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Monograph On RF Transistor Amplifier Design And Matching Networks

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2.1 RF Transistor Amplifier Design

Amplifier designs at RF differ significantly from the conventional low frequency circuit approaches and consequently require special considerations.

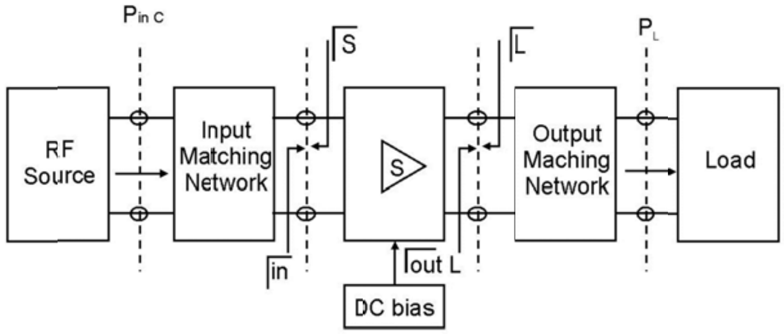

Fig. 2.1 : Generic Amplifier system

The amplifier parameters are,

- Gain (dB)
- Operating frequency and bandwidth (Hz)
- Output power (dBm)
- Power supply requirements (V & A)
- Input and output reflection co-efficient
- Noise figure

In addition Intermodular Distortion Products, harmonics, feedback and heating effects all of which affect the amplifier performance. Christo Ananth et al.[1] discussed about E-plane and H-plane patterns which forms the basis of Microwave Engineering principles.

2.2 Amplifier Power Relations

RF Source

There are various power gain definitions that are critical to the understanding of how an RF amplifier functions.

→ For this reason, power flow relations under the assumption that the two matching networks are included in between the source and load impedances.

Γ_s → Source Ref ection coefficient

Γ_{in} → Input Reflection coefficient

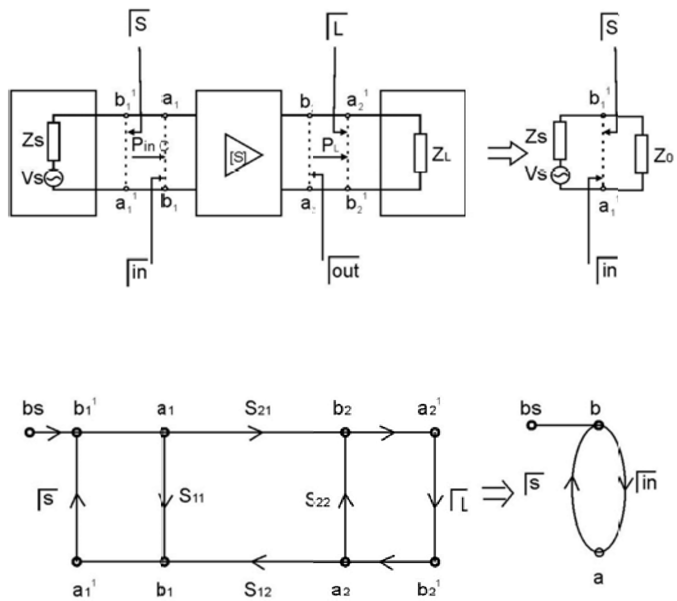

Fig. 2.3 : signal flow graph

The source voltage is given by,

$$b_S = \frac{\sqrt{Z_0}}{Z_S + Z_O} V_S$$

$$b_1^{r} = b_S + a_1^{r} \Gamma_s$$

$$b_S = b_1^{r} - a_1^{r} \Gamma_s$$

From the amplifier diagram, $a_1^r = \Gamma in\, b_1^r$

$b_S = b_1^r - b_1^r\, \Gamma in \Gamma s$

$b_S = b_1^r\ (1 - |\ in|\ s)$ ----- (1)

The incident power wave associated with b_1^r is given as,

$$P_{inc} = \frac{|b_1^r|^2}{2} = \frac{1}{2} = \frac{|b_s|^2}{|\ 1 - /in/s|^2}$$ ----- (2)

The actual input power is the input terminal of the amplifier is composed of the incident and reflected power waves.

$P_{in} = P_{inc}\,(1 - |\Gamma in|^2)$

$$P_{in} = \frac{1}{2}\ \frac{|b_s|^2\,(\ 1 - \Gamma in \Gamma s|^2)}{|\ 1 - |\ in|\ s|^2}$$ ---- (3)

The maximum power transfer from the source to the amplifier is achieved, if the input impedance is complex conjugate matched ($Z_{in} = Zs^*$) or interms of the reflection coefficients ($\Gamma in = \Gamma s^*$).

Under maximum power transfer condition, we define the available power P_A as,

$$P_A = P_{in} /\ \Gamma in = \Gamma s^*$$

$$P_A = \frac{1}{2}\ \frac{|b_s|^2\,(\ 1 - |\ |\ s^*|^2)}{|\ 1 - |\ s|\ s^*|^2}$$

$$= \frac{1}{2}\ \frac{|b_s|^2\,(\ 1 - |\Gamma s|^2)}{(\ |\ 1 - ||\ s\ |)^2}$$

$$P_A = \frac{1}{2}\ \frac{|b_s|^2}{(\ |\ 1 - |\Gamma s|^2)}$$ ----- (4)

If $|\ in = 0$, & $|\ s \neq 0$, it is seen that

$P_{in} = ½\ |b_s|^2$

Transducer Power Gain

The transducer power gain G_T, which quantifies the gain of the amplifier placed between source and load.

$$G_T = \frac{\text{Power delivered to the load}}{\text{Available power from the source}}$$

$$G_T = \frac{P_L}{P_A}$$

The power delivered to the load,

$$P_L = \frac{1}{2} |b_2|^2 (1 - |\Gamma L|^2)$$

$$G_T = \frac{P_L}{P_A} = \frac{|b_2|^2 (1-|\Gamma L|^2)\ 2\ (1-|\Gamma s|^2)}{2\,|b_s|^2}$$

$$G_T = \frac{|b_2|^2 (1-|\Gamma L|^2)(1-|\Gamma s|^2)}{|b_s|^2} \qquad \text{----- (5)}$$

With the help of signal flow, we establish

$$b_2 = \frac{S_{21}\, a_1}{1 - S_{21}\Gamma L} \qquad \text{---- (6)}$$

$$b_S = \left[1 - \left(S_{11} - \frac{S_{21} S_{12}\Gamma L}{1 - S_{22}\Gamma L}\right)\Gamma s\right] a_1 \qquad \text{---- (7)}$$

The required ratio is therefore given by

$$\frac{b_2}{b_s} = \frac{\dfrac{S_{21}\, a_1}{1 - S_{22}\Gamma L}}{\left[1 - \left(S_{11} - \dfrac{S_{21} S_{12}\Gamma L}{1 - S_{22}\Gamma L}\right)\Gamma s\right] a_1}$$

$$= \frac{\dfrac{S_{21}}{1 - S_{22}\Gamma L}}{\dfrac{(1 - S_{22}\Gamma L)(1 - S_{11}\Gamma s) - S_{12} S_{21}\Gamma L\,\Gamma S}{1 - S_{22}\Gamma L}}$$

$$\frac{b_2}{b_s} = \frac{S_{21}}{(1 - S_{22}\Gamma L)(1 - S_{11}\Gamma s) - S_{12} S_{21}\Gamma L\,\Gamma S} \qquad \text{----- (8)}$$

Substitute this value in equation (5)

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_s|^2)}{|(1-S_{22}\Gamma_L)(1-S_{11}\Gamma_s)-S_{12}S_{21}\Gamma_L\Gamma_S|^2} \quad \text{------ (9)}$$

Which can be rearranged by defining the input and output reflection coefficients.

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \quad \text{---- (10)}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} \quad \text{---- (11)}$$

With these two definitions, two more transducer power gain expressions can be derived.

Substitute $S_{12} = 0$ in equation (10)

$\Gamma_{in} = S_{11} + 0$

Substitute these values in equation (9)

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_s|^2)}{|(1-S_{22}\Gamma_L)|^2 |1-\Gamma_{in}\Gamma_S|^2} \quad \text{---- (12)}$$

Substitute $S_{12} = 0$ in equation (11)

$\Gamma_{out} = S_{22}$

Substitute these values in equation (9)

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_s|^2)}{|(1-\Gamma_{out}\Gamma_L)|^2 |1-S_{11}\Gamma_S|^2} \quad \text{---- (13)}$$

The transducer power gain is called unilateral power gain G_{TU} which neglects the feedback effect of amplifier (i.e.) $S_{12} = 0$.

