{n\Delta x}{\lambda_g})} \end{aligned}$$

If $\pi\Delta x \ll \lambda_g$

$$\begin{aligned}
 S &\approx \cos \left(\frac{n\Delta s}{\lambda_g} \right) \\
 &= \frac{1}{\sin \left(\frac{n\Delta x}{\lambda_g} \right)} \\
 &\cong \frac{\lambda_g}{n\Delta s} \quad \text{Where } \Delta x = x_2 - x_1
 \end{aligned}$$

Thus high VSWR can be measured by observing the distance between two successive minima.

The method follows the steps given below.

- Probe is moved to a voltage minimum & the probe depth & gain control can be adjusted to read 3 dB.
- Probe is moved slightly on either side of the minimum to read 0 dB in the meter.
- By moving the probe between two successive minima a distance equal to is found to determine λ_g .
- High VSWR is calculated from

$$S = \frac{\lambda_g}{n(s_1 - s_2)}$$

Impedance measurement

Since impedance is a complex quantity, both amplitude & phase of the test signals are required to be measured. The following techniques are commonly employed.

a) Slotted line Method

The complex impedance Z_L of a load can be measured by measuring the phase \angle angle ϕ_L of the complex reflection coefficient Γ_L from the distance of first voltage standing wave minimum d_{\min} & the magnitude of the same from VSWR.

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

$$\Gamma_L = \delta_L e^{j\phi_L}$$

$$S = (1 + \rho_L)(1 - \rho_L)$$

$$\phi_L = 2 \beta d_{\min} - \pi$$

$$B = \frac{2n}{\beta_g}$$

$\lambda_g = 2 \times \text{distance between two successive minima}$

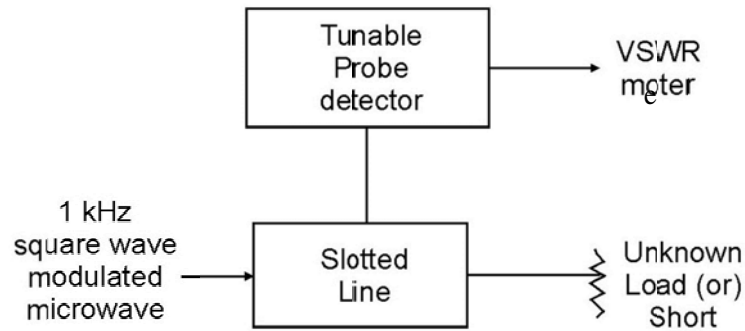


Fig.5. 24 : Determination of load impedance using slotted line

The steps for measurement are summarized below:

- 1) Measure the load VSWR to find ρ_L .
- 2) Measure the distance it, between two successive minima to find $\lambda_g = 2d$ and $\beta = \frac{2\pi}{\lambda_g}$.
- 3) Measure the distance d_{\min} in the following manner.
 - An equivalent load reference plane on the slotted line is established by means of a short circuit at the load reference plane.
 - The load reference plane can shifted to a convenient minimum position.
 - The d_{\min} can then be measured by observing the minimum from this shifted reference.
- 4) Phase angle ϕ_L is calculated
- 5) The unknown impedance Z_L .

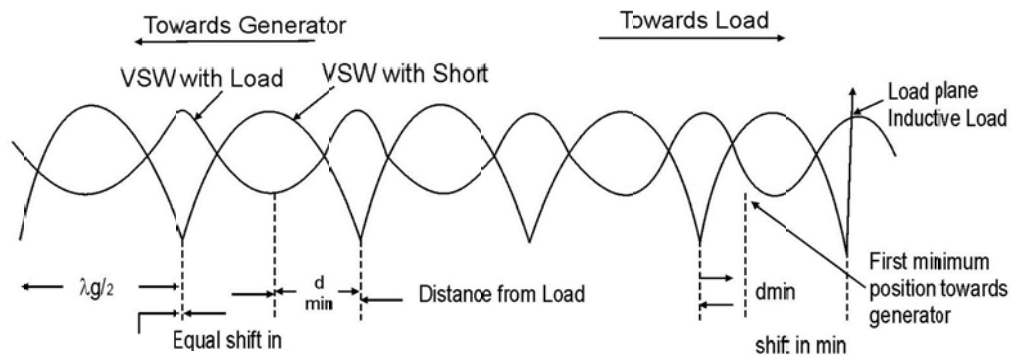


Fig. 5.25 : Determination of d_{min}

b) Impedance measurement by Reflectometer

The reflectometer arranged which is shown previously cannot have ideal conditions of infinite directivity, constant impedance detectors & perfect impedance matching.

