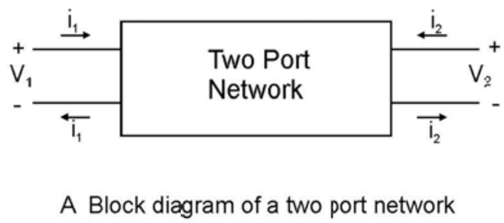
ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. II, Issue IV, April 2015

MONOGRAPH ON TWO PORT RF NETWORKS-CIRCUIT REPRESENTATION

Christo Ananth,
Assistant Professor, Department of ECE, Francis Xavier Engineering College, Tirunelveli, India

1.1 Introduction



A Block diagram of a two port network

Fig. : .1

→ A Network having two pair of terminals is called as two port Network. The network contains dependent sources, one (or) more of the equivalent resistors. Generally network is analyzed in S domain.

Low Frequency Parameters

Low frequency parameters can be classified into four types. They are

1. Impedance Parameter (Z)
2. Admittance Parameter (Y)
3. Hybrid Parameter (h)
4. Transmission parameter (ABCD)

(i) Impedance Parameter (Short Circuit Impedance)

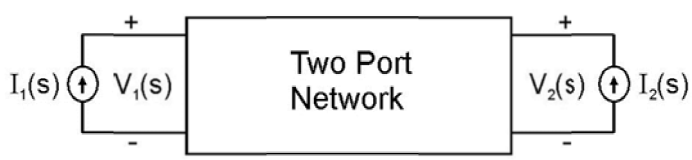


Fig. : 1.2 Block diagram of Imp dance Parameter

The general equation for impedance (Z) parameter is

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad \text{----- (1)}$$

$V_2 = Z_{21}I_1 + Z_{22}I_2$ ----- (2)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

We can also calculate the impedance parameters after making two sets of measurements. Christo Ananth et al.[1] discussed about E-plane and H-plane patterns which forms the basis of Microwave Engineering principles.

If the right side of the network (output port) is open circuit (i.e.) I_2=0, then we can easily solve for two of the impedance parameters then the two port network can be classified as

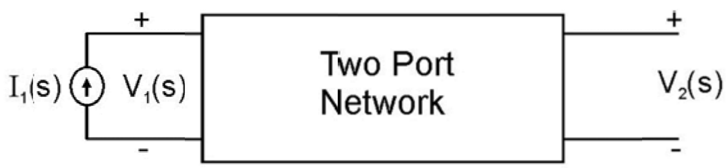


Fig. : 1.3 Port 2 is opened

Substitute I_2=0 in (1) & (2)

(1) $\Rightarrow$ $V_1 = Z_{11} I_1 + 0$ (2) $\Rightarrow$ $V_2 = Z_{21} I_1 + 0$

$$\boxed{Z_{11} = \frac{V_1}{I_1} \quad I_2 = 0}$$

$$\boxed{Z_{21} = \frac{V_2}{I_1} \Big| I_2 = 0}$$

If the left hand side of the network (input port) is open circuited (i.e.) $I_1 = 0$ then we can easily solve for the other two impedance parameters then the network diagram can be modified as

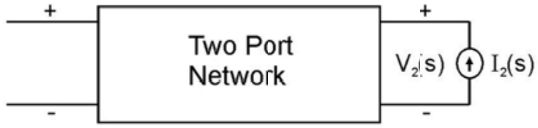


Fig. : 1.4 Port 1 is opened

Substitute $I_1 = 0$ in equation (1) & (2)

(1) $\Rightarrow$ $V_1 = 0 + Z_{12} I_2$ (2) $\Rightarrow$ $V_2 = 0 + Z_{22} I_2$

$$\boxed{Z_{12} = \frac{V_1}{I_2} \quad I_1 = 0}$$

$$\boxed{Z_{22} = \frac{V_2}{I_2} \Big| \; I_1 = 0}$$

The impedance parameters (Z) are

$$Z_{11} = \frac{V_1}{I_1} \quad I_2 = 0 \qquad Z_{21} = \frac{V_2}{I_1} \Big| \; I_2 = 0$$

$$Z_{12} = \frac{V_1}{I_2} \quad I_1 = 0 \qquad Z_{22} = \frac{V_2}{I_2} \Big| \; I_1 = 0$$

(ii) Admittance parameter (open circuit admittance)

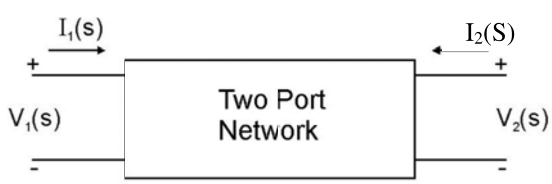


Fig. : 1.5 Block diagram of admittance parameter

The general equations are

$I_1 = Y_{11} V_1 + Y_{12} V_2$ ----- (3)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ ----- (4)

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

If the R.H.S. of the network (output port) short circuited (i.e.) $V_2 = 0$, substitute $V_2 = 0$ in equation (3) & (4).

(3) $\Rightarrow I_1 = Y_{11} V_1 + 0$ (4) $\Rightarrow I_2 = 0 + Y_{21} V_1 + 0$

$$Y_{11} = \frac{I_1}{V_1} \Big| V_2 = 0 \qquad Y_{21} = \frac{I_2}{V_1} \Big| V_2 = 0$$

If the L.H.S. of the network (input port) is short circuited (ie.) $V_1=0$, substitute $V_1=0$ in (3) & (4)

(3) $\Rightarrow I_1 = 0 + Y_{12} V_2$ (4) $\Rightarrow I_2 = 0 + Y_{22} V_2$

$$Y_{12} = \frac{I_1}{V_2} \Big| V_1 = 0 \qquad Y_{22} = \frac{I_2}{V_2} \Big| V_1 = 0$$