This simplifies,

$$G_{TU} = \frac{(1-|\Gamma_L|^2)(1-|\Gamma_s|^2)|S_{21}|^2}{|(1-\Gamma_L S_{22})|^2 |1-S_{11}\Gamma_S|^2} \quad \text{------ (14)}$$

The above equation is often used as a basis to develop approximate designs for an amplifier and its input and output matching networks.

Additional Power Relation

The transducer power gain is a fundamental expression from which additional important power relations can be derived.

For instance, the available power gain for load side matching ($\Gamma L = \Gamma out^*$) is defined as,

$$G_A = \frac{G_T}{\Gamma L} = \Gamma_{out}^{*}$$

$$G_A = \frac{\text{Power available from the amplifier}}{\text{Power available from the source}}$$

Substitute the condition in equation (13)

$$G_A = \frac{|S_{21}|^2 (1-|\Gamma out^*|^2)(1-|\Gamma s|^2)}{|1-\Gamma out \Gamma out^*|^2 |1-S_{11}\Gamma S|^2}$$

$$= \frac{|S_{21}|^2 (1-|\Gamma out|^2)(1-|\Gamma s|^2)}{|1-|\Gamma out|^2)^2 |1-S_{11}\Gamma S|^2}$$

$$G_A = \frac{|S_{21}|^2 (1-|\Gamma s|^2)}{|1-|\Gamma out|^2 |1-S_{11}\Gamma S|^2} \qquad \text{----- (15)}$$

Further, the power, gain is defined as the ratio of the power delivered to the load to the power supplied to the amplifier.

$$G = \frac{\text{Power delivered to the load}}{\text{Power supplied to the amplifier}}$$

$$G = \frac{P_L}{P_{in}}$$

Multiplying and Divide by 'P_A' in the above equation

$$G = \frac{P_L}{P_A} \frac{P_A}{P_{in}}$$

$$G = G_T \frac{P_A}{P_{in}}$$

Substitute equation (3), (4) in equation (12),

$$G = \frac{(1-|\Gamma L|^2)\ |S_{21}|^2\ (1-|\Gamma S|^2)}{|1-\Gamma S\Gamma in|^2 |1-S_{22}\Gamma L|^2} \times \frac{1}{2} \frac{|b_s|^2\ 2\ |1-\Gamma in\Gamma S|^2}{1-|\Gamma S|^2\ |b_s|^2 (1-|\Gamma in|^2)}$$

$$G = \frac{(1-|\Gamma L|^2)\ |S_{21}|^2}{(1-|\Gamma in|)^2\ |1-S_{22}\Gamma L|^2} \quad \text{---- (16)}$$

This example goes through the computation of some of these expressions for an amplifier with given S-parameters.

2.3 Stability Considerations

Stability Circles

One of the first requirements that an amplifier circuit is a stable performance in the frequency range. This is a particular concern when dealing with RF circuits, which tend to oscillate depending on operating frequency and termination on operating frequency and termination.

If $|\Gamma 0| > 1$, then the return voltage increases in magnitude (positive feedback) causing instability. $|\Gamma 0| < 1$ causes a diminished return voltage wave (negative feedback.)

The amplifier as a two-port network, characterized through its S-parameters and external terminations described by ΓL and Γs.

Stability then implies that the magnitudes of the reflection coefficients are less than unity.

$$|\Gamma L| < 1, \ |\Gamma s| < 1 \quad \text{----- (1)}$$

$$|\Gamma in| = \left|\frac{S_{11} - \Gamma L \Delta}{1 - S_{22}\Gamma L}\right| < 1 \quad \text{----- (2)}$$

$$|\Gamma out| = \left|\frac{S_{22} - \Gamma S \Delta}{1 - S_{11}\Gamma S}\right| < 1 \quad \text{----- (3)}$$

Where $\Delta = S_{11} S_{22} - S_{12} S_{21}$

Since the S-parameters are fixed for a particular frequency, the only factors that have a parametric effect on the stability are ΓL and ΓS.

The complex quantities are,

$$\left.\begin{array}{l} S_{11} = S_{11}^R + j S_{11}^I \ ; \ S_{22} = S_{22}^R + j S_{22}^I \\ \Delta = \Delta^R + j \Delta^I \ ; \ \Gamma L = \Gamma L^R + j \Gamma L^I \end{array}\right\} \quad \text{----- (4)}$$

Substitute this complex quantities in equation (2), resulting after some algebra in the output stability circle equation.

$$(|L^R - C_{out}^R)^2 + (\Gamma L^I - C_{out}^R)2 = r^2_{out} \quad \text{------ (5)}$$

Where the circle radius is given by,

$$r_{out} = \frac{|S_{12}S_{21}|}{||S_{22}|^2 - |\Delta|^2|} \quad \text{----- (6)}$$

and the center of this circle is located at,

$$C_{out} = C_{out}^R + j\ C_{out}^I = \frac{(S_{22} - S_{11}^* \Delta)^*}{||S_{22}|^2 - |\Delta|^2|} \quad \text{----- (7)}$$

Substitute the complex quantities (4) in equation (3), resulting after some algebra in the input stability circle equation

$$(\Gamma s^R - C_{in}^R)^2 - (\Gamma s^I - C_{in}^I)^2 = r_{in}^2 \quad \text{------ (8)}$$

$$\text{Where } r_{in} = \frac{|S_{12}S_{21}|}{||S_{11}| - |\Delta|^2|} \quad \text{----- (9)}$$

and the centre of this circle is located at,

$$C_{in} = C_{in}^R + j\,C_{in}^I = \frac{(S_{11} - S_{22}^* \Delta)^*}{|S_{11}|^2 - |\Delta|^2} \quad \text{---- (10)}$$

The response of the stability circle is shown below,

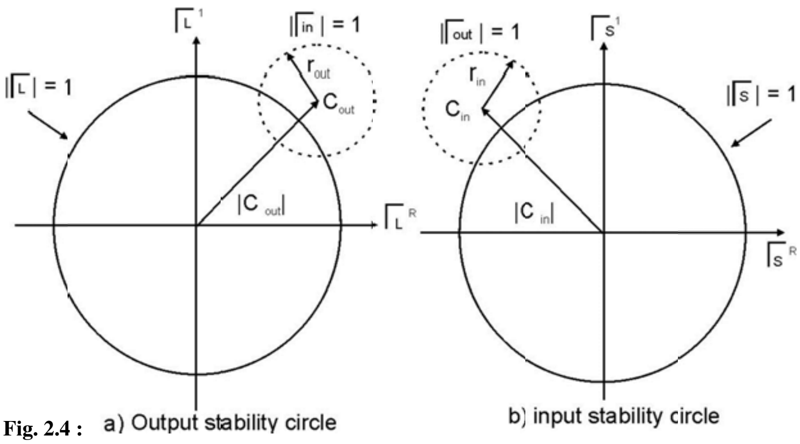

Fig. 2.4 :

If $\Gamma L = 0$, then $|\Gamma in| = |S_{11}|$ and two cases have to be differentiated depending on $|S_{11}| < 1$ or $|S_{11}| > 1$.

Unconditional stability

Unconditional stability refers to the situation where the amplifier remains stable throughout the entire domain of the smith chart at the selected frequency and bias conditions. This applies to both the input and utput ports. For $|S_{11}| < 1$ and $|S_{22}| < 1$ it is stated as,

$$||c_{in}| - r_{in}| > 1 \qquad \text{----- (11)}$$

$$||c_{out}| - r_{out}| > 1 \qquad \text{----- (12)}$$

The stability circles have to reside completely outside the $|\Gamma_s| = 1$ and $|\Gamma_L| = 1$ circles, which is shown in figure (2.5a).

The stability factor (or) Rollett factor (k) is given by,

$$k = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12}|\,|S_{21}|} > 1 \qquad \text{---- (13)}$$

Alternatively, unconditional stability can also be viewed in terms of the Γ_s behavior in the complex $\Gamma_{out} = \Gamma_{out}^R + j\,\Gamma_{out}^I$ plane. Here $|\Gamma_s| \leq 1$ domain must reside completely within the $|\Gamma_{out}| = 1$ circle which is shown in figure (2.5b).