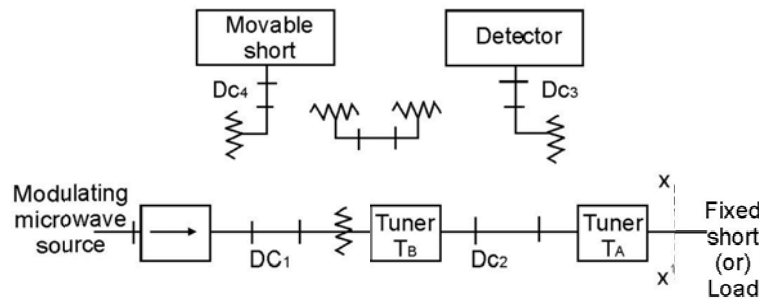


Fig. 5.26 : Reflectometer with tuners for amplitude and phase measurement

Tuner T_A is adjusted to make $|b_3|/|b_4|$ constant while phase of Γ_L is varied.

$$\left| \frac{b_3}{b_4} \right| = \left| \frac{A}{D} \right| |\sqrt{\Gamma}| = k |\Gamma_L|$$

$$|\Gamma_L| = \frac{1}{k} \left| \frac{b_3}{b_4} \right|$$

H is determined by nothing $|b_3|/|b_4|$ using fixed short of known reflection coefficient – 1.

In order to measure the phase of the load reflection coefficient, four identical directional coupler reflectometer can be used. The procedure for phase measurements,

- 1) First a fixed short is placed at xx' plane at port 2. The movable short at port 4 & the attenuator are adjusted to obtain null in the detector output at port 3.
- 2) Fixed shorted at port 2 is replaced by test load & shift x of the movable short at port 4 is measured to obtain null in the detector output at port 3. The phase of Γ_L –

$$\phi_L = \frac{2 n \Delta s}{\beta_g}$$

c) Impedance measurement of Reactive Discontinuity

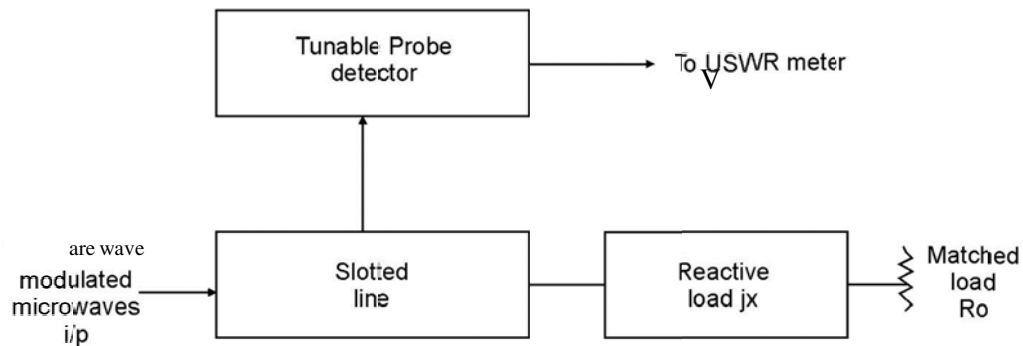


Fig. 5.27 : Experimental set up for the measurement of impedance of discontinuity

The impedance of a shunt reactive discontinuity can be measured using the slotted line method from the measurement of line VSWR & admin at follows.

Let jx be the reactance at load distant point $d = 0$.

The line is terminated by a matched load R_0 at $d = 0$.