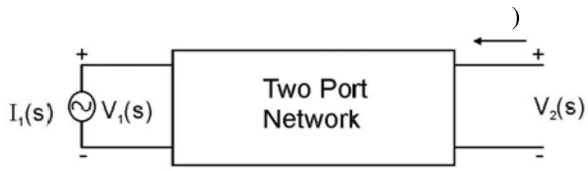
The admittance parameters are

$$Y_{11} = \frac{I_1}{V_1} \Big| V_2 = 0 \qquad Y_{21} = \frac{I_2}{V_1} \Big| V_2 = 0$$

$$Y_{12} = \frac{I_1}{V_2} \Big| V_1 = 0 \qquad Y_{22} = \frac{I_2}{V_2} \Big| V_1 = 0$$

(iii) Hybrid Parameter (h)

→ Hybrid parameter is the combination of impedance (Z) & admittance (Y) parameter. In this the network diagram one side is open circuited & the other side is short circuited and the two port network diagram is shown below.



ircuited

The general equations are

$V_1 = h_{11} I_1 + h_{12} V_2$ ----- (5)

$I_2 = h_{21} I_1 + h_{22} V_2$ ----- (6)

In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

If the R.H.S. of the network is short circuited (i.e.) $V_2 = 0$, then substitute $V_2 = 0$ in equation (5) & (6).

(5) $\Rightarrow V_1 = h_{11} I_1 + 0$ (6) $\Rightarrow I_2 = h_{21} I_1 + 0$

$$h_{11} = \frac{V_1}{I_1} \Big| V_2 = 0 \qquad h_{21} = \frac{I_2}{I_1} \Big| V_2 = 0$$

If the L.H.S. of the network is open circuited (i.e.) $I_1 = 0$, then substitute $I_1 = 0$ in equation (5) & (6)

(5) $\Rightarrow V_1 = 0 + h_{12} V_2$ (6) $\Rightarrow I_2 = 0 + h_{22} V_2$

$$h_{12} = \frac{V_1}{V_2} \Big| I_1 = 0 \qquad h_{22} = \frac{I_2}{V_2} \Big| I_1 = 0$$

The hybrid parameters are

$h_{11} = \frac{V_1}{I_1} | V_2 = 0$ $h_{12} = \frac{V_1}{V_2} | I_1 = 0$

$h_{21} = \frac{I_2}{I_1} | V_2 = 0$ $h_{22} = \frac{I_2}{V_2} | I_1 = 0$

(iv) Transmission Parameter (ABCD)

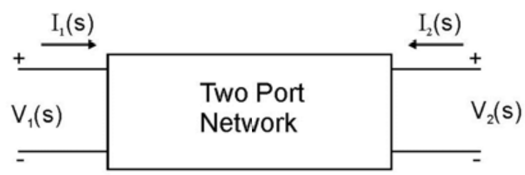


Fig. : 1.7 A Block diagram of ABCD network

In this ABCD parameter there is no separate diagrammatic representation. Because here the R.H.S. (output port) of the network is open as well as short circuited

The general equation of ABCD parameter are

$V_1 = AV_2 — B I_2$ ________ (7)

$I_1 = C V_2 — D I_2$ _______ (8)

In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

R.H.S. of the network is short circuited (i.e.) $V_2 = 0$, then $V_2 = 0$ in (7) & (8).

(7) $\Rightarrow V_1 = 0 - BI_2$ (8) $\Rightarrow I_1 = 0 — DI_2$

$B = \frac{-V_1}{I_2} | V_2 = 0$ $D = \frac{-I_1}{I_2} | V_2 = 0$

Again the R.H.S. of network is open circuited (ie.) $I_2 = 0$, then substitute $I_2 = 0$ in equation (7) & (8)

(7) $\Rightarrow V_1 = AV_2 + 0$ (8) $\Rightarrow I_1 = CV_2 - 0$

$A = \frac{V_1}{V_2} \Big|_{I_2 = 0}$ $C = \frac{I_1}{V_2} \Big|_{I_2 = 0}$

The ABCD parameters are

$A = \frac{V_1}{V_2} \Big|_{I_2 = 0}$ $B = \frac{-V_1}{I_2} \Big|_{V_2 = 0}$

$C = \frac{I_1}{V_2} \Big|_{I_2 = 0}$ $D = \frac{-I_1}{I_2} \Big|_{V_2 = 0}$

1.3 Cascaded Transmission Matrix

→ The ABCD matrix (or) transmission matrix is defined for a two port network in terms of total voltages & currents.

→ The main use of transmission matrix is dealing with the cascaded connection of two port network.

The matrix representation of ABCD parameter for the network 'x' is

$$\begin{bmatrix} V_1 & x \\ I_1 & x \end{bmatrix} = \begin{bmatrix} Ax & Bx \\ Cx & Dx \end{bmatrix} \begin{bmatrix} V_2 & x \\ —I_2 & x \end{bmatrix}$$

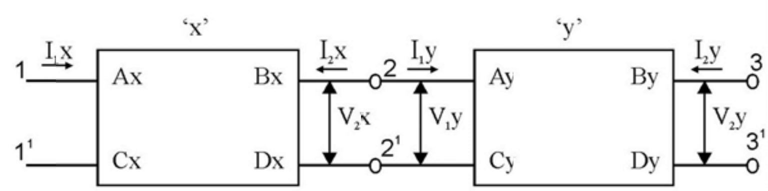


Fig. : 1.8 A Block diagram of cascadded transmission matrix

The matrix representation of ABCD parameter for the network 'y' is

$$\begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

It can be observed that $V_{2x} = V_{1y}$ & $I_{2x} = -I_{1y}$

Combining the results

$V_{1x} = V_{1y} = V_1$, $I_{1x} = I_{1y} = I_1$, $V_{2x} = V_{2y} = V_2$, $I_{2x} = I_{2y} = I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Which shows the ABCD matrix of the cascade connection of the two port network is equal to the product of the ABCD matrices representing the individual two ports.

1.4 High Frequency Parameter

→ The measurements of Z, Y, h & ABCD parameters is difficult at RF and microwave frequencies due to following reasons. (Drawbacks of low frequency parameters)

1) Non availability of terminal voltage & current measuring equipment.
2) Presence of active devices make the circuit unstable for open & short condition.
3) Short circuit & open circuit are not easily measured.

