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Monograph on Microwave Passive Components

Christo Ananth,

Assistant Professor, Department of ECE, Francis Xavier Engineering College, Tirunelveli, India

Microwave Frequencies

→ Microwaves are Electro Magnetic waves (EM) with wavelengths ranging from 30 cm to 1 mm.

→ The corresponding frequency range is 1 GHz to 300 GHz. This means microwave frequencies are upto infrared and visible – light regions.

→ It has the three major bands at the highest end of RF spectrum.

i. Ultra High Frequency (UHF) – 0.3 to 3 GHz
ii. Super High Frequency (SHF) – 3 to 30 GHz
iii. Extra High Frequency (EHF) - 30 to 300 GHz

→ Most applications of microwave technology make use of frequencies in the 1 to 40 GHz range.

Advantages

→ Because of their high operating frequencies, microwave system can carry large quantities of information.

→ Higher frequencies mean short wave lengths, which require relatively small antenna's.

→ Fewer repeaters are necessary for amplification.

→ Minimal cross talk exists between voice channels.

→ Increased reliability and less maintenance.

→ Increased bandwidth availability.

Applications

→ Currently microwave frequency spectrum is used for telephone communications

→ It is used in microwave landing system and microwave-owens.

Significance of microwave frequency range

→ Wide bandwidth

→ Good radar resolution

$$\lambda = \frac{c}{f} \qquad \text{where } c = 3 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{V_p}{€_F} \quad \text{where } Vp = \frac{€}{\sqrt{\varepsilon r}}$$

→ No Interference

→ Speed of operation is high

3.2 Microwave Hybrid Circuits

→ A microwave circuit is formed when several microwave devices and components such as sources, attenuators, resonators, filters, amplifiers etc. are coupled together by transmission lines or waveguides for the desired transmission of microwave signal.

→ The point of interconnection of two (or) more devices is called as junction commonly used microwave junctions include such waveguides tees as the E plane, H plane magic plane Tee, hybrid ring (rat-race circuit), directional coupler and circulator.

→ A waveguide is a hollow metal tube designed to carry microwave energy from one place to another.

Scattering Parameters (S)

Scattering matrix

It is a square matrix which gives all the combination of power relationships between the various input and output port of a microwave junction.

[S] parameters

→ The elements of scattering matrix are called scattering co-efficients (or) scattering parameters.

→ Low frequency circuits can be described by two port networks and the parameters such as Z, Y, h, ABCD etc. Here the network parameters relate the total voltages and total currents.

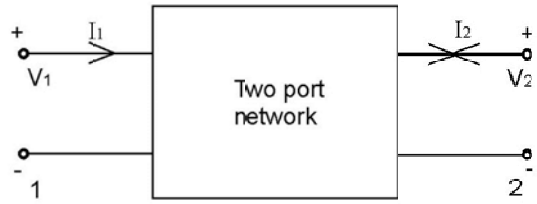


Fig. 3.1 : Two port network

h Parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Y Parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

Z Parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

ABCD Parameters

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

→ These parameters can be measured under short or open circuit condition for use in the analysis of the circuit.

→ The h, Y, Z & ABCD parameters are difficult at microwave frequencies due to following reasons.

- Equipment is not readily available to measure total voltage and total current at the ports of the network.
- Short circuit and open circuit are difficult to achieve over a wide range of frequencies.

Two port Network

→ Microwave circuits are analysed using scattering or S parameters which linearly relate the reflected waves amplitude with those of incident waves.

→ The incident & reflected amplitudes of microwave at any port are used to characterises a microwave circuit.

Input power at the n^{th} port

$P_{in} = ½ \; |a_n|^2$

a_n → represents the normalized incident wave amplitude at the n^{th} port.

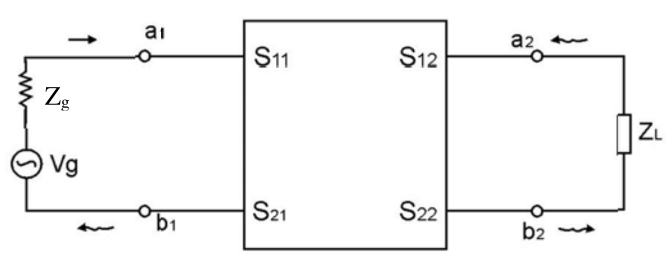


Fig. 3.2 : Two port network

Reflected power at the n^{th} port

$$P_{rn} = \tfrac{1}{2} \, | \, b_n |^2$$

$b_n \rightarrow$ represents the normalized reflected wave amplitude at the n^{th} port.

→ For a two-port network, the relation between incident and reflected waves are expressed interms of scattering parameters S_N is

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_2 + S_{22} a_2$$

where,

S_{11} , S_{22} → reflection co-efficient at port 1 & port 2

S_{12} → transmission co-efficient from port 2 to port 1

S_{21} → transmission co-efficient from port 1 to port 2

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2 = 0} \quad ; \quad S_{22} = \frac{b_2}{a_2}\Big|_{a_1 = 0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a_1 = 0} \quad ; \quad S_{21} = \frac{b_2}{a_1}\Big|_{a_2 = 0}$$

Multiport Networks

→ For a multiport network the S parameters can be expressed by

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots\dots & S_{in} \\ S_{21} & S_{22} & \dots\dots & S_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ S_{n1} & S_{n2} & \dots\dots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_n \end{bmatrix}$$

(Column matrix [b] corresponding to reflected waves (or) outputs) = (Scattering matrix [S] of order nxn) (Column matrix [a] corresponding to incident waves (or) Input)

It can be rewritten as [b] = [s] [a]

Where [a] & [b] are the incident waves and reflected waves.

Losses in Microwave devices

→ In a two port network of power is fed at port 1 is P_i, power reflected at the same port is P_r & the output power at port 2 is Po

i) Insertion loss (dB)

$$\text{Insertion Loss (dB)} = 10\log\frac{p_i}{p_o} = 10\log\frac{|a_1|^2}{|b_2|^2}$$

$$= 20\log\frac{1}{|S_{21}|} \qquad (S_{12} = S_{21})$$

$$= 20\log\frac{1}{|S_{12}|}$$

ii) Transmission (or) Attenuation loss

$$= 10\log\frac{\text{Input energy} - \text{Reflected energy at the input}}{\text{Transmitted energy to the load}}$$

$$= 10\log\left(\frac{p_i}{p_i - p_r}\right)\left(\frac{p_i - p_r}{p_o}\right)$$

$$= 10\log\frac{|a_1|^2 - |b_1|^2}{|b_2|^2} = 10\log\left(\frac{1 - \frac{|b_1|^2}{|a_1|^2}}{\frac{|b_2|^2}{|a_1|^2}}\right)$$

$$= 10\log\frac{1 - |S_{11}|^2}{|S_{12}|^2} \quad \left(\therefore \text{ where } S_{11} = \frac{|b_1|}{|a_1|},\ S_{12} = \frac{|b_2|}{|a_1|}\right)$$

iii) Reflection loss (dB)

$$= 10\log\frac{\text{Input energy}}{\text{Input energy} - \text{Reflected Energy}}$$

$$= 10\log\frac{p_i}{p_i - p_r}$$

$$10\log \frac{|a_1|^2}{|a_1|^2 - |b_1|^2} = 10\log \frac{1}{1 - \frac{|b_1|^2}{|a_1|^2}}$$

$$= 10\log \frac{1}{1 - |S_{11}|^2} \qquad \text{since } S_{11} = \frac{|b_1|}{|a_1|}$$

iv) Return loss (dB)

$$= 10\log \frac{\text{Input energy to the device}}{\text{Reflected energy at the input of device}}$$

$$= 10\log \frac{p_i}{p_r}$$

$$= 10\log \frac{|a_1|^2}{|b_1|^2}$$

$$= 20\log \frac{1}{|S_{11}|}$$

3.3 Properties of S Parameters

a) Zero diagonal elements for perfect matched network

For an ideal N port network with matched termination, $S_{ii} = 0$. Since there is no reflection from any port. Therefore under perfect matched conditions the diagonal elements of [S] are zero.

b) Symmetry of [S] for a reciprocal network

A reciprocal device has the same transmission characteristic in either direction of a pair of ports and it is characterized by a symmetric scattering matrix.

$S_{ij} = S_{ji}$ [$i \neq j$) ----- (1)

Which results $[S]_t = [S]$ ----- (2)

This condition can be proved in the following manner. For a reciprocal network with the assumed normalization the impedance matrix equation is

[V] = [Z] [I]

Substitute [V] = [a] + [b] & [I] = [a] – [b]

$[a] + [b] = [z] [[a] – [b]]$ (or)

$([z] + [U]) [b] = ([z] – [U]) [a]$

$[b] = [[z] + [U]]^{-1} ([z] – [U]) [a]$ ----- (3)

Where [U] is the unit matrix. The S matrix equation for the network is

$[b] = [s] [a]$ ----- (4)

Comparing equation (3) & (4)

$[S] = ([z] + [U])^{-1} ([z] – [U])$ ----- (5)

Let $R = [z] – [U]$ & $[z] + [U] = Q$ ------ (6)

For a reciprocal network, the Z matrix is symmetric. Hence

$[R] [Q] = [Q] [R]$ (or)

$[Q]^{-1} [R] [Q] [Q]^{-1} = [Q]^{-1} [Q] [R] [Q^{-1}]$ (or)

$S = [Q]^{-1} [R] = [R] [Q^{-1}]$ ----- (7)

Now the transpose of [S] is

$[S]_t = ([z] – [U])_t \, , \ ([z] + [U])_t^{-1}$ ----- (8)

Since the Z matrix is symmetrical

$([z] – [U])_t = [z] – [U]$ ----- (9)

$([z] + [U])_t = [z] + [U]$ ----- (10)

Therefore,

$[s]_t = ([z] – [U]) \ . \ ([z] + [U])^{-1}$

$[s]_t = [R] [Q]^{-1} = [s]$ ----- (11)

Thus it is proved that $[S]_t = [S]$ for a symmetrical junction.