∴ The total impedance

$$Z_L = \frac{R_0 jX}{R_0 + jX}$$

Normalized value

$$\begin{aligned} \frac{Z_L}{R_0} &= \frac{jX}{R_0 + jX} = \frac{-x^2}{R_0^2 + X^2} + \frac{jXR_0}{R_0^2 + X^2} \\ &= X + jg, \text{ say} \end{aligned}$$

$$x = \frac{X^2}{R_0^2 + X^2}, \quad y = j \frac{XR_0}{R_0^2 + X^2}$$

With x unknown

$$\frac{X}{R_0} = \frac{s}{y}$$

(or) $\frac{B}{G_0} = \frac{y}{s}$

where $b = \frac{1}{X}$, $G_0 = \frac{1}{R_0}$

Problems :

1. A two cavity klystron amplifiers has following parameters $V_0 = 1000$ V, $R_0 = 35k \Omega$, $I_0 = 20$ mA, $f = 3GHz$, $d = 1mm$, $L = 4$ cm, $R_{sh} = 30 k\Omega$
 - a) Find the input gap voltage to give maximum voltage V_2
 - b) Find the voltage gain neglecting the beam loading in the output cavity
 - c) Find the efficiency, neglecting the beam voltage.

Solution:

$$\begin{aligned} \text{(i) } V_0 &= 0.593 \times 10^6 \sqrt{f V_0} \\ &= 0.593 \sqrt{1000} \\ &= 18.752 \end{aligned}$$

$$V_0 = 1.88 \times 10^7 \text{ m/s}$$

$$\text{Gap transit angle } \theta_g = \frac{m_d}{V_0}$$

$$\omega = 2\pi f = 2\pi (3 \times 10^6)$$

$$\theta_g = 1 \text{ rad.}$$

Beam coupling coefficient

$$\beta_i = \beta_0 = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\left(\frac{\theta_g}{2}\right)}$$

$$\beta_i = \beta_0 = 0.959$$

The DC transit angle between the cavities are,

$$\theta_0 = \omega T_0 = \frac{mL}{V_0} = \frac{m \times 4}{1000}$$

$$\theta_0 = 40 \text{ rad.}$$

- (i) The maximum input voltage is given by,

$$\begin{aligned}
 V_{1 \max} &= \frac{2 V_0 X}{\beta_1 V_0} \\
 &= \frac{2 \times 1000 \times 1.841}{0.959 \times 40} \\
 V_{1 \max} &= 96V
 \end{aligned}$$

(ii) Voltage gain $A_V = \frac{\beta_0^2 R_0}{R_0} \frac{I_1(x)}{X} R_{sh}$

$$\begin{aligned}
 J_1(x) &= 0.582 \\
 x &= 1.841 \\
 A_V &= 9.97
 \end{aligned}$$

(iii) Efficiency $\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$

$$\begin{aligned}
 I_2 &= 2\beta_0 I_0 J_1(x) = 23.3 \text{ mA} \\
 V_2 &= \beta_0 J_2 R_{sh} = 670.34V \\
 \eta &= \frac{0.959 \times 23.3 \text{ mA} \times 670.34}{2 \times 20 \text{ mA} \times 1000} \\
 \eta &= 37.45\%
 \end{aligned}$$

2. A pulse cylindrical magnetron is operated at following parameters. Anode voltage = 25KV, Beam current 25 A, Magnetic density = 0.34 w b/m². Radius of cathode cylinder = 5 cm, Radius of anode cylinder = 10 cm.