Therefore the RF & microwave circuit are analysed using S parameter (scattering parameter).

1.5 Formulation of S parameter

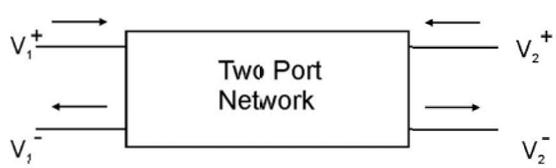


Fig. : 1.9 Block diagram of two port network

Consider a two port network phasor incident voltage $[V_1^+]$ & the phasor reflected voltage $[V_1^-]$.

The scattering (S) parameter is given by

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

In matrix form

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

(or)

$$[V^-] = [S][V^+]$$

The R.H.S. of the network (output port) is short circuited $[V_2^+ = 0]$

$$V_1^- = S_{11} V_1^+ \qquad V_2^- = S_{21} V_2^+ + 0$$

$$S_{11} = \frac{V_1^-}{V_1^+} \Big| V_2^+ = 0 \qquad S_{21} = \frac{V_2^-}{V_1^+} \Big| V_2^+ = 0$$

The L.H.S. of the network (input port) is short circuited (i.e.) $[V_1^+ = 0]$

$$V_1^- = S_{12} V_2^+ \quad ; \quad V_2^- = S_{22} V_2^+$$

$$S_{12} = \frac{V_1^-}{V_2^+} \Big| V_1^+ = 0 \qquad S_{22} = \frac{V_2^-}{V_2^+} \Big| V_1^+ = 0$$

Each element of S matrix is represented by

$$\left. S_{11} = \frac{V_1^-}{V_1^+} \Big| V_2^+ = 0 \right\} \longrightarrow \text{Input Reflection co-efficient } |1$$

$$\left. S_{22} = \frac{V_2^-}{V_2^+} \Big| V_1^+ = 0 \right\} \longrightarrow \text{Output Reflection co-efficient } |2 .$$

$S_{12} = \frac{V_1^-}{V_2^+} \Big| V_1^+ = 0 \Big\} \longrightarrow$ Attenuation of the wave travel from port 1 to port 2.

$S_{21} = \frac{V_2^-}{V_1^+} \Big| V_2^+ = 0 \Big\} \longrightarrow$ Attenuation of the wave travel from port 2 to port 1.

To derive the S-matrix in terms of ABCD parameters of a two port network

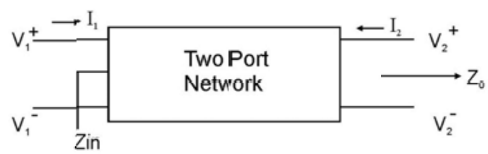


Fig. : .10 Block diagram of ABCD Parameters

Input Reflection co-efficient $| 1 = S_{11} = \frac{Z_{in} - Z_O}{Z_{in} + Z_O}$ ---------- (1)

Where $Z_{in} = \frac{V_1}{I_1}$ ---------- (2)

$V_1 = AV_2 - BI_2$ ---------- (3)

$I_1 = CV_2 - DI_2$ ---------- (4)

From the diagram $V_2 = - I_2 Z_0$ ---------- (5)

Substitute equation (3) & (4) in (2)

$Z_{in} = \frac{ÆV_2 - BI_2}{CV - DI_2}$ --------- (6)

Substitute equation (5) in (6)

$Z_{in} = \frac{Æ(-I_2Z_O) - BI_2}{C(-I_2Z_O) - DI_2} - \frac{-I_2(ÆZ + B)}{-I_2(CZ_O + D)}$

$Z_{in} = \frac{ÆZ_O + B}{CZ_O + D}$ --------- (7)

Substitute equation (7) in (1)

$$(1) \Rightarrow S_{11} = \frac{\left(\frac{\text{Æ}Z_0 + B}{CZ_0 + D}\right) - Z_0}{\left(\frac{\text{Æ}Z_0 + B}{CZ_0 + D}\right) + Z_0}$$

$$S_{11} = \frac{\text{Æ}Z_0 + B - Z_0(CZ_0 + D)}{\text{Æ}Z_0 + B + Z_0 + (CZ_0 + D)} = \frac{Z_0\ (\text{Æ} + B\,F_0 - CZ_0 - D)}{Z_0\ (\text{Æ} + B\,F_0 + CZ_0 + D)} \qquad \left(\therefore\ Y_0 = \frac{1}{Z_0}\right)$$

$$S_{11} = \frac{\text{Æ} + BF_0 - CZ_0 - D}{\Delta}$$

[Where $\Delta = A + BY_0 + CZ_0 + D$]

Similarly

$$S_{12} = \frac{2\ (\text{Æ}D - BC)}{\Delta} \quad ; \qquad S_{21} = \frac{2}{\Delta}$$

$$S_{22} = \frac{-\text{Æ} + BF_0 - CZ_0 + D}{\Delta}$$

1.6 Properties of S Parameters

a) Zero diagonal elements for perfect matched network

For an ideal N port network with matched termination, $S_{ii} = 0$. Since there is no reflection from any port. Therefore under perfect matched conditions the diagonal elements of [S] are zero.

b) Symmetry of [S] for a reciprocal network

A reciprocal device has the same transmission characteristic in either direction of a pair of ports and it is characterized by a symmetric scattering matrix.