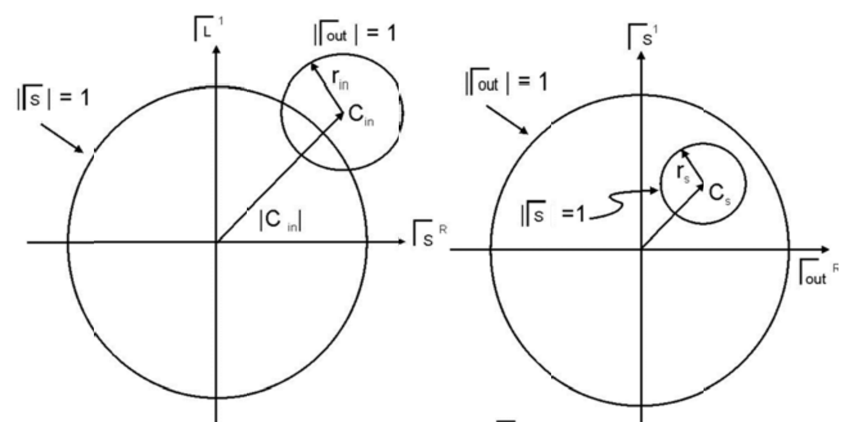

Fig. 2.5 : $|\Gamma_{out}|$ circle must reside outside $|\Gamma_s| = 1$ circle must reside inside

Plotting $|\Gamma_s| = 1$ in the Γ_{out} plane produces a circle whose center is located at

$$C_s = S_{22} + \frac{S_{12}\,S_{21}\,S_{11}^*}{1-|S_{11}|^2} \qquad \text{----- (14)}$$

and the radius of circle is,

$$r_S = \frac{|S_{12} S_{21}|}{1 - |S_{11}|^2} \qquad \text{----- (15)}$$

Stabilization Methods

If the operation of a FET (or) BJT is found to be unstable in the desired frequency range, can be made to stabilize the transistor.

$|\Gamma_{in}| > 1$ and $|\Gamma_{out}| > 1$ can be written in terms of input and output impedances.

$$|\Gamma_{in}| = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right| > 1$$

$$|\Gamma_{out}| = \left| \frac{Z_{out} - Z_0}{Z_{out} + Z_0} \right| > 1$$

Which imply Re $\{Z_{in}\} < 0$ and Re $\{Z_{out}\} < 0$.

One way to stabilize the active device is to add a series resistance (or) shunt resistance to the port.

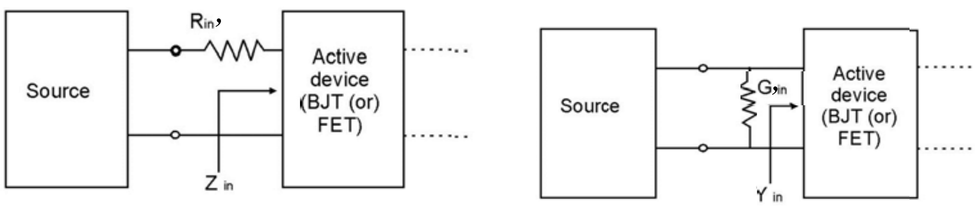

Fig. 2.6: a) Series resistance b) Shurt Conductor

Above figure shows the stabilization configuration of the input port, we require

Re $\{Z_{in} + R_{in}' + Z_S\} > 0$ (or)

Re $\{Y_{in} + G_{in}' + Y_S\} > 0$

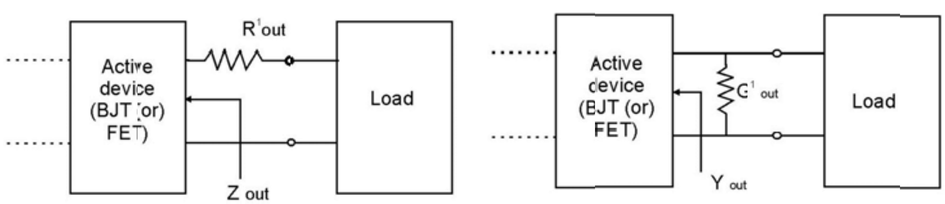

Fig. 2.7 : a) Series resistance b) Shurt resistance

Above figure shows the stabilization configuration of output port.

The condition is,

Re {Zout + R'out + Z_L } > 0 or

Re { Yout + G'out + Y_L } > 0

Stabilization through the addition of resistors comes at high expensive, impedance matching can suffer, increase the loss in power flow and increasing the noise figure.

2.4 Gain consideration

Constant Gain

Generally gain is defined as the ratio of output power to input power

$$\text{Gain} = \frac{\text{Output power}}{\text{Input power}}$$

Unilateral Design

Besides ensuring stability, there is need to obtain a desired gain performance is another important consideration in the amplifier design.

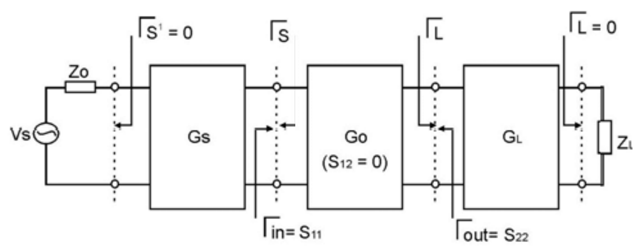

Fig. 2.8 : Unilateral power gain System arrangement.

The influence of the transistor's feedback is neglected (S_{12} = 0). We can employ the unilateral power gain described by G_{TU}, is rewritten such that the individual contributions of the matching network.

$$G_{TU} = \frac{1-|\Gamma_s|^2}{|1- S_{11}\Gamma_s|^2} \; |S_{21}|^2 \; \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2} \qquad \text{----- (1)}$$

$$G_{TU} = G_S \; G_O \; G_L \qquad \text{----- (2)}$$

Where G_S = Gain associated with input matching network

$$G_S = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2} \qquad \text{----- (3)}$$

G_L = Gain associated with output matching network

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \qquad \text{----- (4)}$$

Go = Insertion gain of transistor

$$G_O = |S_{21}|^2 \qquad \text{----- (5)}$$

Equation (2) can be expressed interms of dB,

$$G_{TU}\,(dB) = G_S\,(dB) + G_O\,(dB) + G_L\,(dB) \qquad \text{----- (6)}$$

From equation (4), the network gain can be greater than unity, since they do not contain any active devices. For this reason, without any matching a significant power loss can occur at the input and output sides of the amplifier. G_S and G_L are used to reduce these losses.

If $|S_{11}|$ and $|S_{22}|$ are less than unity, the maximum unilateral power gain G_{TUmax} results when both input and output are matched.

(i.e.) $\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$

Maximum gain associated with input matching network is,

$$G_{Smax} = \frac{\overline{1-|S_{11}^*|^2}}{|1-S_{11}S_{11}^*|^2} = \frac{\overline{1-|S_{11}|^2}}{|1-|S_{11}|^2|^2}$$

$$G_{Smax} = \frac{1}{1-|S_{11}|^2} \qquad \text{----- (7)}$$

$$G_{Lmax} = \frac{1}{1-|S_{22}|^2} \qquad \text{----- (8)}$$

Normalized gain associated with input matching network is,

$$g_s = \frac{G_S}{G_{smax}} = \frac{1-|\Gamma_s|^2}{|1-|S_{11}\Gamma_s|^2}\; 1 - |S_{11}|^2 \qquad \text{----- (9)}$$

$$g_L = \frac{G_L}{G_{Lmax}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}\; 1 - |S_{22}|^2 \qquad \text{----- (10)}$$

Normalized gain is given in both cases as

$0 \leq g_i \leq 1$ with i = S, L

Normalized gain is expressed as,

$$g_i = \frac{1-|\Gamma_i|^2}{|1-|S_{ii}\Gamma_i|^2}(1-|S_{ii}|^2) \qquad \text{----- (11)}$$

Here ii = 11, 22 depending on i = S, L

The result is a set of circles with centre locations at,

$$d_{gi} = \frac{g_i S_{ii}^*}{1-|S_{ii}|^2(1-g_i)} \qquad \text{----- (12)}$$

radious of size is given by,

$$r_{gi} = \frac{\sqrt{1-g_i}\,(1-|S_{11}|^2)}{1-|S_{ii}|^2(1-g_i)} \qquad \text{----- (13)}$$

The equation (12) and (13) are the unilateral constant gain equation.

Unilateral Figure of Merit

The unilateral design approach involves the approximation that the feedback effect (or) reverse gain of the amplifier is negligible (i.e.) $S_{12} = 0$.