Calculate (i) Angular frequency
(ii) cutoff voltage
(iii) cutoff magnetic flux density

Solution:

(i) Angular frequency $\omega_c = \frac{e B_0}{m}$

$$\omega_c = 0.5981 \times 10^{11} \text{ rad}$$

(ii) Cutoff voltage $= r \left(\frac{e B_0^2 h^2}{8 m} \right) \left(1 - \frac{a^2}{h^2} \right)^2$

$$\begin{aligned}
 a &= 5 \text{ cm ; } b = 10 \text{ cm} \\
 \text{cutoff voltage} &= 142.97 \text{ kV}
 \end{aligned}$$

(iii) Cutoff magnetic flux density $= \frac{(8 V_0 \frac{m}{e})^{1/2}}{b \left(1 - \frac{a^2}{b^2} \right)}$

$$\text{Cutoff magnetic flux density} = 142.2 \text{ m wb/m}^2$$

3. An x band magnetron has the following parameter. Anode voltage = 5.5 kV, Beam current $I_0 = 4.5\text{A}$, $f = 9\text{GHz}$, $G_r = 2 \times 10^{-4}\text{ mho}$, $G_L = 2.5 \times 10^5\text{ mho}$, $C = 2.5\text{ pf}$. Duty cycle = 0.02, $P_{\text{Loss}} = 18.5\text{ kW}$.

- Compute
- (i) The angular frequency
 - (ii) unloaded Quality factor
 - (iii) Loaded θ - factor
 - (iv) External θ - factor
 - (v) Circuit efficiency
 - (vi) Electronic efficiency

Solution:

$$\begin{aligned}
 \text{(i)} \quad \omega_0 &= 2\pi f_0 \\
 &= 56.55 \times 10^9 \text{ rad} \\
 \text{(ii)} \quad \theta_{\text{un}} &= \frac{m_0 C}{G_r} \\
 &= 707 \\
 \text{(iii)} \quad \theta_l &= \frac{m_0 C}{G_r + G_l} \\
 &= 628.3 \\
 \text{(iv)} \quad \theta_{\text{ex}} &= \frac{m_0 C}{G_l} \\
 &= 5655 \\
 \text{(v)} \quad \eta_e &= \frac{1}{1 + \frac{g_{\text{ex}}}{g_{\text{un}}}} \\
 &= 11.11\% \\
 \text{(vi)} \quad \eta_e &= \frac{P_{\text{gen}}}{P_{\text{dc}}} = \frac{V_0 I_0 - P_{\text{loss}}}{V_0 I_0} \\
 &= 25.25\%
 \end{aligned}$$

- 4) A Reflex Klystron has following parameters $V_0 = 500\text{ v}$; $L = 1\text{mn}$; $\text{RSG} = 10\text{ k}\Omega$, $f_r = 8\text{GHz}$, $\frac{e}{m} = 1.759 \times 10^{11}$ the tube is osculating at f_r at the peak of the $n=2$ (or) $1.75v$. Assume that the transit time through the gap and beam loading can be neglected. Find the
- (i) Value of repeller voltage
 - (ii) The direct current necessary to gain a microwave gap voltage of 200 V. What is the electronic efficiency under this condition.

Solution:

$$\begin{aligned} \text{a)} \quad \frac{V_0}{(V_r + V_0)^2} &= \frac{e}{m} \frac{(2\pi n - \frac{u}{2})^2}{8 m^2 C} \\ \frac{500}{(V_r + 500)^2} &= 1.759 \times 10^{11} \left(\frac{(2\pi n - \frac{\pi}{2})^2}{8 \times 2 \pi^2 \times 8 \times 10^9 \times 8 \times 10 \times 1 \times 10^{-3}} \right) \\ (V_r + 500)^2 &= \frac{500}{1.05 \times 10^{-3}} \Rightarrow V_r + 500 = 89.409 \\ V_r &= 189.92 \text{ V} \end{aligned}$$

(b) The direct current

$$\begin{aligned} I_0 &= \frac{V_2}{2 I_1 (s^F) R_{sh}} \\ V_2 &= I_2 \cdot R_{sh} = 2 I_0 J_1(x') R_{sh} = \frac{200}{2 \times 10 \times 10^3 \times (0.582)} \\ I_0 &= 17.18 \text{ mA} \\ \text{(c)} \quad \eta_e &= \frac{2 s J_1(s^F)}{2\pi n - \frac{u}{2}} = \frac{V_0 I_0 - I_{loss}}{V_0 I_0} = \frac{2 \times 1.84 \times 0.582}{2 \times 2 - \frac{u}{2}} \\ &= 19.48\% \end{aligned}$$

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