$S_{ij} = S_{ji}$ [$i \neq j$) ------- (1)

Which results $[S]_t = [S]$ -------- (2)

This condition can be proved in the following manner. For a reciprocal network with the assumed normalization the impedance matrix equation is

$[V] = [Z]\ [I]$

Substitute $[V] = [a] + [b]$ & $[I] = [a] - [b]$

$[a] + [b] = [z]\ [\ [a] - [b]\ ]$ (or)

$(\ [z] + [U]\ )\ [b] = (\ [z] - [U]\ )\ [a]$

$$[b] = [\ [Z] + [U]\ ]^{-1}\ (\ [Z] - [U])\ [a] \qquad \text{-------- (3)}$$

Where [U] is the unit matrix. The S matrix equation for the network is

$$[b] = [s]\ [a] \qquad \text{-------- (4)}$$

Comparing equation (3) & (4)

$$[S] = (\ [Z] + [U]\ )^{-1}\ (\ [Z] - [U]\ ) \qquad \text{-------- (5)}$$

$$\text{Let } R = [z] - [U]\ \&\ [Z] + [U] = Q \qquad \text{-------- (6)}$$

For a reciprocal network, the Z matrix is symmetric. Hence

$[R]\ [Q] = [Q]\ [R]$ (or)

$[Q]^{-1}\ [R]\ [Q]\ [Q]^{-1} = [Q]^{-1}\ [Q]\ [R]\ [Q^{-1}]$ (or)

$$S = [Q]^{-1}\ [R] = [R]\ [Q^{-1}] \qquad \text{-------- (7)}$$

Now the transpose of [S] is

$$[S]_t = (\ [Z] - [U]\ )_t\ ,\ (\ [Z] + [U]\ )_t^{-1} \qquad \text{-------- (8)}$$

Since the Z matrix is symmetrical

$$(\ [Z] - [U]\ )_t = [Z] - [U] \qquad \text{-------- (9)}$$

$$(\ [Z] + [U]\ )_t = [Z] + [U] \qquad \text{-------- (10)}$$

Therefore,

$[s]_t = (\ [Z] - [U]\ )\ .\ (\ [Z] + [U]\ )^{-1}$

$$[s]_t = [R]\ [Q]^{-1} = [S] \qquad \text{-------- (11)}$$

Thus it is proved that $[S]_t = [S]$ for a symmetrical junction.

(c) Unitary property for a lossless junction

For any lossless network the sum of the products of each term of any one row (or) of any column of the 'S' matrix multiplied by its complex conjugate is unity.

For a lossless 'n' port device, the total power leaving N ports must be equal to the total power input to these ports, so that,

$$\sum_{n=1}^{N} |b_n|^2 = \sum_{n=1}^{N} |a_n|^2$$

$$\sum_{n=1}^{N} \left|\sum_{i=1}^{n} S_{ni}\ a_i\right|^2 = \sum_{n=1}^{N} |a_n|^2 \qquad \text{-------- (1)}$$

If only i^{th} port is excited and all other ports are matched terminated all $a_n=0$ except a_i,

So that $$\sum_{n=1}^{N} |S_{ni}\ a_i|^2 = \sum_{n=1}^{N} |a_i|^2 \qquad \text{--------- (2)}$$

$$\sum_{n=1}^{N} |S_{ni}|^2 = 1 = \sum_{n=1}^{N} S_{ni}\ S_{ni}^* \qquad \text{--------- (3)}$$

Therefore, for a loseless junction

$$\sum_{n=1}^{N} S_{ni}\ S_{ni}^* = 1 \qquad \text{-------- (4)}$$

If all $a_n = 0$ except a_i & a_k

$$\sum_{n=1}^{N} S_{nk} \cdot S_{nk}^* = 0\ ;\ i \neq k \qquad \text{-------- (5)}$$

In matrix notation, these relations can be expressed as

$[S^*]\ [S]_t = [U]$

$[S^*] = [S]_t^{-1}$ ------- (6)

Here [U] is the Identity (or) unit matrix. A matrix [S] for lossless network which satisfies the above three conditions equation (4), (5) & (6) is called a unitary matrix.

(d) Phase Shift property

→ Complex S-parameters of a network are defined with respect to the positions of port or reference planes. For a two port network with unprimed reference planes 1 & 2 as shown in figure the parameters have definite complex values.

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \text{-------- (1)}$$

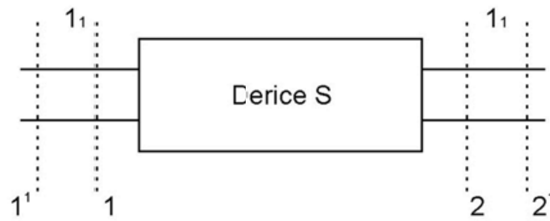


Fig. 1.11 : Phase shift property of S

If the reference planes 1 & 2 are shifted outward to $1'$ & $2'$ by electrical phase shifts. $\phi_1 = \beta_1 l_1$, $\phi_2 = \beta_2 l_2$ respectively, then the new wave variables are $a_1 e^{j\phi 1}$, $b_1 e^{-j\phi 1}$, $a_2 e^{j\phi 2}$, $b_2 e^{-j\phi 2}$. The new 'S' matrix S^1 is given by

$$[S^1] = \begin{bmatrix} e^{-j\emptyset 1} & 0 \\ 0 & e^{-j\emptyset 2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\emptyset 1} & 0 \\ 0 & e^{-j\emptyset 2} \end{bmatrix} \quad \text{------ (2)}$$

This property is valid for any number of ports & is called the phase shift property applicable to a shift of reference planes.

1.7 Reciprocal Networks

Reciprocal network is defined to be a network that satisfies the reciprocity theorem.

Reciprocity theorem

In a linear time-invarient system, the ratio of the response measured at a point to an excitation at some other point is unchanged if the measurements and the excitation points are interchanged.

$$Z_{12} = \frac{E_{12}}{I_2}$$

$$Z_{21} = \frac{E_{21}}{I_1}$$

$$Z_{12} = Z_{21}$$

$$\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}$$

It can be shown that for all reciprocal networks, the [S] matrix is symmetrical

$$S_{12} = S_{21}$$

For N-port network,

$$S_{ij} = S_{ji} \text{ for } i \neq j$$

Where i = 1,2,3,……,N

j = 1,2,3,……N

Lossless Networks

The network which satisfies the lossless condition is called lossless network. For a lossless passive network, the power entering circuit will always be equal to the power leaving the network.