(c) Unitary property for a lossless junction

For any lossless network the sum of the products of each term of any one row (or) of any column of the 'S' matrix multiplied by its complex conjugate is unity.

For a lossless 'n' port device, the total power leaving N ports must be equal to the total power input to these ports, so that,

$$\sum_{n=1}^{N} |b_n|^2 = \sum_{n=1}^{N} |a_n|^2$$

$$\sum_{n=1}^{N} \left|\sum_{i=1}^{n} S_{ni} \; a_i\right|^2 = \sum_{n=1}^{N} |a_n|^2 \qquad \text{----- (1)}$$

If only i^{th} port is excited and all other ports are matched terminated all a_n=0 except a_i,

So that
$$\sum_{n=1}^{N} |S_{ni} \, a_i|^2 = \sum_{n=1}^{N} |a_i|^2 \qquad \text{----- (2)}$$

$$\sum_{n=1}^{N} |S_{ni}|^2 = 1 = \sum_{n=1}^{N} S_{ni} \, S_{ni}^* \qquad \text{----- (3)}$$

Therefore, for a loseless junction

$$\sum_{n=1}^{N} S_{ni} \, S_{ni}^* = 1 \qquad \text{---- (4)}$$

If all $a_n = 0$ except a_i & a_k

$$\sum_{n=1}^{N} S_{nk} \, . \, S_{nk}^* = 0 \; ; \; i \neq k \qquad \text{---- (5)}$$

In matrix notation, these relations can be expressed as

$$[S^*] \, [S]_t = [U]$$

$$[S^*] = [S]_t^{-1} \qquad \text{----- (6)}$$

Here [U] is the Identity (or) unit matrix. A matrix [S] for lossless network which satisfies the above three conditions equation (4), (5) & (6) is called a unitary matrix.

d) Phase Shift property

→ Complex S-parameters of a network are defined with respect to the positions of port or reference planes. For a two port network with unprimed reference planes 1 & 2 as shown in figure the parameters have definite complex values.

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad \text{------ (1)}$$

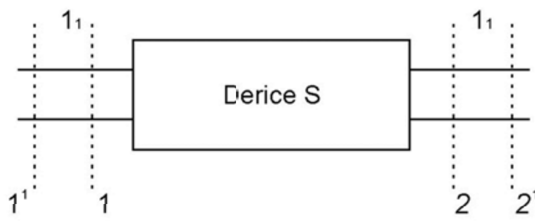


Fig. 3. : Phase shift property of S parameters

If the reference planes 1 & 2 are shifted outward to $1^{'}$ & $2^{'}$ by electrical phase shifts. $\phi_1 = \beta_1 I_1$, $\phi_2 = \beta_2 I_2$ respectively, then the new wave variables are $a_1 e^{j\phi 1}$, $b_1 e^{-j\phi 1}$, $a_2 e^{j\phi 2}$, $b_2 e^{-j\phi 2}$. The new 'S' matrix S^1 is given by

$$[S^{'}] = \begin{bmatrix} e^{-j\emptyset 1} & 0 \\ 0 & e^{-j\emptyset 2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\emptyset 1} & 0 \\ 0 & e^{-j\emptyset 2} \end{bmatrix} \quad \text{----- (2)}$$

This property is valid for any number of ports & is called the phase shift property applicable to a shift of reference planes.

Microwave Tee junctions

A Tee-junction is an intersection of three waveguides.

Tee – junctions:

In microwave circuits a waveguide or co-axial line with independent ports is commonly referred to as a Tee junction.

→ Waveguide tees are three port components. They are used to connect branch (or) section of the waveguide in series or parallel with the main waveguide transmission line for providing means of splitting and also of combining power in a waveguide system.

Types

1. E-plane Tee (series) 2. H – plane Tee (shunt)

3.4.1 E-plane Tee (Series Tee)

→ Side arm is in the direction of electric field. Axis of the side arm is parallel to the axis of main frame.

→ The wave enters a side arm and leaves the main arm will have equal magnitude and opposite phase. ∴ $S_{13} = -S_{23}$

→ Port 1 and port 2 are the collinear arms and port 3 is the E - arm.

→ The output port 3 is proportional to the difference between instantaneous powers entering from ports 1 & 2.

Difference arm

→ If two input waves are fed into port 1 and port 2 of the collinear arm the output wave at port 3 will be opposite in phase and subtractive. Sometimes the third port is called as difference arm.

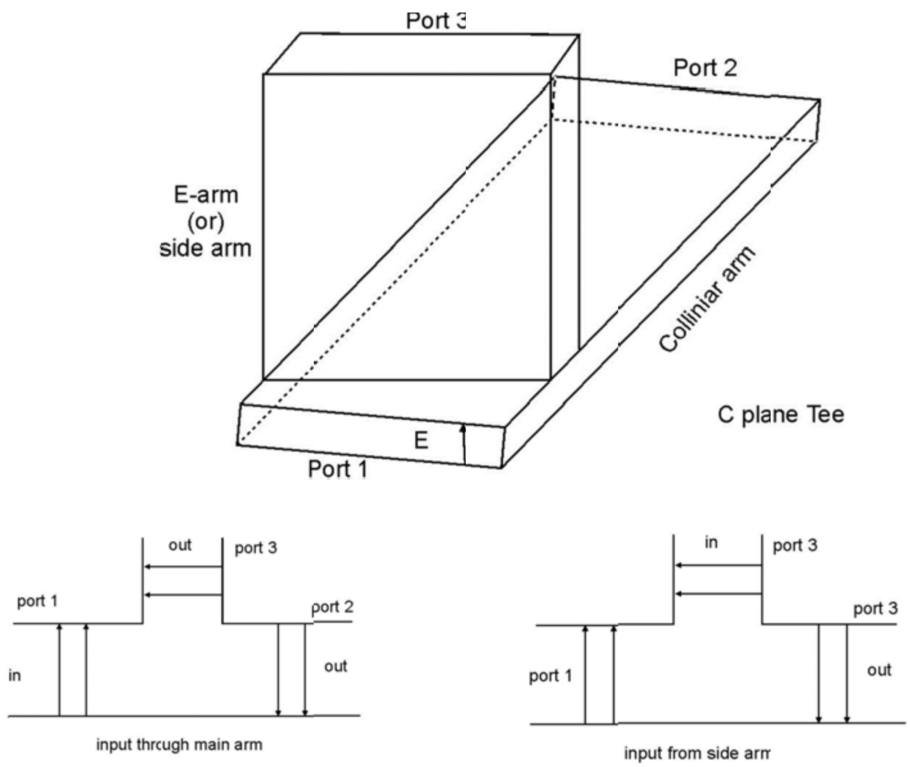


Fig. 3.4 : E-plane Tee

[S] is a 3x3 matrix since there are 3 ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{---- (1)}$$

The scattering co-efficient $S_{23} = -S_{13}$ ---- (2)

Since output at port 1 & port 2 are out of phase by 180^0 with an input port 3.

→ If port 3 is perfectly matched to the junction, $S_{33} = 0$ ---- (3)

→ From symmetry property $S_{ij} = S_{ji}$

$S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$ ---- (4)

Substitute equation (2), (3), (4) in (1)

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \quad \text{---- (5)}$$

From the unitary property $[S] . [S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Unitary Property

The unitary property of S matrix, the sum of products of each term of any one row (or) column multiplied by its complex conjugate is unity.

R_1C_1 : $S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} S_{13}^* = 1$

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ ---- (6)

R_2C_2 : $S_{12} S_{12}^* + S_{22} S_{22}^* + S_{13} S_{13}^* = 1$

$\Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ --- (7)

R_3C_3 : $S_{13} S_{13}^* + S_{13} S_{13}^* = 1$

$$\Rightarrow \quad |S_{13}|^2 + |S_{13}|^2 = 1$$

$$|S_{13}|^2 = ½ \Rightarrow |S_{13}| = \frac{1}{\sqrt{2}} \qquad \text{---- (8)}$$

Equate equation (6) & (7)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$$|S_{11}|^2 = |S_{22}|^2 \qquad \Rightarrow \qquad S_{11} = S_{22} \qquad \text{---- (9)}$$

R_3C_1 : $S_{13}\,S_{11}^* - S_{13}\,S_{12}^* = 0$

$$S_{13}\,(S_{11}^* - S_{12}^*) = 0 \qquad S_{11} = S_{12} = S_{22} \qquad \text{--- (10)}$$

Substitute equation (8) , (9) & (10) , in (6)

$$|S_{11}|^2 + |S_{11}|^2 + ½ = 1 \qquad \Rightarrow \qquad 2\,|S_{11}|^2 = ½$$

$$|S_{11}|^2 = ¼$$

$$S_{11} = ½$$

Substitute all the values in S matrix (equation (5))

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

3.4.2 H – Plane Tee (Shunt Tee)

→ A H-plane Tee junction is formed by cutting a rectangular slot along the width of a main waveguide and attaching another waveguide the side arm called the H-arm.

→ Port 1 and port 2 of the main waveguide are called collinear ports and port 3 H-arm (or) side arm.