To estimate the error due to this assumption, the ratio between transducer gain (G_T) and transducer gain (G_{TU}) is formed.

$$\frac{G_T}{G_{TU}} = \frac{1}{\left|1-\frac{S_{12}S_{21}\Gamma_L\Gamma_s}{(1-S_{11}\Gamma_s)(1-S_{22}\Gamma_L)}\right|^2} \quad \text{Where } G_T < G_{TU} \qquad \text{---- (14)}$$

The maximum error is obtained for the input and output matching conditions $\Gamma_s = S_{11}^*$, $\Gamma_L = S_{22}^*$

$$\frac{G_T}{G_{TU\max}} = \frac{1}{\left|1-\frac{S_{12}S_{21}S_{11}^*S_{22}^*}{(1-S_{11}S_{11}^*)(1-S_{22}S_{22}^*)}\right|^2} \qquad \text{---- (15)}$$

$$\frac{G_T}{G_{TU\max}} = \frac{1}{\left|1-\frac{S_{12}S_{21}S_{11}^*S_{22}^*}{(1-|S_{11}|^2)(1-|S_{22}|^2)}\right|^2} \qquad \text{--- (16)}$$

This can be used to set bounds on the error fluctuation.

$$(1+U)^{-2} \leq \frac{G_T}{G_{TU}} \leq (1-U)^{-2} \quad \text{---- (17)}$$

Where U → frequency dependent unilateral figure of merit.

$$U = \frac{|S_{12}||S_{21}||S_{22}||S_{11}|}{(1-|S_{11}|^2(1-|S_{22}|^2)} \quad \text{---- (18)}$$

To justify a unilateral amplifier design approach this figure of merit should be as small as possible. In the limit G_T approaches G_{TU} for the ideal case of $S_{12} = 0$, so the error is vanished (i.e.) U = 0.

Bilateral Design

For many practical situations the unilateral approach may not be appropriate because the error committed by setting $S_{12} = 0$. So the bilateral design approach is used.

Instead of unilateral matching $\Gamma S^* = S_{11}$ and $\Gamma L^* = S_{22}$, it deals with the complete equations for the input and output reflection coefficients.

$$\Gamma s^* = S_{11} + \frac{S_{12} S_{21} \Gamma L}{1-S_{22}\Gamma L} = \frac{S_{11}-\Gamma L\,\Delta}{1-S_{22}\Gamma L} \quad \text{----- (19)}$$

$$\Gamma L^* = S_{22} + \frac{S_{12} S_{21} \Gamma s}{1-S_{11}\Gamma s} = \frac{S_{22}-\Gamma s\,\Delta}{1-S_{11}\Gamma s} \quad \text{----- (20)}$$

Where $\Delta = S_{11} S_{22} - S_{12} S_{21}$

Which require simultaneous conjugate match that means matched source and load reflection coefficients ΓM_S & ΓM_L.

$$\Gamma M_S = \frac{B_1}{2C_1} - \frac{1}{2}\sqrt{\left(\frac{B_1}{C_1}\right)^2 - \frac{4C_1^*}{C_1}} \quad \text{------ (21)}$$

$$\text{Where} \left.\begin{array}{l} C_1 = S_{11} - S_{22}^*\Delta \\ B_1 = 1 - |S_{22}|^2 - |\Delta|^2 + |S_{11}|^2 \end{array}\right\} \quad \text{------ (22)}$$

$$\Gamma M_L = \frac{B_2}{2C_2} - \frac{1}{2}\sqrt{\left(\frac{B_2}{C_2}\right)^2 - \frac{4C_2^*}{C_2}} \quad \text{------ (23)}$$

$$\text{Where} \quad \left.\begin{aligned} C_2 &= S_{22} - S_{11}^{*}\Delta \\ B_2 &= 1 - |S_{11}|^2 - |\Delta|^2 + |S_{22}|^2 \end{aligned}\right\} \qquad \text{------- (24)}$$

The optimal matching equation (21) & (23) is given by,

$$\Gamma M_S^{*} = S_{11} + \frac{S_{12}\, S_{21}\, \Gamma M_L}{1 - S_{22}\Gamma M_L}$$

$$\Gamma M_L^{*} = S_{22} + \frac{S_{12}\, S_{21}\, \Gamma M_S}{1 - S_{11}\Gamma M_S}$$

From this it is noted that the unilateral approach which decouples input and output ports is a subset of the bilateral design approach.

Operating and Available Power Gain Circles

For the situation where the reverse gain of S_{12} cannot be neglected, the input impedance is dependent on the load reflection coefficient. The output impedance becomes the function of the source reflection coefficient.

In bilateral case, which takes the mutual coupling between input and output ports there are 2 alternative methods are to develop amplifiers with a specified gain.

The first method is based on the use of operating power gain G. To find the load reflection coefficient ΓL, assume the source is complex conjugate matched to the input reflection coefficient ie., $\Gamma s = \Gamma in^{*}$. If the input voltage standing wave ratio is unity, the first method is preferable.

The second method is based on the use of available power gain G_A. In this case, we assume the load is complex conjugate matched to the output reflection coefficient. (i.e.) $\Gamma L = \Gamma out^{*}$. If the output standing wave ratio is unity, the second method is preferable.

Operating Power Gain

Operating power gain is given by,

$$G = \frac{(1 - |\Gamma L|^2)\, |S_{21}|^2}{(1 - |\Gamma in|^2\, |1 - S_{22}\Gamma L|^2} \qquad [\therefore \Gamma in = S_{11} + \frac{S_{22}\, S_{12}\, \Gamma L}{1\, S_{22}\, |\, L}]$$

$$= \frac{(1-|\Gamma_L|^2)\,|S_{21}|^2}{\left(1-\left|S_{11}+\frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}\right|^2\right)|1-S_{22}\Gamma_L|^2}$$

$$G = g_o|S_{21}|^2 \qquad \text{------ (1)}$$

Where,

$$g_o = \frac{1-|\Gamma_L|^2}{\left(1-\left|S_{11}+\frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}\right|^2\right)|1-S_{22}\Gamma_L|^2}$$

The circle equation for load reflection coefficient ΓL; that is,

$$|\Gamma L - d_{go}| = r_{go} \qquad \text{----- (2)}$$

Where the center position d_{go} is,

$$d_{go} = \frac{g_0\,(S_{22}-\Delta\,\Omega^*_1)}{1+g_0\,(|S_{22}|^2-|\Delta|^2} \qquad \text{------ (3)}$$

The radius r_{g0} is defined as,

$$r_{go} = \frac{\sqrt{1-2\,k g_0\,|S_{12}S_{21}|+g_0^2|S_{12}S_{21}|^2}}{|1+g_0(|S_{22}|^2-|\Delta|^2)|} \qquad \text{----- (4)}$$

where,

k → Roulette stability factor

Constant gain circle in ΓL plane into a circle in Γs plane, i.e.,

$$|\Gamma s - d_{gS}| = r_{gS} \qquad \text{----- (5)}$$

Circle radius r_{gs} and its center d_{gs} are obtained from the requirement that $\Gamma s = \Gamma in^*$. This can be written as,

$$\Gamma s^* = \frac{S_{11}-\Delta\Gamma L}{1-S_{22}\Gamma L} \qquad \text{----- (6)}$$

$$\Gamma L = \frac{S_{11}-\Gamma S^*}{\Delta - S_{22}\Gamma S^*} \qquad \text{----- (7)}$$

Substitute (7) in (2)

$$\left| \frac{S_{11} - \Gamma_S^*}{\Delta - S_{22}\Gamma_S^*} - d_{go} \right| = r_{go} \qquad \text{----- (8)}$$

The circle radius is,

$$r_{gS} = \frac{r_{go}\,|S_{12}S_{21}|}{\left|\,|1 - S_{22}d_{g0}|^2 - r_{g0}^2|S_{22}|^2\right|} \qquad \text{----- (9)}$$

Center is given by,

$$d_{gS} = \frac{(1 - S_{22}d_{g0})(S_{11} - \Delta d_{g0})^* - r_{go}^2\Delta^* S_{22}}{|1 - S_{22}d_{g0}|^2 - r_{go}^2|S_{22}|^2|} \qquad \text{---- (10)}$$

Available Power Gain

Circle equation which relates the source reflection coefficient to the desired gain.

$|\Gamma_S - d_{ga}| = r_{ga}$

Where the center position d_{ga} is,

$$d_{ga} = \frac{g_a(S_{11} - \Delta S_{22}^*)^*}{1 + g_a(|S_{11}|^2 - |\Delta|^2)} \qquad \text{---- (11)}$$

And the radius is defind as,

$$r_{ga} = \frac{\sqrt{1 - 2kg_a|S_{21}S_{12}| + g_a^2|S_{12}S_{21}|^2}}{|1 + g_a(|S_{11}|^2 - |\Delta|^2)|} \qquad \text{---- (12)}$$

The proportionality factor g_a is given by,

$$g_a = \frac{G_A}{|S_{21}|^2}$$

Where G_A is desired power level.