1.8 Transmission matrix for high frequency parameters

→ At high RF & micro wave frequencies, the transmission matrix (T) is expressed interms of the input incident & reflected waves as the independent variables & output incident & reflected waves as the dependent variables.

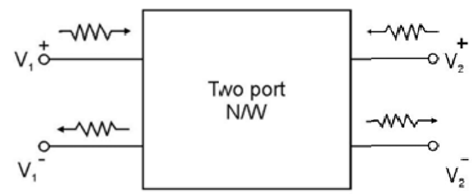


Fig. 1.12 : Block diagram of two port network

$$\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^- \\ V_2^+ \end{bmatrix}$$

The relationship between S & T can be derived using the above basic definition as follows.

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{S_{21}} & \frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11} S_{22}}{S_{21}} \end{bmatrix}$$

The reverse relationship is given by,

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{21}T_{12}}{T_{11}} \\ \frac{1}{T_{11}} & \frac{-T_{12}}{T_{11}} \end{bmatrix}$$

For a cascaded connection of 2 – port network the overall T-matrix can be obtained as

$$\begin{bmatrix} V_1^{+} \\ V_1^{-} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^{-} \\ V_2^{+} \end{bmatrix}$$

$$\begin{bmatrix} V_1^{+} \\ V_1^{-} \end{bmatrix} = \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} V_2'^{-} \\ V_2'^{+} \end{bmatrix}$$

But $V_1'^{-} = V_2^{+}$

$V_2^{-} = V_1'^{+}$

$$\begin{bmatrix} V_1^{+} \\ V_1^{-} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} V_2'^{-} \\ V_1'^{+} \end{bmatrix}$$

Thus the total T-matrix is the multiplication of the two T-matrices

$$[\,T\,]_{tot} = [\,T\,]\ [\,T^{1}\,]$$

RF Microwaves versus DC (or) low AC signals

The following 4 effects provide a brief summary of the effects of RF/MW signals in a circuit that are not present at DC (or) low AC signals.

Effect # 1. Presence of stray capacitance. This is the capacitance that exists.

1. Between conductors of the circuit.
2. Between conductors (or) components & ground
3. Between components.

Effect # 2. Presence of stray inductance.

This is the inductance that exists due to

- Inductance of the Conductors that connect components.
- The parasitic inductance of the components themselves.

Effect # 3. Skin effect

→ This is due to the fact that AC signals penetrate a metal partially and flow in a narrow band near the outside surface of each conductor. This effect is in contrast to DC signal that flow through the whole cross section of the conductor.

→ For AC signals, the current density falls off exponentially from the surface of the conductor toward the center. At a critical length (δ) called the skin depth (or) depth of penetration, signals amplitude is $\frac{1}{e}$ (or) 3.8% of its surface amplitude.

The skin depth is given by $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$

Where μ s the permeability (H/m)

σ is the conductivity of conductor.

Effect # 4 : Radiation

Radiation can occur outside or within a circuit. The radiation factor causes coupling effect to occur as follows.

- Coupling between elements of the circuit
- Coupling between the circuit & its environment
- Coupling from the environment to the circuit.

1.10 S parameters in terms of T-parameter

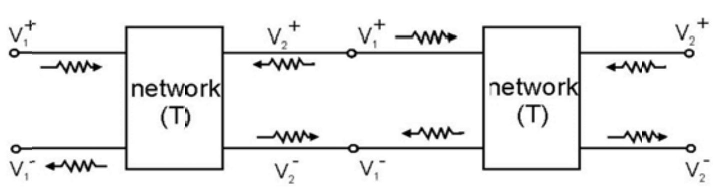


Fig. : 1.13 Block diagram of S parameter in t rms of T parameter

$V_1^r = S_{11} V_1^+ + S_{12} V_2^+$; $\quad S_{11} = \frac{V_1^-}{V_1^+} | V_2^+ = 0 \quad S_{12} = \frac{V_1^-}{V_2^+} | V_1^+ = 0$

$V_2^r = S_{21} V_1^+ + S_{22} V_2^+ \quad S_{21} = \frac{V_2^-}{V_1^+} | V_2^+ = 0 \quad S_{22} = \frac{V_2^-}{V_2^+} | V_1^+ = 0$

T – parameter

$$V_1^+ = T_{11} V_2^- + T_{12} V_2^+ \qquad \text{----- (1)}$$

$$V_1^- = T_{21} V_2^- + T_{22} V_2^+ \qquad \text{----- (2)}$$

Sub $V_2^+ = 0$ in equation (1) & (2)

$$V_1^+ = T_{11} V_2^- \qquad \text{------ (3)}$$

$$V_1^- = T_{21} V_2^- \qquad \text{------ (4)}$$

$$\frac{(4)}{(3)} \Rightarrow \frac{V_1^-}{V_1^+} = \frac{T_{21} V_2^-}{T_{11} V_2^-}$$

$$S_{11} = \frac{T_{21}}{T_{11}}$$

Equation 3 becomes

$$\frac{1}{T_{11}} = \frac{V_2^-}{V_1^+}$$

$$S_{21} = \frac{1}{T_{11}}$$

Substituted $V_1^+ = 0$ in equation (1) & (2)

$$0 = T_{11} V_2^- + T_{12} V_2^+ \qquad \text{------- (5)}$$

$$V_1^- = T_{21} V_2^- + T_{22} V_2^+ \qquad \text{------- (6)}$$

$$(5) \Rightarrow T_{11} V_2^- = -T_{11} V_2^+$$

$$S_{22} = \frac{V_2^-}{V_2^+} = \frac{-T_{12}}{T_{11}}$$

Equation (6) is divided by V_2^+

(6) $\Rightarrow \quad \frac{V_1^-}{V_2^+} = T_{21} \frac{V_2^-}{V_2^+} + T_{22}$

$S_{12} = T_{21} \left(\frac{-T_{12}}{T_{11}}\right) + T_{22}$

$S_{12} = \frac{-T_{12}+T_{21}+T_{11}T_{22}}{T_{11}} = \frac{T_{11}T_{22}-T_{12}T_{21}}{T_{11}}$