→ An H-plane tee is a waveguide tee in which the axis of its side arm is shunting the E-field (or) parallel to the H-field of the main guide.

Sum arm

→ In a H-plane tee if two input waves are fed into port 1 and port 2 of the collinear arm the output wave at port 3 will be in phase and additive. So that the third port is called the sum arm.

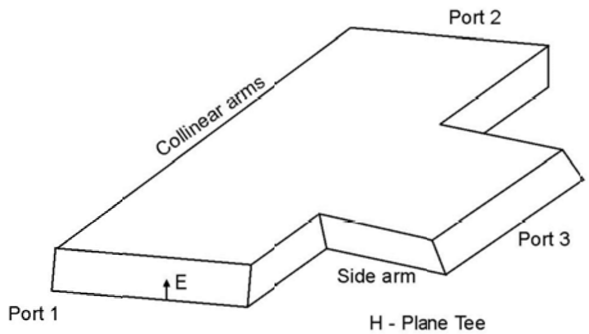


Fig. 3.5 : H-plane Tee

[S] is a 3x3 matrix since there are 3 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad \text{---- (1)}$$

→ If input is fed into port 3, the wave split equally into port 1 and port 2 in phase and same magnitude. Scattering co-efficients must be equal

$$S_{13} = S_{23} \qquad \text{---- (2)}$$

→ From the symmetry property $S_{ij} = S_{ji}$

$$S_{12} = S_{21}, \; S_{13} = S_{31}, \; S_{23} = S_{32} = S_{13} \qquad \text{---- (3)}$$

→ Port 3 is perfectly matched to the junction $S_{33} = 0$ ---- (4)

Substitute equation (2), (3), (4) in (1)

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \qquad \text{---- (5)}$$

→ From the unitary property $[S] . [S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \quad \Rightarrow \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ ----- (6)

$R_2C_2 \quad \Rightarrow \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ ----- (7)

$R_3C_3 \quad \Rightarrow \quad |S_{13}|^2 + |S_{13}|^2 = 1$

$\Rightarrow \quad 2|S_{13}|^2 = 1 \qquad |S_{13}| = \frac{1}{\sqrt{2}}$ ---- (8)

Equate (6) & (7)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$|S_{11}|^2 = |S_{22}|^2 \qquad \Rightarrow \quad S_{11} = S_{22}$ ---- (9)

$R_3C_1 \quad : \quad S_{13} S_{11}^* + S_{13} S_{12}^* = 0$

$\Rightarrow \quad S_{13}(S_{11}^* + S_{12}^*) = 0$

$S_{11}^* = -S_{12}^* \quad$ (or) $\quad S_{21} = -S_{11}$ ---- (10)

From equation (6)

$|S_{11}|^2 + |S_{11}|^2 + ½ = 1 \qquad \Rightarrow \qquad 2|S_{11}|^2 = ½$

$\Rightarrow \qquad S_{11} = ½$ ---- (11)

Substitute (9), (10) and (11)

$S_{12} = -½, \quad S_{22} = ½$ ---- (12)

Substitute all the values in (5)

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$ ----- (13)

→ Because of mismatch at any two ports, the VSWR at the mismatch port of either E (or) H Tee junction is ery high.

$$\text{VSWR} = \frac{1+S_{11}}{1-S_{11}} = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

3.4.3 Hybrid Junction

A hybrid Junction is a four port network in which a signal incident on any one of the ports divides between two output ports with the remaining ports being isolated.

Magic Tee (Hybrid or E-H plane Tee)

→ Here rectangular slots are cut both along the width and breadth of a long waveguide and side arms are attached.

→ A magic tee is a combination of E-plane & H-plane tee.

→ Port 1 and 2 are collinear arms, port 3 is the E-arm and port 4 is H-arm.

Characteristics of Magic Tee

→ If two in phase waves of equal magnitude are fed into port 1 and 2, the output at port 3 is subtractive and hence zero and total output will appear additively at port 4. Hence port 3 is called the difference (or) E-arm & port 4 the sum (or) H-arm.

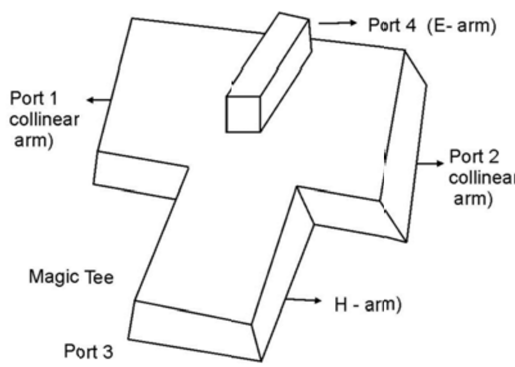


Fig. 3.6 : Magic Tee

→ A wave incident at port 3 divides equally between port 1 and port 2 but opposite in phase with no coupling to port 4.

$$S_{43} = S_{34} = 0 \qquad \text{---- (1)}$$

→ A wave incident at port 4 divides equally between port 1 and port 2 in phase no coupling to port 3.

→ A wave fed into one collinear port 1 or 2 will not appear in the other collinear port 2 or 1. Hence two collinear ports 1 and 2 are isolated from each others making.

$$S_{12} = S_{21} = 0 \qquad \text{----- (2)}$$

→ A magic-T can be matched by putting screws suitably in the E & H arms without destroying the symmetry of the junction.

→ Therefore for an ideal lossless magic Tee matched

$$S_{33} = S_{44} = 0 \qquad \text{----- (3)}$$

[S] is a 4 x 4 matrix, since there are 4 ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \qquad \text{---- (4)}$$

→ From the Symmetry property $S_{ij} = S_{ji}$

$$\left.\begin{array}{l} S_{14} = S_{41}\ ,\ S_{24} = S_{42} \\ S_{31} = S_{13}\ ,\ S_{23} = S_{32} \end{array}\right\} \qquad \text{---- (5)}$$

Substitute equation (1), (2), (3) & (5) in (4)

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \qquad \text{----- (6)}$$

From the unitary property $[S] . [S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* & S_{14}^* \\ S_{13}^* & -S_{13}^* & 0 & 0 \\ S_{14}^* & S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ ------- (7)

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ ------- (8)

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1$ ------- (9)

$$|S_{13}| = \frac{1}{\sqrt{2}}$$ ------ (10)

R_4C_4 : $|S_{14}|^2 + |S_{14}|^2 = 1$ ------- (11)

$$|S_{14}| = \frac{1}{\sqrt{2}}$$ ----- (12)

Equating (7) & (8)

$$|S_{11}| = |S_{22}|$$ ------ (13)

From equation (11)

$$|S_{13}|^2 + |S_{13}|^2 = 1 \ , \ |S_{13}| = \frac{1}{\sqrt{2}}$$ ------- (14)

From equation (12)

$$|S_{14}| = \frac{1}{\sqrt{2}}$$ ------- (15)

Substituting equations (10) & (12) in equation (7)

$$|S_{11}|^2 + |S_{12}|^2 + ½ + ½ = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

Which is valid if $S_{11} = S_{12} = 0$ ------- (16)

From equation (13) & (14) $S_{22} = 0$

So the S matrix is given by

$$[S] = \begin{bmatrix} 0 & 0 & S_{3} & S_{14} \\ 0 & 0 & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

Where $|S_{13}| = |S_{14}| = \frac{1}{\sqrt{2}}$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

→ Hence in any 4 port junctions, if any 2 ports are perfectly matched to the junction, the remaining two ports are automatically matched to the junction. Since all the 4 ports are perfectly matched to the junction is called a Magic tee.

Applications

→ Measurement of impedance, duplexer, mixer and an isolate.

3.4.4 Hybrid rings (Rat-Race circuits)

→ It is a four port junction, which is obtained by adding a fourth port to the normal three port tee junction.

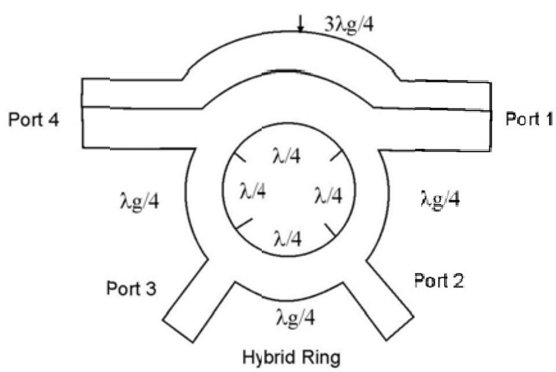


Fig. 3.7 : Hybrid Ring

Construction

→ By means of series or parallel junction, the four ports are connected in the form an angular ring at proper intervals.

i) The mean circumference of total race must be $\frac{3\,hg}{4}$.

ii) Each of the four ports are separated by a distance of $\frac{hg}{4}$.

iii) The characteristics impedance of hybrid ring is equal to $\sqrt{2}\,Z_O$. Where Z_O is the characteristic impedance of the connecting lines.

Properties

→ When power is fed into port 1 it splits equally into port 2 and 4 and nothing enters port 3.

→ At port 2 and 4 the power combine in phase but at port 3 cancellation occurs because the difference of phase shifts for the wave travelling in the clockwise and anticlockwise direction is 180^0.

→ Input applied to port 3 is equally divided between ports 2 and 4 but the output at port 1 will be zero.

→ S matrix for hybrid ring.

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Application

→ It is used for combining two signals (or) dividing a single signal into two equal halves.

3.4.5 Waveguide Bends

→ Bends are used to alter the direction of propagation in a waveguide system.