The constant available power circle in Γ_L plane,

$|\Gamma_L - d_{g1}| = r_{g1}$. ----- (13)

The circle radius is given by,

$$r_{g1} = \frac{rg_a \,|\, S_{12} S_{21}|}{|\,|1 - S_{11}\, dg_a\,|^2 - r_{ga}^2 |\, S_{11}\,|^2\,|} \qquad \text{----- (14)}$$

The center location is given by,

$$d_{g1} = \frac{(1 - S_{11}\; dg_a)\,(S_{22} - \Delta\, dg_a)^* - r_{ga}^2 \Delta^* S_{11}}{|\,1 - S_{11\,dga}\,|^2 - r_{ga}^2 |\; S_{11}\,|^2}$$

2.5 Noise Figure Circles

In many RF amplifiers, the need for signal amplification at low noise level becomes an essential system requirement. Unfortunately, designing a low noise amplifier competes with such factor as stability and gain.

For instance, a minimum noise performance at maximum gain cannot be obtained. It is important to develop a method that allows us to display the influence of noise as part of the smith chart and observe trade-offs between gain and stability.

The key ingredient of a noise analysis is the noise figure of a two-port amplifier in the admittance form.

$$F = F_{min} + \frac{R_n}{G_S} \,|\, Y_S - Y_{opt}\,|^2 \quad \text{(or)}$$

$$F = F_{min} + \frac{G_n}{R_S} \,|\, Z_S - Z_{opt}\,|^2$$

Where $Z_s = \frac{1}{Y_S}$ is the source impedance.

When using transistors, four noise parameters are known either through datasheets from the FET or BJT manufacturers. They are,

- The minimum noise figure F_{min} whose behavior depends on biasing condition and operating frequency. If the device were noise free $F_{min} = 1$.
- The equivalent noise resistance
  $R_n = \frac{1}{G_n}$ of the device.

The optimum source admittance $Y_{opt} = G_{opt} + j\, B_{opt} = \frac{1}{Z_{opt}}$. Instead of impedance or admittance, the optimum reflection coefficient Γ_{opt} is often listed.

The relationship between Y_{opt} and Γ_{opt} is given by

$$Y_{opt} = Y_0 \ \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

Since the S Parameter representation is suitable for high frequency.

$$Y_S = Y_0 \ \frac{1-\Gamma_s}{1+\Gamma_s}$$

G_S can be written as $G_S \ = \ Y_0 \ \dfrac{(1-|\Gamma_s|^2)}{|1+\Gamma_s|^2}$

$$F = F_{min} + \frac{R_n}{\dfrac{Y_0(1-|\Gamma_s|^2)}{|1+\Gamma_s|^2}} \left| \frac{Y_0(1-\Gamma_s)}{(1+\Gamma_s)} - Y_0 \frac{(1-\Gamma_{opt})}{(1+\Gamma_{opt})} \right|^2$$

$$F = F_{min} + \frac{R_n}{\dfrac{Y_0(1-|\Gamma_s|^2)}{|1+\Gamma_s|^2}} Y_0^2 \left| \frac{Y_0(1-\Gamma_s)}{(1+\Gamma_s)} \ \frac{(1-\Gamma_{opt})}{(1+\Gamma_{opt})} \right|^2$$

$$F = F_{min} + \frac{R_n}{\dfrac{(1-|\Gamma_s|^2)}{|1+\Gamma_s|^2}} Y_0 \left| \frac{(1+\Gamma_s)(1+\Gamma_{opt}) - (1-\Gamma_{opt})(1+\Gamma_s)}{(1+\Gamma_s)(1+\Gamma_{opt})} \right|^2$$

$$F = F_{min} + \frac{R_n}{\dfrac{(1-|\Gamma_s|^2)}{|1+\Gamma_s|^2}} Y_0 \left| \frac{1+\Gamma_{opt}-\Gamma_s-\Gamma_s\Gamma_{opt}-1-\Gamma_s+\Gamma_{opt}+\Gamma_{opt}\Gamma_s}{(1+\Gamma_s)^2(1+\Gamma_{opt})^2} \right|^2$$

$$F = F_{min} + \frac{R_n F_0}{1-|\Gamma_s|^2} \ \frac{|2\Gamma_{opt}-2\Gamma_s|^2}{(1+\Gamma_{opt})^2}$$

$$F = F_{min} + \frac{4 R_n F_0}{1-|\Gamma_s|^2} \ \frac{|\Gamma_{opt}-\Gamma_s|^2}{(1+\Gamma_{opt})^2}$$

$$F = F_{min} + \frac{4 R_n}{Z_0(1-|\Gamma_s|^2)} \ \frac{|\Gamma_s-\Gamma_{opt}|^2}{(1+\Gamma_{opt})^2}$$

For $\Gamma_s = \Gamma_{opt}$ lowest possible noise figure is achieved $F = F_{min}$.

$$\frac{4 R_n}{Z_0(1-|\Gamma_s|^2)} \ \frac{|\Gamma_s-\Gamma_{opt}|^2}{(1+\Gamma_{opt})^2} = F - F_{min}$$

$$\frac{|\Gamma_s - \Gamma_{opt}|^2}{(1+\Gamma_{opt})^2} = \frac{(F-F_{min})(1-|\Gamma_s|^2)}{\frac{4R_n}{Z_O}}$$

$$|\Gamma_s - \Gamma_{opt}|^2 = \frac{(F-F_{min})(1-|\Gamma_s|^2)(1+\Gamma_{opt})^2}{\frac{4R_n}{Z_O}}$$

$$|\Gamma_s - \Gamma_{opt}|^2 = Q_K(1 - |\Gamma_s|^2)$$

Where,

$$Q_K = |1+\Gamma_{opt}|^2 \left(\frac{F - F_{min}}{\frac{4R_n}{Z_o}}\right)$$

Let $F = F_K$

$$Q_K = |1+\Gamma_{opt}|^2 \left(\frac{F_K - F_{min}}{\frac{4R_n}{Z_o}}\right)$$

$$|\Gamma_s - \Gamma_{opt}|^2 = Q_K - Q_K|\Gamma_s|^2$$

$$\Gamma_s^2 - 2\Gamma_s\Gamma_{opt} + \Gamma_{opt}^2 = Q_K - Q_K|\Gamma_s|^2$$

$$\Gamma_s^2 - 2\Gamma_s\Gamma_{opt} + \Gamma_{opt}^2 + Q_K|\Gamma_s|^2 = Q_K$$

$$|\Gamma_s|^2[1 + Q_K] - 2\Gamma_s\Gamma_{opt} + |\Gamma_{opt}|^2 = Q_K$$

After some algebra

$$\left|\Gamma_s - \frac{\Gamma_{opt}}{1+Q_K}\right|^2 = \frac{Q_K^2 + Q_K(1 - |\Gamma_{opt}|^2)}{(1+Q_K)^2}$$

This is the circle equation in standard form that can be displayed as part of the smith chart.

$$|\Gamma_s - d_{FK}|^2 = (\Gamma_S^R - d_{FK}^R)^2 + (\Gamma_S^I - d_{FK}^I)^2 = r_{FK}^2$$

The circle centre location d_{FK} denoted by the complex number.

$$d_{FK} = d_{FK}^R + j\,d_{FK}^I = \frac{\Gamma_{opt}}{1+Q_K}$$

The associated radius

$$r_{FK}^2 = \left| \Gamma_s - \frac{\Gamma opt}{1 + Q_K} \right|^2$$

$$r_{FK}^2 = \frac{Q_K^2 + Q_K (1 - |\Gamma opt|^2}{(1 + Q_K)^2}$$

$$r_{FK} = \frac{\sqrt{Q_K^2 + Q_K (1 - |\Gamma opt|^2)}}{1 + Q_K}$$

- The minimum noise figure is obtained for $F_K = F_{min}$, which coincides with the location $d_{FK} = \Gamma opt$ and radius $r_{FK} = 0$.
- All constant noise circles have their centres located along a line drawn from the origin to point Γopt.

2.6 Impedance Matching Using Discre e Components

Two-Component Matching Networks

To analyze and design the simplest possible type of matching networks, called two-component networks also known as L – sections due to their element arrangement.

These networks use two reactive components to transform the load impedance (Z_L) to the desired input impedance (Z_{in}). In conjunction with the load and source impedances, the components are alternatively connected in series and shunt configuration shown below, which depicts eight possible arrangements of capacitor and inductors.