T – parameter in terms of S – parameter

$V_1^+ = T_{11} V_2^- + T_{12} V^+_2$

$V_1^- = T_{21} V_2^- + T_{22} V^+_2$

$T_{11} = \frac{V_1^+}{V_2^-} \mid V_2^+ = 0, \quad T_{12} = \frac{V_1^+}{V_2^+} \mid V_2^- = 0$

$T_{21} = \frac{V_1^-}{V_2^-} \mid V_2^+ = 0, \quad T_{22} = \frac{V_1^-}{V_2^+} \mid V_2^+ = 0$

S – parameter

$V_1^- = S_{11} V^+_1 + S_{12} V^+_2$ ----- (1)

$V_2^- = S_{21} V^+_1 + S_{22} V^+_2$ ----- (2)

Substitute $V_2^+ = 0$ in equation (1) & (2)

$V_1^- = S_{11} V_1^+$ ------ (3)

$V_2^- = S_{21} V_1^+$ ------ (4)

From (4) $\Rightarrow \frac{V_1^+}{V_2^-} = T_{11} = \frac{1}{S_{21}}$

$\frac{(3)}{(4)} \Rightarrow \frac{S_{11}}{S_{21}} = T_{21}$

Sub $V_2^- = 0$ in equation (1) & (2)

$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$ ------(5)

$0 = S_{21} V_1^+ + S_{22} V_2^+$ ------- (6)

From (6) $\Rightarrow$ $S_{21} V_1^+ = -S_{22} V_2^+$

$$\frac{V_1^+}{V_2^+} = T_{12} = \frac{-S_{22}}{S_{21}} \quad , \; T_{12} = \frac{-S_{22}}{S_{21}}$$

Divide equation (5) by V_2^+

$$(5) \Rightarrow \frac{V_1^-}{V_2^+} = S_{11} \frac{V_1^+}{V_2^+} + S_{12}$$

Substitute $\frac{V_1^+}{V_2^+} = \frac{-S_{22}}{S_{21}}$ in above equation

$$\frac{V_1^-}{V_2^+} = \frac{-S_{11}S_{22}}{S_{21}} + S_{12} = T_{22} \;, \quad T_{22} = \frac{V_1^-}{V_2^+} = \frac{-S_{11}S_{22}}{S_{21}} + S_{12}$$

Application of RF

i) Communication

This application includes satellite, space, long distance telephone, marine, cellular telephone, data, mobile phone, aircraft vehicle, personal and Wireless Local Area Network (WLAN).

ii) TV and Radio Broadcast

In this application, RF/MW are used as the carrier signal for audio and video signals. An example is the Direct Broadcast system which is designed to link satellites directly to users.

iii) Optical Communication

In this application microwave modulator is used in the transmitting sides of a low-loss optical fiber with a microwave demodulator as the other end. The microwave signal acts as a modulating signal with the optical signal as the carrier optical communication is useful in cases where a much large number of frequency channels and less interfere from outside electromagnetic radiation are desired current applications include telephone cables computer network links, low noise transmission lines and soon.

iv) Radar

It includes air defense, aircraft/ship, guidance smart weapons, police, weather, collision avoidance & imaging.

v) Navigation

It is used for orientation guidance of aircraft, ships and land vehicles.

vi) Domestic & Industrial Applications

It includes microwave ovens, clothes dryers, fluid heating systems, moisture sensors, tank gauges, automatic door openers, highway traffic monitoring & control.

vii) Medical applications

It includes cautery, selective heating, heart simulation, hemorrhage control, and sterilization and imaging.

1.12 Introduction to Component Basics

i) Wire

A wire is the simplest element having zero resistance which makes it appear as a short circuit DC and low AC frequencies yet at RF/MW frequencies, it becomes a very complex element wire in a circuit can take on many forms, such as

- Wire wound resistors
- Wire wound inductors
- Leaded capacitors
- Element to element interconnect applications

→ Problems associated with a peace of wire. Problems associated with a wire can be traced to two major areas. They are skin effect and straight wire inductance.

Skin effect in a wire

→ As frequency increases the electric signal propagate less and less in the inside of the conductor. The current density increases near the outside parameter of the wire. The resistance of the wire is given by

$$R = \frac{\eth_l}{\AE}$$

→ If effective cross sectional Area (A) decreases, this leads to an increase in resistance (R).

Straight wire Inductance

→ In the medium surrounding any current carrying conductor there exists a magnetic field. This produces an induced voltage in the wire that opposes any change in the current flow. This opposition to change is called self-inductance.

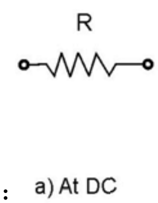
ii) Resistors:

→ An element specializing in the resistance property of the material. The resistance of a material is a property whose value determines the rate at which electrical energy is converted into thermal energy when an electric current passes through it. Resistors are used for different purposes such as

- In transistor bias network to establish an operating point.
- In alternators to control the flow of power.
- In signal combiners to produce a higher output power.
- In transmission lines to create matched conditions etc.

At DC, V = IR

At low AC , V~IR at high RF/MW



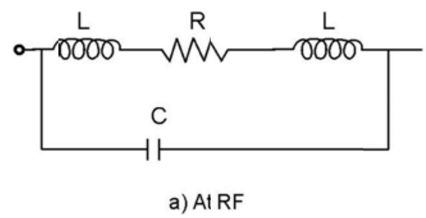


Fig. 1.14 :

→ At RF/MW frequencies, a resistor (R) appears as a combination of several elements. As frequency increases, the wire inductance (L) increased resistor (R'>R) due to skin effect and the parasitic capacitors become prominent.

iii) Capacitors:

→ A device that consists of two conducting surfaces separated by an insulating material (or) dielectric.

→ The capacitance is the property that permits the storage of charge when the potential difference exists between the conductors.

→ A practical capacitor has several parasitic elements that become important at higher frequencies. The equivalent circuit of a real capacitor is shown below.