→ The reflection due to th bend is a function of its radius, the larger the radius, the lower will be the SWR.

→ The bends can be H bend (or) E-bend.

→ If the bend is in the direction of the wide direction the H lines are affected. If the bend is in the direction, of narrow dimension, the E lines are affected.

→ The bendin radius must be at least $2\lambda_g$ to av id SWR’s greater than 1.05 and mean length as long as possible.

R min = 1.5b for an E-bend where a & b are dimensions of waveguide bend.

R min = 1.5 a for an H bend.

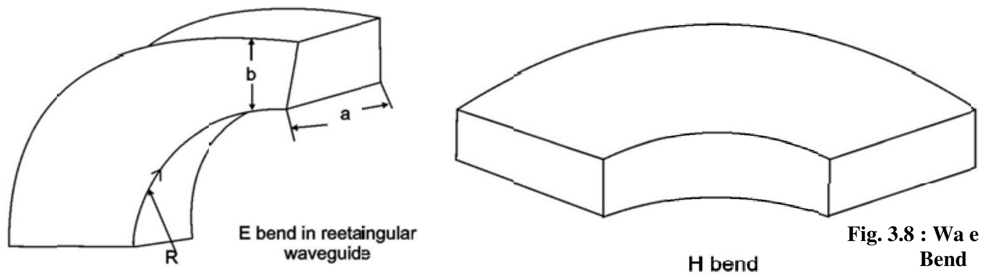


Fig. 3.8 : Wa e guide Bend

3.4.6 Corners

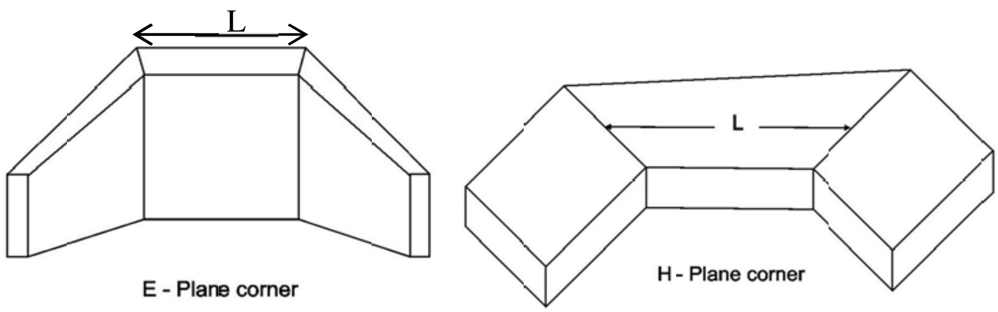


Fig. 3.9 : Waveguide Corner

→ The lower frequencies a bend may have to be very long and in such cases a corner would be preferred.

→ In order to minimize the reflections, from the discontinuities the mean length L must be odd no of quarter wavelength. So that the reflected wave from both ends of waveguide are completely cancelled. $L = (2n + 1)\ \lambda g/4$

Where n = 0, 1, 2, 3…….. λ g → wavelength of waveguide

3.4.7 Twists

→ Twists are used to change the plane of polarizat on of a propagating wave.

→ Twists such as 90^0 & 45^0 twists are helpful in converting vertical to horizontal polarizations or vice versa.

→ Twists can be incorporated along with bends also.

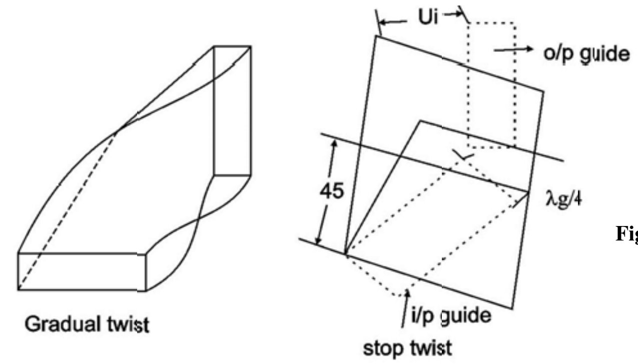


Fig. 3.10 : Waveguide Twist

Gradual Twists

→ It changes the plane of polarization in a continuous fashion.

Step Twists:

→ Step twists are used when the space is available in the propagation direction is limited.

→ It contains a rectangular waveguide section (i.e.) oriented at 45^0 with respect to input & output guides. Thus the polarization takes place in two 45^0 steps.

→ Multi step twists can be used when broader band performance is required.

3.5 Directional Couplers

→ It is a four port passive device commonly used for coupling a known fraction of the microwave power to a port in the auxiliary line while flowing from input port to output port in the main line. The remaining port is ideally isolated port & matched terminated.

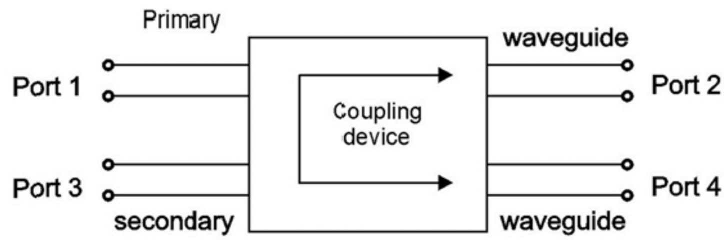


Fig. 3.11 : Directional coupler

→ This can be designed to measure incident/reflected power, Standing Wave Ratio provide a signal path to a receiver or perform other desirable operations.

→ This can be unidirectional (measuring only ncident power) or bidirectional (measuring both incident & reflected powers).

Properties of Directional Coupler

→ A portion of power traveling from port 1 to port 2 is coupled to port 4 and not to port 3.

→ A portion of power traveling from port 2 to port 1 is coupled to port 3 and not to port 4.

→ A portion of power incident on port 3 is coupled to port 2 but not to port 1 and a portion of power incident on port 4 is coupled to port 1 but not to port 2. Also port 1 and 3 are decoupled as are ports 2 & 4.

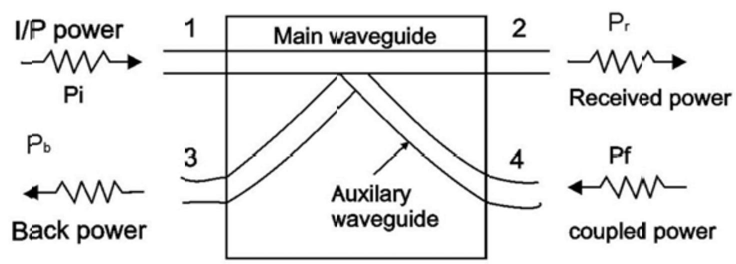


Fig. 3.12 : Directional coupler indicating powers

P_i → Incident power at port 1, P_r → Received power at port 2

P_b →back power at port 3, P_f → forward coupled power at port 4.

→ The performance of DC is described interms of its directivity and coupling.

Coupling (C)

→ It is the ratio of the incident power (P_i) to the forward power (P_f) in dB

$$C\ (dB) = 10 \log \frac{P_i}{P_F}.$$

i) Directivity (D)

→ It is ratio of forward power (P_f) to the back power (P_b).

$$D(dB) = 10 \log \frac{(P_f)}{(P_b)}$$

ii) Isolation

It is ratio of incident power (P_i) to back power (P_b)

$$\text{Isolation (dB)} = 10 \log \frac{(P_i)}{(P_b)}$$

Types of DC

i) Two hole DC ii) Four hole DC , iii) Reverse coupling DC & iv) Bethe hole DC.

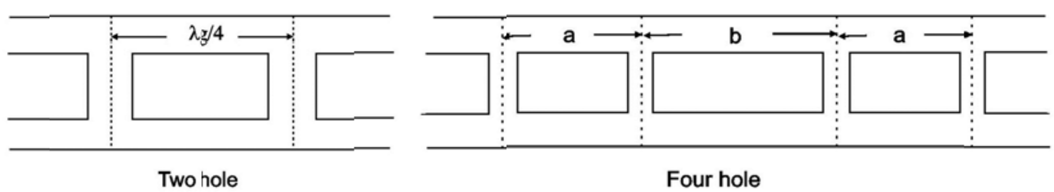


Fig. 3.13 : Types of DC

Two hole DC

→ It consists of two guides, the primary and secondary with two tiny holes common between them.

→ The no of holes one or more than two.

→ The degree of coupling can be determined by size and location of the holes.

→ The two holes are at a distance $\frac{hg}{4}$, λg is guide wave length.

The spacing between the centers of two holes must be $L = \frac{(2n+1)\, h g}{4}$.

n → positive integer

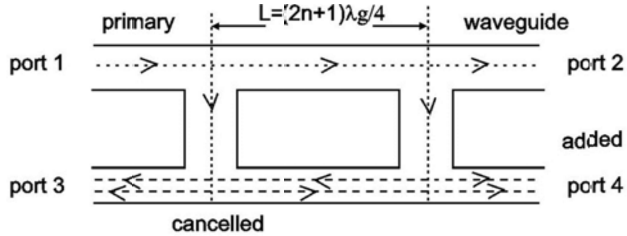


Fig. 3.1 : Two hole Directional coupler

→ A fraction of the wave entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as the slot antennas.

→ The forward waves in the secondary guide are in the same plane of the hole space and added to port 4.

→ The backward waves in the secondary guide are out of phase by 180^0 at the position of 1^{st} hole and are cancelled at port 3.

→ The coupling is given by $C = -20 \log_2 |B_f|$.

B_f → Amplitude in the forward direction.