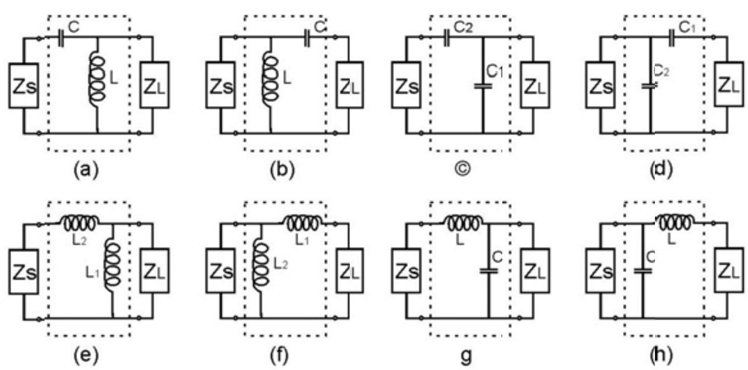

Fig. 2.9 : Eight possible configurations of the discrete two-component matching networks

In designing a matched network, two approaches,

i) To derive the values of the elements analytically
ii) To rely on the smith chart as a graphical design tool.

The first approach yields very precise results and is suitable for computer synthesis. Alternatively, the second approach is more intuitive, easier to verify and faster for an initial design. Since it does not require complicated computations.

Instead of the method, we can use the smith chart for rapid and relatively precise designs of the matching circuits. The appeal of this approach is that its complexity remains almost the same independent of the number of components in the network.

The generic solution procedure for optimal power transfer includes the following steps,

i) Find the normalized source and load impedances.
ii) In the smith chart plot circles of constant resistance and conductance that pass through the point denoting the source impedance.
iii) Plot circles of constant resistance and conductance that pass through the point of the complex conjugate of load impedance.
iv) Identify the intersection points between the circles in steps ii & iii. The number of intersection points determines the number of possible L-section matching networks.
v) Find the values of the normalized reactance and susceptances of the inductors and capacitors.
vi) Determine the actual values of inductors and capacitors for a given frequency.

Frequency Response and Quality Factor

The frequency responses of these two matched networks in terms of input reflection coefficient.

$$\Gamma_{in} = \frac{Z_{in} - Z_S}{Z_{in} + Z_S}$$

and the transfer function $H = \frac{V_{out}}{V_S}$

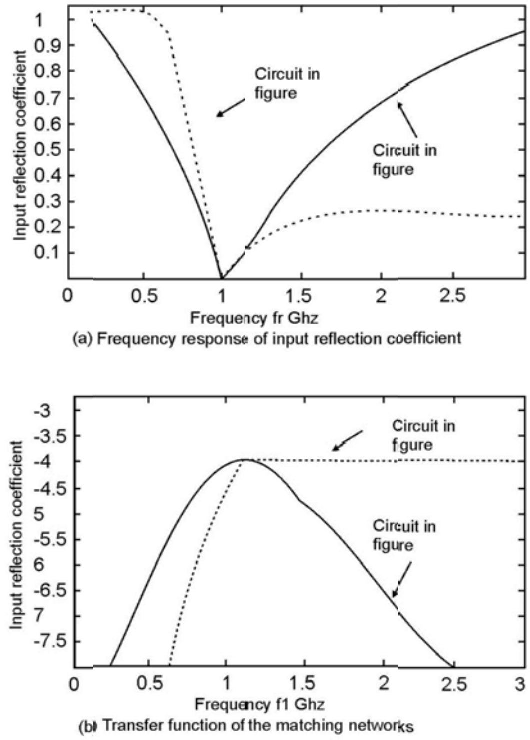

Fig. 2.10 : Frequency response of the two matching network realizations.

In both networks exhibit perfect matching only at a particular frequency $f_0 = 1$ GH_Z and begin to deviate quickly when moving away from f_0. The networks may be described by a loaded quality factor Q_L, which is equal to the ratio of the resonance frequency f_0' over the 3 dB bandwidth (BW).

$$Q_L = \frac{f_0}{BW}$$

Where bo h 'f_0' and BW are expressed in Hz. For frequencies close to f_0 the matching network can be redrawn as a band pass filt r with a loaded quality factor.

The impedance transformation move from one node of the circuit to another. At each node of the matching network, the impedance can be expressed interms of an equivalent series impedance,

$Z_S = R_S + j X_S$ (or)

Admittance $Y_P = G_P + j B_P$

Hence at each node, we can find Q_n as the ratio of the absolute value of the reactance 'X_S' to the corresponding resistance R_S.

$$Q_n = \frac{|X_S|}{R_S}$$

(or) as the ratio of the absolute value of susceptance B_P to the conductance G_P,

$$Q_n = \frac{|B_P|}{G_P}$$

To relate the nodal quality factor Q_n to Q_L and find

$$Q_L = \frac{Q_n}{2}$$

This result is true for any L-type matching network is usually estimated as simply the maximum nodal quality factor.

To simplify the matching network design process even further, we draw constant – Q_n contours in the smith chart. The below figure 2.11 shows such contours for Q_a valued 0.3, 1, 3, and 10.

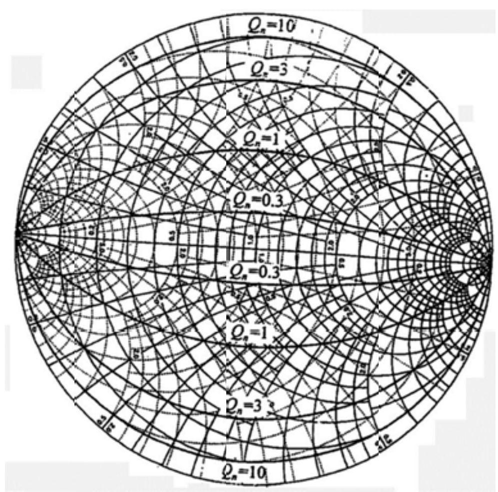

Fig. 2.11 : Constant Qn contours displayed in the Smith chart.

The normalized impedance can be written as,

$$Z = r + jx = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} + j \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Thus the nodal quality factor can be written as,

$$Qn = \frac{|x|}{r} = \frac{2|\Gamma_i|}{1 - \Gamma_r^2 - \Gamma_i^2}$$

Rearranging the above equation the circle equation is found in the form.

$$\Gamma i^2 + (\Gamma r \pm \frac{1}{Qn})^2 = 1 + \frac{1}{Qn^2}$$

Where the '+' sign is taken for positive reactance x, and the '-' sign for negative reactance x.

With these constant Q_n circles in the smith ch rt it is possible to find the loaded quality factor of an L-type matching network by simply reading the corresponding Q_n and dividing it by 2.

In Many ractical applications the quality factor of a matching network is of importance. The L-type matching networks provide no control over the value of Q_n and we must either accept (or) reject the resulting quality factor. By choosing the values of Q, that affect the bandwidth behavior of the circuit. Thus we introduce a third element in the matching network. The addition of this third element results in either the 'T' (or) Pi (∏) network.

2.8 T and ∏ Matching Networks

The loaded quality factor of the matching network can be estimated from the maximum nodal Qn. The addition of the third element into the matching network produces an additional node in the circuit allows us to control the value of Q_L by choosing an appropriate impedance at that node. The design of T and pi – type matching networks is shown below.

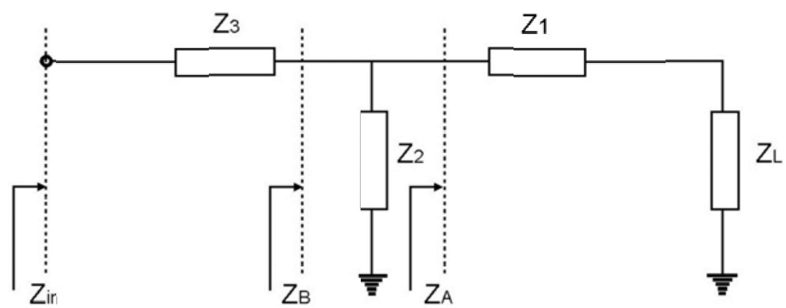

Fig. 2.12 : General topology of a T-type matching network

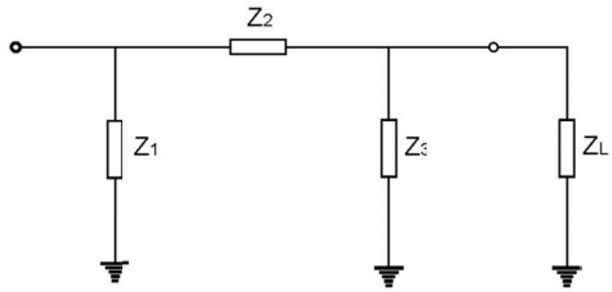

Fig. 2.13 : General topology of a π - type matching network

The design of a pi-type matching network is developed with the intent to achieve a minimum nodal quality factor. A low quality factor design directly translates into a wider bandwidth of the network as required in broadband FET and BJT amplifiers.