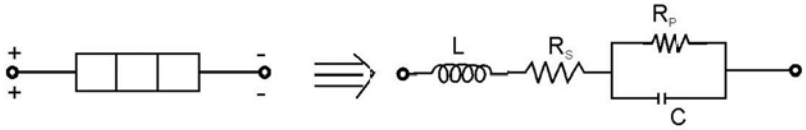


Fig. 1.15 : Parasitic elements in capacitor

Capacitance (C) = $\frac{Q}{V}$, C is the actual capacitance

L is the lead inductance

R_S is the series resistance, Rp is the insulation resistance. The behavior of capacitor Vs frequency.

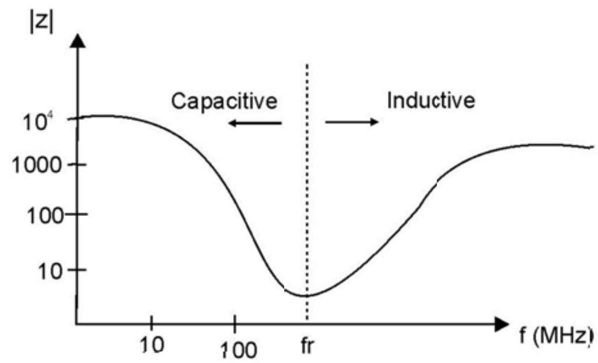


Fig. : 1.16 Frequency responce

→ If the frequency is larger than the resonance frequency ie. $f > f_r$ then it acts like an inductor.

→ If the frequency is smaller than the resonance frequency ie. $f < f_r$ then it acts like a capacitor.

iv) Inductors:

→ A wire that is wound (or) coiled in such a manner as to increase the magnetic flux linkage between the turns of the coil.

→ The increased flux linkage increases the wires self-inductance.

→ There is no such thing as a perfect component. Among all the components inductors are most suitable to very drastic changes over frequency. This is due to the fact that the distributed capacitance (C_d) and the series resistance (Rs) in an inductor at RF/MW play a major role in the performance of an inductor shown below.

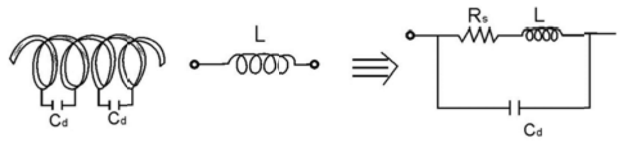


Fig. 1.17 : Equivalent circuit of an inductor, at RF/MW frequencies

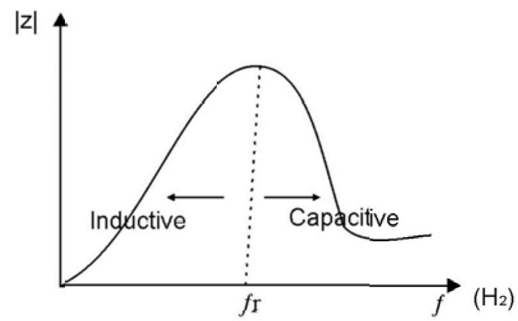


Fig. : 1.18 Block iagram Frequency response

→ If the frequency ‘f’ is greater than the resonance frequency ($f > f_r$) then it acts like capacitive.

→ If the frequency ‘f’ is l ss than the resonance frequency ($f < f_r$) then it acts like inductive.

Quality Factor (Q)

→ It is defined as the measure of the ability of an element to store energy equal to 2π times the average energy stored divided by the energy dissipated per cycle.

For capacitor $Q = \frac{\text{Reactance}}{\text{Series resistance}} = \frac{X_C}{R_{EQ}} = \frac{1}{M_C\, R_{EQ}}$

For Inductor $Q = \frac{M_L}{R_S} = \frac{M_L}{R_S}$

****

Problems :

1. What are the S Parameters of a series element (Z) as shown below.

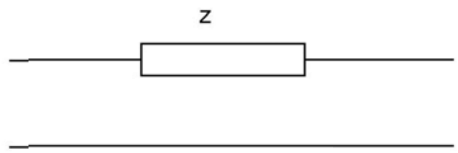
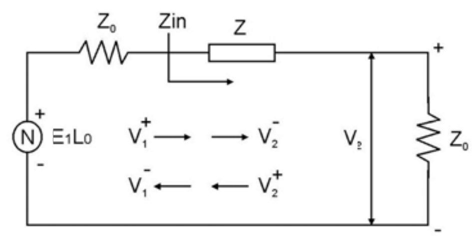


Fig. : 1.19 Block iagram of a seri s element

Solution :



This is a reciprocal & symmetrical network, for $S_{11} = S_{22}$

$S_{12} = S_{21}$

We know that $S_{11} = \frac{V_1^-}{V_1^+} \mid V_2^+ = 0 = \Gamma_{in}$, $\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$ & $Z_{in} = Z + Z_0$

Here $S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z + Z_0 - Z_0}{Z + Z_0 + Z_0} = \frac{Z}{Z + 2Z_0} = S_{22}$

$S_{21} = \frac{V_2^-}{V_1^+} \mid V_2^+ = 0$, $I = \frac{E}{R} = \frac{E_1}{Z_{in} + Z_0}$

We know that $V_1 = V_1^+ + V_1^- = V_1^+ \left(1 + \frac{V_1^-}{V_1^+}\right) = V_1^+ \, (1 + S_{11})$ ---------- (1)

$V_1 = I Z_{in}$ --------- (2)

Equation (1) and (2)

Where, $I\,Z_{in} = V_1^+\,(1 + S_{11})$

$$V_1^+ = \frac{I\,Z_{in}}{1 + S_{11}} \qquad \text{--------- (3)}$$

Because the load is matched, $V_2^+ = 0 \;\therefore\; V_2^- = I\ Z_0$ --------- (4)

$$\therefore S_{21} = \frac{V_2^-}{V_1^+} = \frac{I\,Z_0}{I\,Z_{in}/1+S_{11}} = \frac{Z_0(1 + S11)}{Z_{in}} = \frac{Z_0\left(1 + \frac{Z}{2Z_0 + Z}\right)}{Z_{in}}$$

$$= \frac{\frac{Z_0\,(Z+2Z_0+Z)}{(Z+2Z_0)}}{Z_{in}} = \frac{2Z_0(Z+Z_0)}{Z+2ZO} \times \frac{1}{Z_{in}}$$

$$S_{21} = \frac{2Z_0(Z+Z_0)}{Z+2ZO} \times \frac{1}{Z+Z_0} = \frac{2Z_0}{Z+2ZO} = S_{12}$$

For a series Z – network, we can see that $S_{21} = 1 - S_{11}$

2) Find the [ABCD] matrix for a series impedance element (Z).