Scattering matrix of a DC

→ Directional coupler is a four port netwrok. Hence 4x4 matrix.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{----- (1)}$$

→ Here all the four ports are perfectly matched to the junction

$S_{11} = S_{22} = S_{33} = S_{44} = 0$ ----- (2)

→ From symmetry property $S_{ij} = S_{ji}$

$S_{23} = S_{32}$, $S_{13} = S_{31}$, $S_{24} = S_{42}$, $S_{34} = S_{43}$, $S_{41} = S_{14}$ ----- (3)

→ There is no coupling between port 1 and port 3

$S_{13} = S_{31} = 0$ ----- (4)

Also there is no coupling between port 2 and port 4

$$S_{24} = S_{42} = 0 \qquad \text{----- (5)}$$

Substitute equation (2), (3) & (4) (5) in (1)

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{13} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \qquad \text{------- (6)}$$

From unitary property $[S]\,[S]^* = [I]$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{12}|^2 + |S_{14}|^2 = 1$ ----- (7)

R_2C_2 : $|S_{12}|^2 + |S_{23}|^2 = 1$ ----- (8)

R_3C_3 : $|S_{23}|^2 + |S_{34}|^2 = 1$ ----- (9)

R_1C_3 : $S_{12} S_{23}^* + S_{14} S_{34}^* = 0$ ----- (10)

Comparing equation (7) & (8)

$$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2 \quad \Rightarrow S_{14} = S_{23} \qquad \text{----- (11)}$$

Comparing equation (8) & (9)

$$|S_{12}|^2 + |S_{23}|^2 = |S_{23}|^2 + |S_{34}|^2 \quad \Rightarrow S_{12} = S_{34} \qquad \text{----- (12)}$$

Let us assume that S_{12} is real & positive $= P$

$$S_{12} = S_{34} = P = S_{34}^* \qquad \text{----- (13)}$$

Substitute equation (13) in equation (10)

$S_{12}\, S^*_{23} + S_{14}\, S^*_{34} = 0$, $S_{14} = S_{23}$ $\Rightarrow$ $P\,(S^*_{23} + S_{23}) = 0$

Since $P \neq 0$, $S_{23} + S^*_{23} = 0$ ----- (14)

$S_{23} = jq$, $S^*_{23} = -jq$, S_{23} must be imaginary.

Let $S_{23} = jq = S_{14}$, $S_{12} = S_{34} = P$, & $S_{23} = S_{14} = jq$ ------ (15)

Substitute equation (15) in equation (7)

$P^2 + q^2 = 1$ ----- (16)

Substitute all the values in (6)

[S] matrix of a directional coupler is reduce to

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix} \quad \text{------ (17)}$$

3.6 Ferrite Devices

→ It is a device that is composed of materials that causes useful magnetic properties.

Ferrites

→ Ferrites are non-metallic materials with resistivity ρ around 10^{14} times greater than the metal and with dielectric constants ε_0 around 10 - 15 and relative permeabilities of the order of 1000.

→ It is used for designing non-reciprocal components.

→ It does not have same electrical characterstics in all directons.

Properties

→ It possess strong magnetic properties.

→ Ferrites are most suitable for use in microwave devices in order to reduce the reflected power.

→ It possess high resistivity, hence they can be used up to 100 GHz.

→ It also exhibit non-reciprocal property.

Types

(i) Circulator (ii) Isolator (iii) Gyrator.

3.6.1 Circulator

→ It is a multiport device that has the property that a wave incident in port 1 is coupled into port 2 only, wave incident in port 2 is coupled into port 3 only and so on.

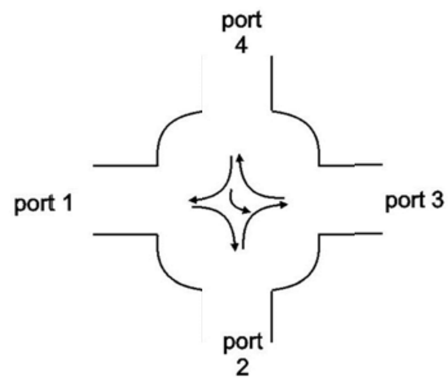
→ It can travel one port to next immediate in one direction only.

→ A four port circulator is very similar to the 3 port circulator.

→ All the four ports are matched and transmission of power takes place in cycles order only.

(i.e.) Port 1 to port 2, port 2 to port 3, port 3 to port 4 , port 4 to port 1.

→ It may be constructed by two single magic T and non-reciprocal network 180^0 phase shifter or a combination of two 3-dB side hole directional coupler and rectangular waveguide with two non-reciprocal phase shifters.



Fi . 3.15 : Four port circulator

Four Port Circulator

→ Each of the two 3-dB couplers in the circular introduces a phase shift of 90^0 and each of the two phase shifter produces a certain amount of phase change in a certain direction.

→ When a wave is incident to port 1, the wave is split into two components by coupler 1.

→ The wave is in primary guide arrives at port 2 with a relative phase change of 180^0.

→ The second wave propagates through the two coupler and secondary wave arrives at port 2 with a phase shift of 180^0.

→ Since the two waves reaching port 2 are in phase the power transmission is obtained from port 1 to port 2. However the wave propagate through the primary guide, phase shifter and coupler 2 and arrives at port 4 with a phase change of 270^0.

→ The wave travels through coupler 1 and the secondary guide and it arrives at port 4 with a phase shift of 90^0.

→ The two waves reaching port 4 are out of phase by 180^0 the power transmission from port 1 to port 4 is zero.

→ In general the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be.

$\omega_1 - \omega_3 = (2m + 1)\pi$ rad / s

$\omega_2 - \omega_4 = 2n\pi$ rad / s where m & n are integers including zeros.

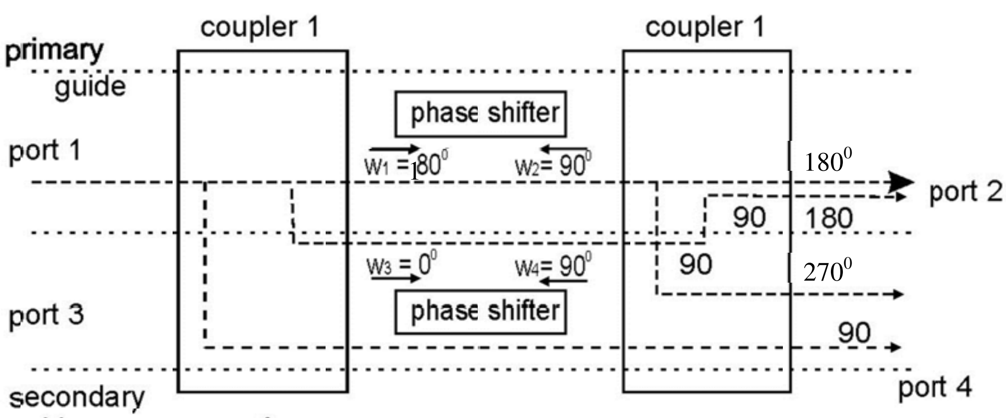


Fig. 3.16 : Schematic diagram of a 4 port circulator

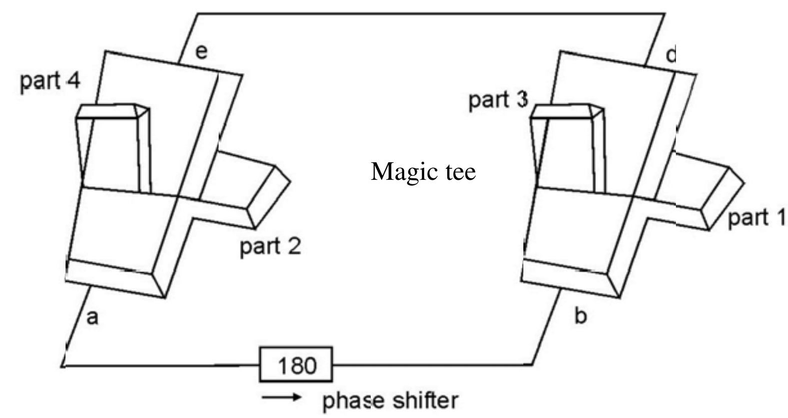


Fig. 3.17 : Four port circulator

→ In an input signal at port 3 will emerge from 4 and at port 4 will appear at port 1. The circulator property is exhibited. The phase shifter produces a phase shift of 180^0.

S matrix of circulator

→ A perfectly matched, lossless and non-reciprocal four port circulator has an S matrix of the form

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

→ S_{12} means port 2 is the output port and port 1 is the input port but in circulator property output of port 1 it given as input to port 2 so $S_{12} = 0$. This property is applicable to all other S parameter and the diagonal element are zero.

$$[S] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

S matrix for 3 port circulator

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

→ The performance of microwave is finite duration.

→ Insertion LOSS < 1dB, solation = 30 – 40 db, & VSWR < 1.5.

Applications

→ It can be used as a duplexer for a radar antenna system.

→ Two three port circulators can be used in tunnel diode or parametric amplifier.

→ It can be used at low power devices.

3.6.2 Microwave Isolators

→ An Isolator is a two port non-reciprocal device which produces a minimum attenuation in wave propagation in one direction and very high attenuation in the opposite direction.

→ When Isolator inserted between a signal source and load almost all the signal power can be transmitted to the load and reflected power from the load is not fed back to the generator output ports. Then the isolator is actually called uniline.