2.9 Microstrip Line Matching Networks

The design of matching networks involving discrete components. However with increasing frequency and correspondingly reduced wavelength.

Micro strip lines are used extensively to interconnect high speed logic circuits in digital computers. Such several interconnect makes a network and that is called micro strip line network. It is also used as an alternative lumped elements and distributed components.

Discrete Components to Micro strip Lines

In the mid GH_Z range, mixed approach by combining lumped and distributed elements. In this matched network contains a number of transmission lines connected in series and capacitors connected in parallel.

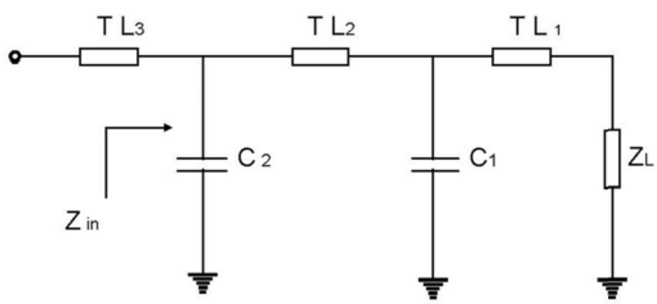

Fig. 2.14 : Mixed design cf matching network

Usually inductors are avoided in such designs, because they have a tendency to have high resistive losses than capacitors.

Generally only one shunt capacitor with two transmission lines connected in series on both sides to transform any given load impedance to input impedance.

It shows wide range of flexibility, when the capacitor value is changed and placed at different locations along the transmission lines. There are two types of networks,

1) Single stub matching network
2) Double stub matching network

1. Single stub Matching Network

Here the transmission from lumped to distributed elements network is the complete elimination of all lumped components. That can be constituted by open and/or short circuit stub lines.

Consider the matching networks that consist of a series transmission lines connected to a parallel open circuit (or) short circuit stub. Under this consideration there are two topologies.

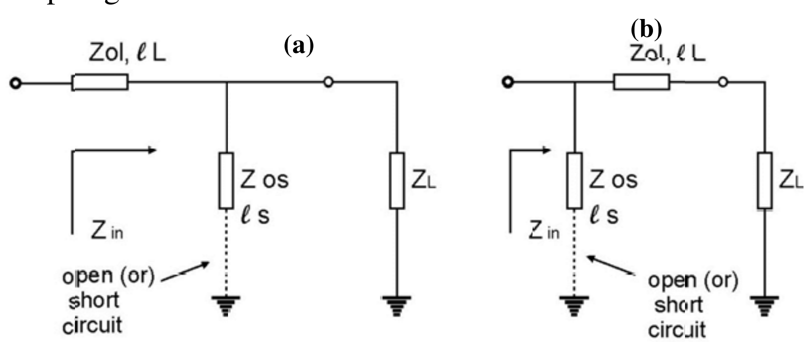

Fig. 2.15 : Two topologies of single - stub matching networks

For (a) :

Involves a series transmission line connected to the parallel combination of load and stub. The adjustable parameters are,

Ls → length of the stub

Zo*s* → characteristic impedance of the stub.

For (b) :

Involves a parallel stub connected to the series combination of load and transmission line. The adjustable parameters are,

l_L → length of the transmission line

Z_{ol} → Characteristic impedance of transmission line

2. Double stub matching networks

Double stub devices consists of two short circuited stub connected in parallel with a fixed length between them.

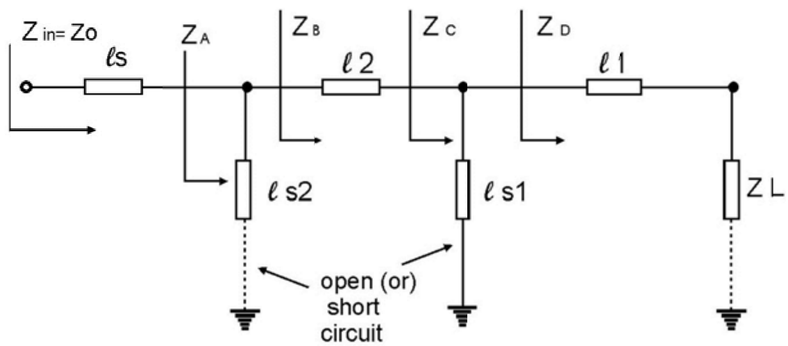

Fig. 2.16 : Double stub matching network arrangement

The length of the fixed section is usually one–eighth, three–eighth (or) five-eighth of wa elength. These are used to simplify the tuner construction.

Assume, the length of line segment between the two stubs is 'l_2'

$l_2 = \left(\frac{3}{8}\right)\lambda$

For a perfect match, it is required that $Z_{in} = Z_o$.

$Y_A = 1$

Since the lines are lossless, the normalized admittance,

$Y_B = Y_A - jbS_2$

Which is located som where on the constant conductance circle, g = 1 in smith chart.

Where bS_2 → susceptance of the stub

$l S_2 \rightarrow$ Associated length of the stub

For an $l_2 = (\frac{3}{8})\ \lambda$ lines the g = 1 circle is rotated by,

$2\beta l_2 = 3\ \frac{n}{2}$ radians (or) 270^0.

The admittance, Y_C needs to reside on this rotated circle g = 1 in order to ensure matching. In varying the length of $l s_1$ stub can transform point Y_D in such a way that the resulting Y_C is needed to located on the circle g = 1.

This procedure can be done for any load impedance Y_0 which is located inside the circle g = 2. This represents the forbidden region that has to be avoided.

To overcome this problem in practical applications, commercial double-stub tuners have input and output transmission lines, where the lengths are related according to $l_1 = l_3 + \frac{ß}{4}$.

Christo Ananth et al. [2] discussed about Improved Particle Swarm Optimization. The fuzzy filter based on particle swarm optimization is used to remove the high density image impulse noise, which occur during the transmission, data acquisition and processing. The proposed system has a fuzzy filter which has the parallel fuzzy inference mechanism, fuzzy mean process, and a fuzzy composition process. Christo Ananth et al.[3] presented a short overview on two port RF networks. They widely used microwave and RF applications and the denomination of frequency bands. The monograph start outs with an illustrative case on wave propagation which will introduce fundamental aspects of high frequency technology.

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Problems :

1) An RF amplifier has the following S- Parameter:

$S_{11} = 0.3 < -70^0$; $S_{21} = 3.5 < 85^0$

$S_{12} = 0.2 < -10^0$; $S_{22} = 0.4 < -45^0$

Furthermore, the input side of the amplifier is connected to a voltage source with $V_S = 5\ V < 0^0$ and source impedance $Z_S = 40\Omega$. The output is utilized to drive an antenna which has an impedance of $Z_L = 73\Omega$. Assuming that the S-Parameters of the amplifier are measured with reference to a $Z_0 = 50\Omega$ characteristic impedance. Find the following quantities.

a) Transducer gain GT, unilateral transfer gain GTU, available gain G_A, operating power gain G and

b) Power delivered to a load P_L, available power P_A and incident power to the amplifier Pinc.