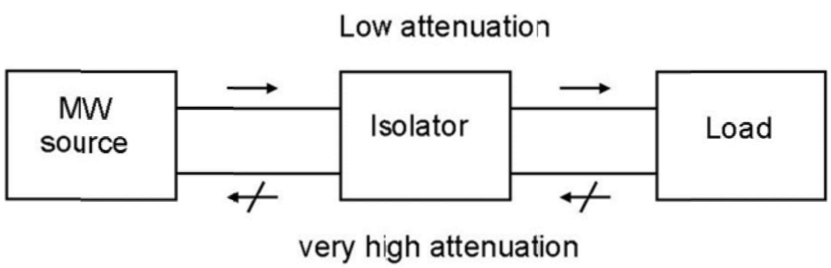


Fig. 3.18 : Isolator in ie wave transmission

→ Isolators can be made by inserting a ferrite rod along the axis of rectangular waveguide. Here the isolator is called as Faraday – Rotation Isolator.

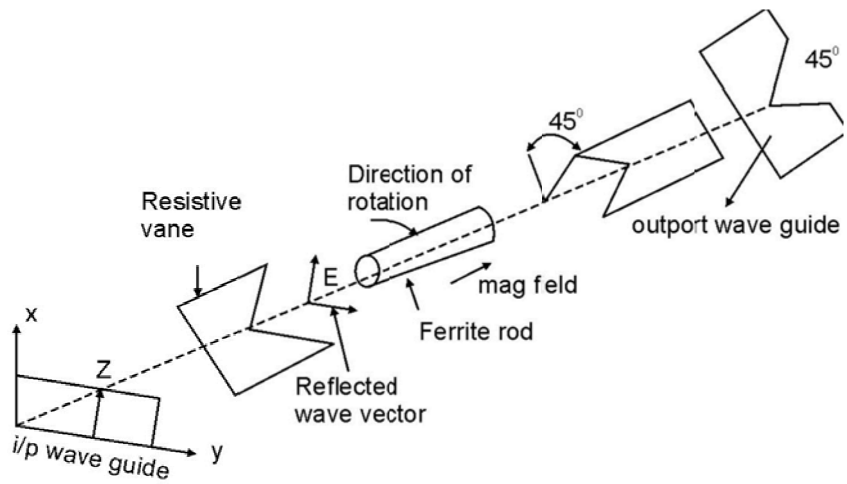
Faraday Rotation Isolator

→ The input resistive card is in the Y – Z plane and output resistive card is displaced 45^0 with respect to the input card.

→ The dc magnetic field which is applied longitudinally in all to the ferrite rod rotates the wave plane of polarization by 45^0.

→ Degree of rotation depend on the length & diameter of the rod and on the applied dc magnetic field.

→ TE_{10} mode wave is perpendicular to the input resistive card the wave passes through the ferrite is without attenuation.



olator

→ The wave is the ferrite rod section is isolated clockwise by 45^0 and is normal to output resistive card. As a result of rotation the wave arrives at the output and without attenuation.

→ A reflected wave from the output end is similarly rotated clockwise 45^0 by the ferrite rod since the reflected wave is parallel to the input resistive load the wave is thereby absorbed by the input card.

S matrix for Isolator

Ideal losseless matched Isolator.

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

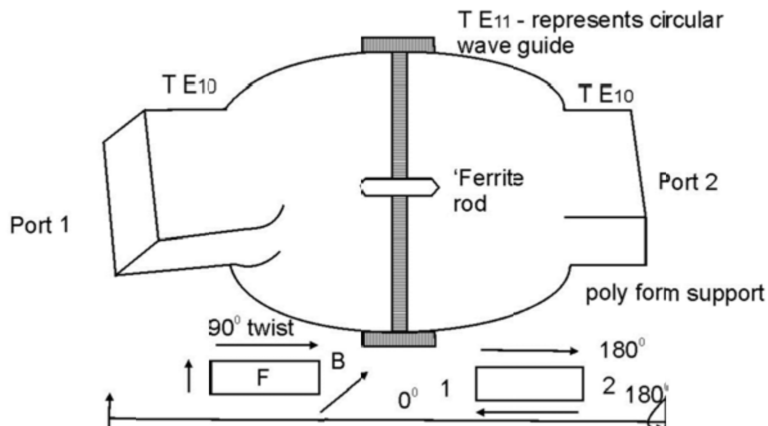
→ Typical performance of these isolators of about 1 dB insertion loss for forward transmission.

→ About 20 to 30 dB isolation in reverse d rection.

Applications

→ It is used to improve the frequency stability of microwave generators, such as klystrons & magnetrons.

3.6.3 Gyrator



Fi . 3.20 : Two port Gyrator

It is a two-port device, which provides a relative phase shift of 180^0 for transmission from port 1 to port 2 as compared to the phase for transmission from port 2 to port 1.

Construction

It consists of circular waveguide supporting TE_{11} mode with transitions to a standard rectangular waveguide TE_{10} mode at both ends. The rectangular to circular waveguide transition is twisted by 90^0.

→ A thin ferrite red is placed inside a circular waveguide supported by polyform & the waveguide is surrounded by a permanent magnet, which generates dc magnetic field in the ferrite rod co e. The ferrite rod is tapered at both the ends to reduce the attenuation & also for smooth rotation of the polarized wave.

Operation

→ When a wave enters port 1 plane of polarization rotates by 90^0 because of the twist in the waveguide. It again undergoes faraday rotation through 90^0 and the wave comes out of port 2 which will have a phase shift of 180^0 compared to wave entering port 1.

→ But the same wave enters port 2 undergoes faraday rotation through 90^0 in the same direction because of the twist in the waveguide the wave again rotates back through 90^0 and works out of port 1 with zero phase shift.

→ Gyrator provides 180^0 phase shift in forward direction and 0^0 phase shift in reverse direction.

3.6.4 Attenuators

→ Attenuators are passive devices used to control power levels in a microwave system by partially absorbing the transmitted signal wave. Both fixed & variable attenuator are designed.

→ A coaxial fixed attenuator use a film with losses on the centre conductor to absorb some of the power. It consists of a thin dielectric strip coated with resistive film and placed at the centre of the wave guide parallel to the maximum Electric field. The dielectric strip is tapered at both ends up to a length of more than half wavelength to reduce reflections.

→ A variable type attenuator can be constructed by moving the resistive vane by means of micro meter screws from one side of the narrow wall to the centre where the Electric field is maximum. A maximum of 90 dB attenuation is possible with VSWR of 1.05.

→ A precision type variable attenuator makes use of a circular waveguide section (c) containing a very thin tapered resistive card (R_2) of both sides of which are connected axisymmetric sections of circular to rectangular waveguide tapered transistions (RC_1 & RC_2).

→ The incident TE_{10} dominant wave in the rectangular waveguide is converted into the dominant TE_{11} mode in the circular waveguide.

→ The attenuation of the incident wave is

$$\alpha = \frac{E}{E \sin^2 0} = \frac{1}{\sin^2 0} = \frac{1}{|S_{21}|} \qquad \text{(or)}$$

$$\alpha \text{ (dB)} = -40 \log (\sin\theta) = -20 \log |S_{21}| \qquad \text{----- (1)}$$

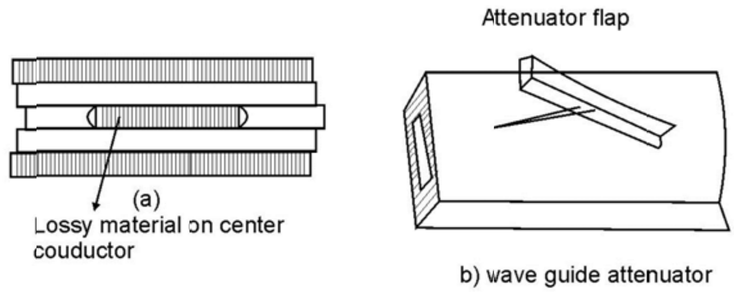
Attenuators are normally matched reciprocal devices.

$$|S_{21}| = |S_{12}| \qquad \text{----- (2)}$$

$$\text{and } |S_{11}| \text{ (or) } |S_{12}| = \frac{VSWR - 1}{VSWR + 1} \ll 0.1. \qquad \text{----- (3)}$$

The S matrix of an ideal precision rotary attenuator is

$$[S] = \begin{bmatrix} 0 & \sin^2\theta \\ \sin^2\theta & 0 \end{bmatrix} \qquad \text{----- (4)}$$



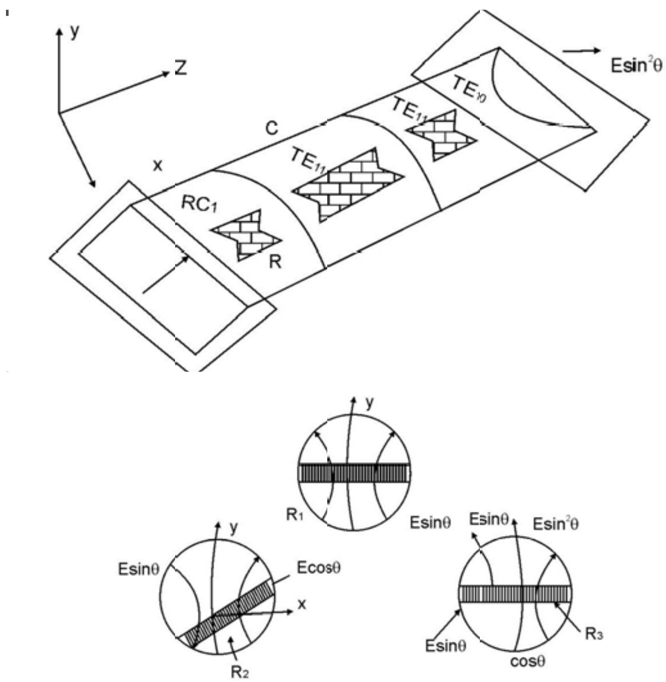


Fig. 3.21 : Precessio type variable attenuator

R_1, R_2, R_3 → Tapered resistive cards, RC_1 & RC_2 → Rectangular to circular waveguide transitions

C_1 → Circular waveguide section.

3.6.5 Phase Shifters

→ It is a two-port passive device that produces a variable change in phase of the wave transmitted through it. It can be realized by placing a lossless dielectric slab within a waveguide parallel to and at the position of maximum Electric field.

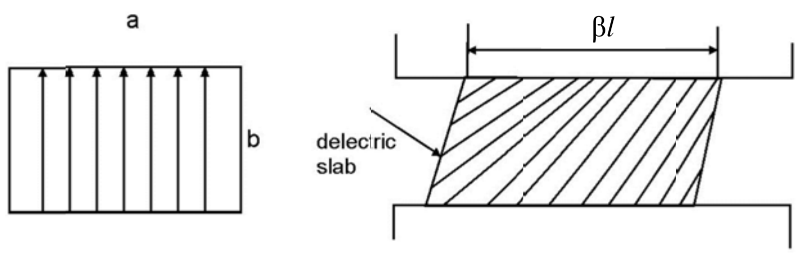


Fig. 3.22 : phase shifters

The propagation constant through a length *l* of a dielectric slab & of an empty guide are respectively.

$$\beta_1 = \frac{2u}{h_{g1}} = \frac{2u\sqrt{\left[1-\left(\frac{h_0}{2a\sqrt{\epsilon_r}}\right)^2\right]}}{\frac{h_0}{s_r}} \qquad \text{----- (1)}$$

$$\beta_2 = \frac{2u}{h_{g2}} = \frac{2u\sqrt{\left[1-\left(\frac{h_0}{2a}\right)^2\right]}}{\frac{h_0}{s_r}} \qquad \text{----- (2)}$$

Thus the differenti l phase shift produced by the phase shifter is $\Delta\phi = (\beta_1 - \beta_2)$ L. By adjusting the length *l*, difference phase shifts can be produced. The S matrix of an ideal phase shift can be expressed by

$$[S] = \begin{bmatrix} 0 & e^{-j\Delta\emptyset} \\ e^{-j\Delta\emptyset} & 0 \end{bmatrix} \qquad \text{----- (3)}$$

i) Precision Phase Shifter

→ It can be designed by rotary type the TE_{11} mode incident field E_1. In the input quarter wave section can be decomposed into two transverse components one E_1 is parallel & E_2 is perpendicular to quarter wave plane.

The quarter wave components are

$$E_1 = E_i \cos 45^0 e^{-j\beta_1 l} = E_0 e^{-j\beta_1 l} \quad \text{----- (1)}$$

$$E_2 = E_i \sin 45^0 e^{-j\beta_2 l} = E_0 e^{-j\beta_2 l} \quad \text{----- (2)}$$

Where $E_0 = \frac{E}{\sqrt{2}}$. The length l is adjusted such that these two components will have equal magnitude but a different phase change as $(\beta_1 - \beta_2)\, l = 90^0$. ∴ after propagation through the quarter wave phase these field components become

$$E_1 = E_0 e^{-i\beta_1 l} \quad \text{----- (3)}$$

$$E_2 = jE_0 e^{-j\beta_1 l} = jE_1 = E_1 e^{j\pi/2} \quad \text{---- (4)}$$

Thus the quarter wave sections convert a linearly polarized TE_{11} wave to circular wave guide & vice versa. The field components parallel & perpendicular to the half ware plate can be expressed as

$$E_3 = (E_1 \text{Cos}\theta - E_2 \text{Sin}\theta)\, e^{-j_2\beta_1 l} = E_0 e^{-j\theta} e^{-j_3\beta_1 l} \quad \text{----- (5)}$$

$$E_4 = (E_2 \text{Cos}\theta + E_1 \text{Sin}\theta)\, e^{-j_2\beta_2 l} = E_0 e^{-j\theta} e^{-j_3\beta_1 l} e^{-j\pi/2} \quad \text{----- (6)}$$

Since, $2(\beta_1 - \beta_2)l = \pi$ (or) $-2\, \beta_2\, l = \pi \;\; x\; -2\, \beta_1 l$ ----- (7)

The field components E_3 & E_4 may be decomposed into two TE_{11} modes polarized parallel & perpendicular to the output quarter wave plate.

→ At the output end of this quarter wave plate the field components parallel & perpendicular to the quarter wave plate can be written as

$$E_5 = (E_3 \text{Cos}\theta + E_4 \text{Sin}\theta)\, e^{-j\beta_1 l} = E_0 e^{-j2\theta} e^{-j_4\beta_1 l} \quad \text{----- (8)}$$

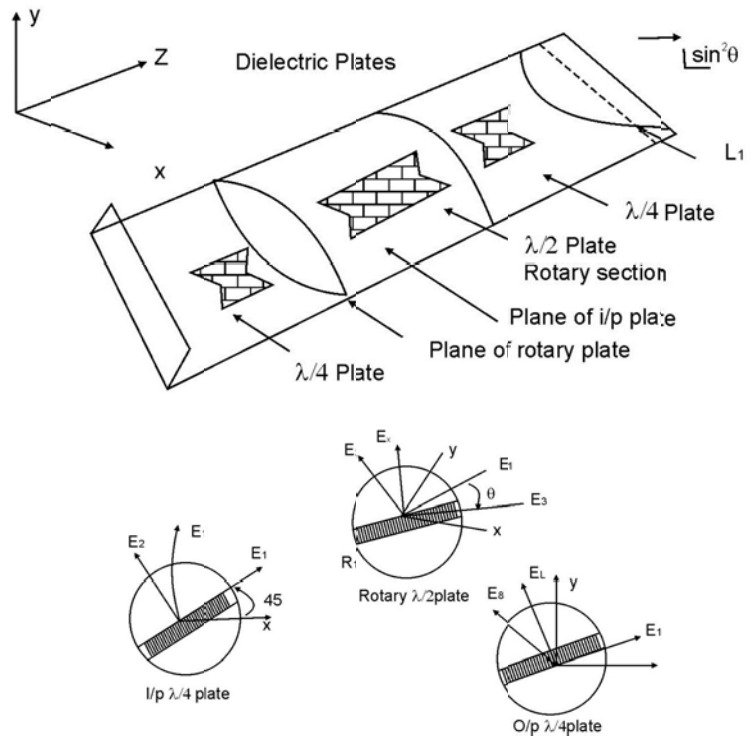
$$E_6 = (E_4 \text{Cos}\theta - E_3 \text{Sin}\theta)\, e^{-j\beta_2 l} = E_0 e^{-j2\theta} e^{-j_4\beta_1 l} \quad \text{----- (9)}$$

Thus the parallel component E_5 & perpendicular component E_6 at the output end of the quarter wave plate are equal in magnitude & in phase to produce a resultant field which is a linearly polarized TE_{11} wave.

$$E_{out} = \sqrt{2}\; E_0 e^{-j2\theta} e^{-j_4\beta_1 l}$$

$$= E_1 e^{-j2\theta} e^{-j4\beta_1 l} \quad \text{----- (10)}$$

$E_1 \rightarrow$ Incident field with a phase change of $2\theta + 4\beta_1 l$, since θ can be varied. & $4\beta_1 l$ is fixed at a given frequency.



ig. 3.23 : Precision rotary phase shifter

3.7 Microwave cavities

→ Microwave resonators are tunable circuits used in microwave oscillators, amplifiers, wave meters and filters. As the tuned frequency the circuit resonates where the average energies in the electric field W_e and magnetic field W_m are equal and the circuit impedance becomes purely real.

→ The total energy is twice the electric (or) magnetic energy stored in the resonator.

Resonant frequency

→ Resonant frequency ‘f’ at which the energy in the cavity attains maximum value $= 2\ W_e$ (or) $2\ W_m$.

→ Quality factor ‘Q’ which is measure of the frequency selectivity of a cavity.

$$Q = \frac{2\pi \times \text{maximum energy stored}}{\text{Energy dissipated per cycle}}$$

Cylindrical cavity resonators

→ It is a circular wave guide with two ends closed by a metal wall.

→ The field component inside the cavity are described interms of TE_{npq} and TM_{npq} as follows.

TE_{npq} mode field

$$H_z = H_{0z} J_n \left(\frac{X_{np} r}{a}\right) \cos(n\phi) \sin\left(\frac{q\pi Z}{d}\right) \qquad \text{------ (1)}$$

Where

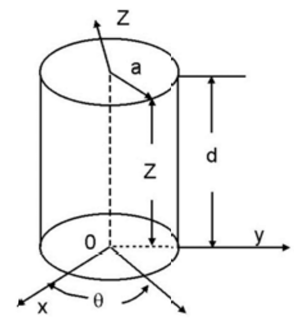


Fig. 3.24 : cylindrical cavity resonator

$n = 0, 1, 2, 3$ is the number of periodicity in the ϕ direction.

$p = 1, 2, 3,$ is the number of zeros is the field in radial direction.

$q = 1, 2, 3$ is the number of half waves in axial direction.

J_n → Bessel’s function of the first kind.

$h = \frac{X_{np}}{a}$ and X_{np} → p^{th} root of equation $J_n(ha) = 0$

H_{0Z} = Amplitude of the magnetic field.