Solution:

Source reflection coefficient $\Gamma s = \frac{Z_s - Z_O}{Z_s + Z_O}$

$= \frac{40-50}{40+50}$

$= -0.111$

Load reflection coefficient $\Gamma L = \frac{Z_L - Z_O}{Z_L + Z_O}$

$= \frac{73-50}{73+50}$

$= 0.187$

Input impedance $\Gamma in = S_{11} + \frac{S_{21} S_{12} \Gamma L}{1 - S_{22} \Gamma L}$

$S_{11} = 0.3 < - 70^0 = 0.102 - j\, 0.282$

$S_{21} = 3.5 < 85^0 = 0.305 + j\, 3.49$

$S_{12} = 0.2 < - 10^0 = 0.197 - j\, 0.035$

$S_{22} = 0.4 < - 45^0 = 0.283 - 0.283\, j$

$$\Gamma in = 0.103 - 0.282\, j + \frac{(0.305+3.4aj)(0.197-0.035j)(0.187)}{1-(0.283-0.283\, j)(0.187)}$$

$$= 0.103 - 0.282\, j + \frac{(0.182+0.677\, j)(0.187)}{1-(0.053-0.053\, j)}$$

$$= \frac{(0.947+0.053j)(0.103-0.282j)+(0.182+0.677j)(0.187)}{0.947+0.053\, j}$$

$$= \frac{0.112-0.262\, j+0.034+0.127\, j}{0.947+0.053\, j}$$

$$= \frac{0.146-0.135\,j}{0.947+0.053\,j}$$

$$\boxed{\Gamma_{in} = 0.146 - 0.151\,j}$$

Output impedance $\Gamma_{out} = S_{22} + \frac{S_{12}\,S_{21}\,\Gamma_S}{1-S_{11}\Gamma_S}$

$$\Gamma_{out} = 0.283 - 0.283\,j + \frac{(0.197-0.035\,j)0.305+3.49\,j)\,(-0.111)}{1-(0.103-0.282\,j)(-0.111)}$$

$$= 0.283 - 0.283\,j + \frac{(-0.0202-0.0751\,j)}{1-(-0.011+0.031\,j)}$$

$$= \frac{(1.011-0.031\,j)(0.283-0.283\,j)+(-0.0202-0.0751\,j)}{1.011-0.031\,j}$$

$$= \frac{0.\quad 7-0.295\,j-0.0202-0.0751\,j}{1.011-0.031\,j}$$

$$= \frac{0.257-0.37\,j}{1.011-0.031\,j}$$

$$\boxed{\Gamma_{out} = 0.265 - 0.358\,j}$$

$$G_T = \frac{(1-|\Gamma_L|^2\,|S_{21}|^2\,(1-|\Gamma_S|^2)}{|1-\Gamma_L\Gamma_{out}|^2\,|1-S_{11}\Gamma_S|^2}$$

$$= \frac{(1-|0.187|^2)\,|0.305+3.49\,j|^2\,(1-|-0.111|^2)}{|1-(0.187)(0.265-0.358\,j)|^2\,|1-(0.103-0.282\,j)(-0.111)|^2}$$

$$= \frac{0.965\,[f(0.305)^2+(3.49)^2]^2\,0.988}{[f(0.95)^2+(0.067)^2]^2 \times [f(1.0114)^2+(-0.031)^2]^2}$$

$$= \frac{11.701}{0.931}$$

$$\boxed{GT = 12.56 \text{ (or) } 10.98\text{ dB}}$$

$$G_{TU} = \frac{(1-|\Gamma_L|^2)\,|S_{21}|^2\,(1-|\Gamma_S|^2)}{|1-\Gamma_L S_{22}|^2\,|1-S_{11}\Gamma_S|^2}$$

$$= \frac{(1-|0.187|^2)10.305+3.49\,j|^2\,(1-|-0.111|^2)}{|1-(0.187)(0.283-0.283\,j)|^2\,|1-(0.103-0.282\,j)(-0.111)|^2}$$

$$= \frac{11.701}{0.920} = 12.71$$

$$\boxed{G_{TU} = 12.7 \text{ (or) } 11.04\text{ dB}}$$

$$G_A = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)}{|1-|\Gamma_{out}|^2||1-S_{11}\Gamma_S|^2}$$

$$= \frac{10.305+3.49\, j\,|^2 (1-(-0.111)^2)}{|1-|0.265-00.265-058\, j\,|^2||1-(0.103-0.282\, j)(-0.111)|^2}$$

$$= \frac{12.273 \times 0.988}{0.802 \times 1.024}$$

$$= \frac{12.126}{0.821}$$

$$G_A = 14.76$$

G_A = 14.76 (or) 11.69 dB

$$G = \frac{(1-|\Gamma_L|^2)|S_{21}|^2}{|1-|\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}$$

$$= \frac{(1-|0.187|^2)|0.305+3.49\, j\,|^2}{|1-|0.146-0.151\, j\,|^2||1-(0.283-0.283\, j)(0.187)|^2}$$

$$= \frac{0.965 \times 12.273}{0.956 \times 0.899}$$

$$= \frac{11.84}{0.859}$$

$$= 13.78$$

G = 13.78 (or) 11.39 dB

$$P_{inc} = \frac{|b_S|^2}{|1-\Gamma_{in}\Gamma_S|^2} \quad \text{where } b_S = \frac{\sqrt{Z_0}}{Z_S+Z_O} V_S = \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{Z_O}{(Z_S-Z_O)^2} \frac{|V_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}$$

$$= \frac{50 \times 5^2}{2(40+50)^2|1-(0.146-0.151\, j)(-0.111)|^2}$$

$$= \frac{1250}{16200 \times 1.033}$$

$$= 0.0747 \text{ w}$$

Pinc = 74.7 mω (or) 18.73 dbM

$$P_A = \frac{1}{2}\frac{|b_S|^2}{|1-\Gamma S|^2} = \frac{1}{2}\frac{Z_0}{(Z_S - Z_0)^2}\frac{|\sqrt{S}|^2}{(1-|\Gamma S)^2}$$

$$= \frac{1}{2}\frac{50 \times 5^2}{90^2|1-(-0.111)^2|}$$

$$= 0.0781$$

$$\boxed{\mathbf{P_A = 78.1\ m\omega\ (or)\ 18.92\ dBm}}$$

$P_L = P_A G_T$

$$= 78.1 \times 10^{-3} \times 12.56 = 0.9809$$

$$\boxed{\mathbf{P_l = 980.9\ m\omega\ (or)\ 29.91\ dbM}}$$

2. Investigate the stability regions of a transistor whose S – parameters are recorded as follows

$S_{11} = 0.7 < -70^0$; $S_{12} = 0.2 < -10^0$

$S_{21} = 5.5 < 85^0$; $S_{22} = 0.7 < -45^0$

Solution:

$S_{11} = 0.239 - 0.658\ j$

$S_{12} = 0.197 - 0.035\ j$

$S_{21} = 0.479 + 5.48\ j$

$S_{22} = 0.495 - 0.495\ j$

Compute values of K, $|\Delta|$, c_{in}, r_{in}, c_{out}, v_{out}

$$K = \frac{|-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|}$$

$$\Delta = S_{11} S_{22} - S_{12} S_{21}$$

$$= (0.239 - 0.658\ j)(0.495 - 0.495\ j) - (0.197 - 0.035\ j)(0.479 + 5.48\ j)$$

$$\boxed{\mathbf{\Delta = -0.491 - 1.507\ j}}$$

$$K = \frac{1-|0.239-0.658\ j|^2 - |0.495 - 0.495\ j|^2 + (1.58\ )^2}{2\,|0.197-0.035\ j|\,|0.479+5.48\ j|}$$

$$= \frac{1-0.49-0.49+2.512}{0.22}$$

$$= \quad 1.15$$

$$\boxed{\mathbf{K = 1.15}}$$

$$c_{in} = \frac{(S_{11} S_{22}^{*} 6)^{*}}{| S_{11} |^2 - |6|^2}$$

$$= \frac{[(0.239-0.658j)-(0.495+0.495j)(-0.491-1.507 j)]^{*}}{10.239-0.658 j |^2 - | 1.58 |^2}$$

$$= \frac{(-0.264+0.331 j)^{*}}{-2.022}$$

$$\boxed{\mathbf{c_{in} = 0.21 < 52^0}}$$

$$r_{in} = \frac{| S_{12} S_{21} |}{| |S_{11}|^2 - | 6 |^2 |}$$

$$= \frac{1 (0.197-0.35 j)(0.479+5.48 j)|}{| 10.239-0.658 j |^2 - (1.58)^2 |}$$

$$= \frac{| 0.286+1.063 j |}{0.49-2.496}$$

$$= \frac{1.1}{-2.006}$$

$$\boxed{\mathbf{r_{in} = 0.54}}$$

$$c_{out} = \frac{(S_{22} S_{11}^{*} 6)^{*}}{| S_{22} |^2 - |6|^2}$$

$$= \frac{((0.495-0.495 j)-(0.239+0.6758 j)(-0.491-1.501 j))^{*}}{| 0.495-0.495 j |^2 - 1- 0.491- 1.507 j |^2}$$

$$= \frac{(-0.379+0.188 j)^{*}}{-2.022}$$

$$= 0.187 + 0.093$$

$$\boxed{\mathbf{c_{out} = 0.21 < 26.4^0}}$$

$$r_{out} = \frac{| S_{12} S_{21} |}{| |S_{22}|^2 - | 6 |^2 |}$$

$$= \frac{| (0.197-0.35 j)(0.479+5.48 j)|}{| |0.495-0.495 j |^2 - | 1.58 |^2 |}$$

$$= \frac{|\,0.286+1.063\,j\,|}{-2.022}$$

$$= \frac{1.1}{-2.022}$$

$$\boxed{r_{out} = 0.54}$$

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