TM_{npq} mode field

$$E_z = E_{0z} - J_n \left(\frac{X_{nq} r}{a}\right) \cos(n\phi) \cos\left(\frac{q\pi Z}{d}\right) \qquad \text{------ (3)}$$

E_{0z} Amplitude of the electric field. The separation of TE & TM modes are given by,

$K^2 = \left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2$ for TE mode, $K^2 = \left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2$ for TM mode

$K^2 = \omega^2 \mu\varepsilon$

$\omega^2 = 2\pi f_r$

Resonant frequencies of TE & TM modes:

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \quad \text{(TE) mode}$$

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \quad \text{for TM mode}$$

The dominant mode in a circular cavity will depend on the dimensions of the cavity.

i) For $2a \geq d$, the dominant mode is TM_{110}.

ii) For $d \geq 2a$, the dominant mode is TE_{111}.

Semicircular cavity resonator

The wave function of TE_{npq} mode is $H_Z = H_0 J_n\left(\frac{X'_{np}\, r}{a}\right) \cos(n\phi) \sin\left(\frac{q\pi Z}{d}\right)$

Where n = 0, 1, 2, 3, 4 ……….

p = 1, 2, 3, 4 …………

q = 1, 2, 3, 4, …………….

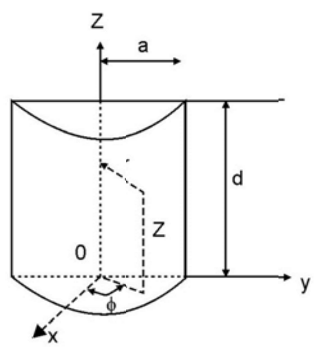


Fig. 3.25 : semicircular cavity resonasor

a → radius of the semicircular cavity resonator.

d → length of the resonator

$$E_Z = E_0 J_n\left(\frac{X_{np}\, r}{a}\right) \cos(n\phi) \cos\left(\frac{q\pi Z}{d}\right)$$

With the separation equations the resonant frequencies for TE and TM modes are

$$f_r = \frac{1}{2\pi a\sqrt{\mu\varepsilon}}\sqrt{(X'_{np})^2 + \left(\frac{q\pi a}{d}\right)^2} \quad \text{for } TE_{npq} \text{ mode}$$

$$f_r = \frac{1}{2\pi a\sqrt{\mu\varepsilon}}\sqrt{(X_{np})^2 + \left(\frac{q\pi a}{d}\right)^2} \quad \text{for } TM_{npq} \text{ mode}$$

3.8 Termination

Two types of terminations are used. They are,

i) Matched Load
ii) Variable Short Circuit

1. Matched Load

→ Matched load provides a termination that absorbs all the incident power and hence equivalent to terminating the line in its characteristic impedance.

→ The usual matched load for a waveguide is a tapered wedge (or) slab of lossy material inserted into the guide.

→ Reflections are avoided by tapering the lossy material into a wedge.

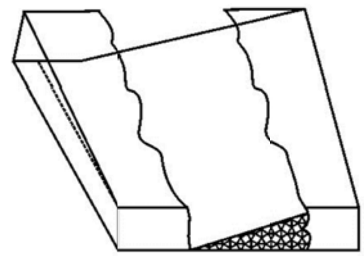


Fig. 3.26 : Lossy wedge

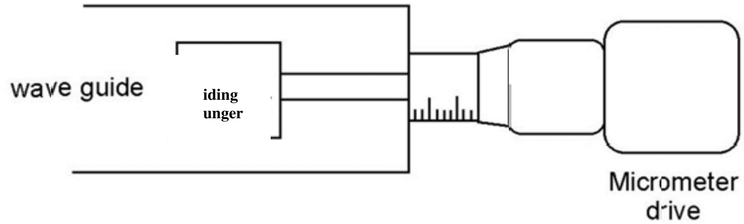
→ Thus the termination may be viewed as a lossy tapered transmission line.

2. Variable short Circuit

→ It is a termination that eflects all the incident power. The phase of the reflected wave is viewed by changing the position of the short circuit.

→ The simple form of adjustable short circuit for use in a waveguide is a sliding block of copper (or) some other good conductor.

→ The position of the block is varied by means of micrometer device.



ig. 3.27 : A simple short circuit for a wave guide

→ The simple form of adjustable short circuit is not very satisfactory in its electrical performance.

→ A choke type plunger may overcome these problems.

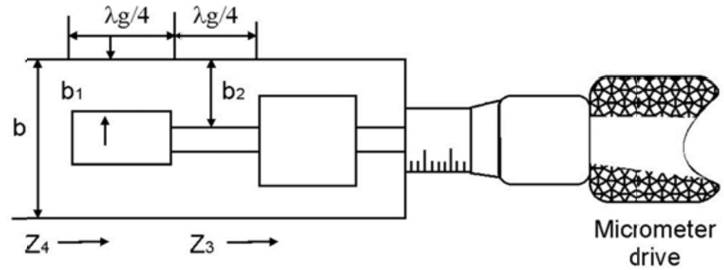


Fig. 3.28 : Choke type adjustable short circuit (side view)

→ It works on the principle of a quarter wave transformer.

→ In a choke type plunger, the width of the plunger is uniform and slightly less than the interior guides width.

→ The height of the plunger is made non-uniform.

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Problems

1. Calculate the coupling factor of a directional coupler when incident power is 600 mω & power in auxiliary wave guide is 350 μω.

Solution:

$$C = \log \frac{P_1}{P_4}$$

$$C = \log \frac{600 X 10^{-3}}{350 X 10^{-6}}$$ **C = 32.34 dB**

2. For a DC the incident power is 550 mω calculate the power in main arm & auxiliary arm. C = 30 dB

Solution:

$$P_1 = 550 \text{ m}\omega$$

$$C = \log \left(\frac{P_1}{P_4}\right)$$

$$30 = 10 \log \left(\frac{550 \text{ X } 10^{-3}}{P_4}\right)$$

$$\frac{30}{10} = \log \frac{550 \text{ X } 10^{-3}}{P_4}$$

$$P_4 = \frac{550 \text{ X } 10^{-3}}{\text{anti log}(3)}$$

$$\mathbf{P_4 = 350\ \mu\omega}$$

Power in main arm = Input power + output power

Power in auxiliary arm + output power = input power

Output power = $P_1 - P_4$

Output power = 549.45 mω

3. Calculate the Scattering coefficients by using isolation loss of 25 dB and Insertion loss of 0.5 dB.

Solution:

$$\text{Insertion loss} = 20 \log \frac{1}{|S_{21}|}$$

$$= 20 \log |S_{21}|^{-1}$$

$$0.5 = -20 \log |S_{21}|$$

$$\text{Antilog } 0.5 = -20 |S21|$$

$$S_{21} = 0.944$$

$$\text{Insertion loss} = 20 \log \frac{1}{|S_{12}|}$$

$$25 = -20 \log |S_{12}|$$

$$S_{12} = 0.0563$$

$$[S] = \begin{bmatrix} 0 & 0.0563 \\ 0.944 & 0 \end{bmatrix}$$

4. A wave guide termination having VSWR of w is used dissipate 100w of power. Find the reflected power.

Solution:

$S = 1.1 \quad p_i = 100$ w

$|\Gamma| = \frac{S-1}{S+1}$

$P_{ref} = |\Gamma|^2 P_i$

$|\Gamma| = \frac{1.1-1}{1.1+1} = \frac{0.1}{2.1} = 0.0476$

$|\Gamma|^2 = 2.26 \times 10^{-3}$

$P_{ref} = 2.26 \times 10^{-3} \times 100$

$P_{ref} = 0.226$ w

5. A 20 mω signal is fed into one of collinear part 1 of a loseless µplave T junction calculate the power delivered through each port when other ports are terminated in matched load.

Solution:

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

The total effective power input to port 1 is

$P_1 = |a_1|^2 - |a_1|^2 |S_{11}|^2$

$= |a_1|^2 (1 - |S_{11}|^2)$

$P_1 = 20 (1-0.5^2) = 15$ mω

Power terminated to port 3 is

$P_3 = |a_1|^2 |S_{31}|^2 = 20 \left(\frac{1}{\sqrt{2}}\right)^2 = 10$mω

Power transmitted to port 2 is

$P_2 = |a_1|^2 \, |S_{21}|^2 = 20 \, (½)^2 = 50 \text{ m}\omega$

In H-plane

$P_1 = P_2 + P_3$

$15 \text{ m}\omega = 10 + 5 \text{ m}\omega$

$15 \text{ m}\omega = 15 \text{ m}\omega$

If E-plane means

$P_1 = P_2 - P_3$.

6. A three port circulator has an insertion loss of 1 dB isolation loss 30 dB & VSWR = 1.5. Find the S-matrix.

Solution:

Insertion loss $= 1 \text{ dB} = 20 \log \frac{1}{S_{21}}$

$= - \log |S_{21}|$

$S_{21} = \text{Antilog } (-0.05)$

$S_{21} = 0.891$

For some Insertion loss between ports 1 & 2, 2 & 3, 3 &1

$S_{12} = S_{32} = S_{13} = 0.891$

Isolation loss $= 20 \log \frac{1}{|S_{31}|}$

$30 = -20 \log |S_{31}|$

$\frac{-30}{20} = \log S_{31}$

$\text{Antilog } (-1.5) = S_{31}$

$S_{31} = 0.032$

$S_{12} = S_{23} = S_{31} = 0.032$

VSWR = 1.5

$| = \frac{S-1}{S+1} = 0.2$

$S_{11} = S_{22} = S_{33} = 0.2$

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