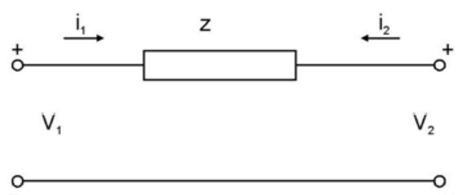


Fig. : 1.20 Block di gram of series element

ABCD network representation of an impedance element

Solution:

$$\left.\begin{aligned} V_1 &= AV_2 - Bi_2 \\ I_1 &= CV_2 - Di_2 \end{aligned}\right\} \qquad \text{----- (1)}$$

Apply KCL $i_1 = -\,i_2$ ------- (2)

By Applying KVL $V_1 - i_1Z + i_2Z - V_2 = 0$

$$V_1 = V_2 - i_2Z + i_1 Z \qquad \text{------- (3)}$$

Step 1) Port 2 open circuit $i_2 = 0$

$(3) \Rightarrow V_1 = V_2$

$(1) \Rightarrow V_1 = AV_2$ $\qquad$ $(2) \Rightarrow i_1 = CV_2$

$A = \frac{V_1}{V_2} = 1$ $\qquad$ $C = \frac{i_1}{V_2} - \frac{-i_2}{V_2} = 0$ | $\qquad$ $[\therefore i_2 = 0]$

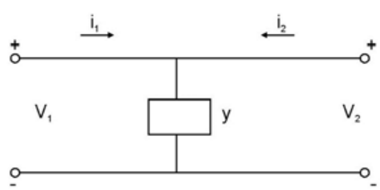
Step 2) Port 2 Short circuit $V_2 = 0$

$(1) \Rightarrow V_1 = -Bi_2 \Rightarrow B = \frac{-V_1}{i_2} = \frac{-i_1Z}{i_2} = Z$ $\qquad$ $[i_1 = -i_2]$

$(2) \Rightarrow i_1 = -Di_2 \Rightarrow D = \frac{-i_1}{i_2} = \frac{i_2}{i_2} = 1$

The [ABCD] matrix is given by $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

3) Find the [ABCD] matrix for a shunt element (Y).



F g. : 1.21 Block diagram of Shunt element

Solution:

Port 2 open circuit, $V_2 = V_1$, $i_2 = 0$

$V_1 = AV_2$ $\qquad$ $i_1 = CV_2$

$\boxed{A = 1}$ $\qquad$ $C = \frac{i_1}{V_2} = \frac{i_1}{\frac{i_1}{Y}} = Y$

$\boxed{C = Y}$

Port 2 short circuit $V_2 = 0$

$V_1 = -Bi_2$ $\qquad\qquad$ $i_1 = -Di_2$

$V_2 = -Bi_2$ $\qquad\qquad$ $D = \frac{-i_1}{i_2}$

$B = \frac{-V_2}{i_2}$ $\qquad [\therefore V_2 = 0]$ $\qquad D = \frac{-i_1}{-1_1}$

$\boxed{B = 0}$ $\qquad\qquad$ $\boxed{D = 1}$

Thus the [ABCD] matrix is given by $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$

4) Find the [ABCD] matrix for a circuit consisting of a series element (Z) & shunt element (Y).

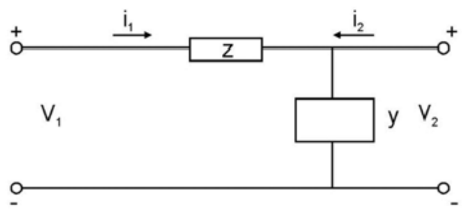


Fig. : 1.22 Block diagram of series and shunt element

Solution:

The [ABCD] matrix for the whole circuit which is a cascade of a series & shunt element is a multiplication of 2 matrices.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + ZY & Z \\ Y & 1 \end{bmatrix}$$

5) Considering a copper as the conductive medium, what is the skin depth at 60 H_Z and 1 MH_Z?

Solution:

For copper we have $\mu = 4\pi \times 10^{-7}$ H/m

$\sigma = 5.8 \times 10^{7}$ S/m

At 60 H_Z $$\delta = \left(\frac{1}{n\ s\ 60\ s\ 5.8\ s\ 10^{7}\ s\ 4n\ s\ 10^{-7}}\right)^{½}$$

$\delta = 0.85$ cm

At f = 1 MH_Z $\delta = 0.007$ cm

Which is a substantial reduction in penetration depth.

Relations of Z, Y, ABCD parameters with S parameter

$$A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{-(Z_{11}Z_{22} - Z_{12}Z_{21})}{Z_{21}}, \quad C = \frac{1}{Z_{21}}, \quad D = \frac{-Z_{22}}{Z_{21}}$$

$$Y_{11} = \frac{D}{B}, \quad Y_{22} = \frac{-\text{Æ}}{B}, \quad Y_{12} = \frac{1}{B}, \quad Y_{21} = \frac{C+\text{Æ}D}{2}$$

$$S_{11} = \frac{\text{Æ}-B-C+D}{\text{Æ}-B+C-D}, \quad S_{12} = \frac{-2(\text{Æ}D+BC)}{\text{Æ}-B+C-D}, \quad S_{21} = \frac{2}{\text{Æ}-B+C-D}, \quad S_{22} = \frac{-\text{Æ}-B-C-D}{\text{Æ}-B+C-D}$$

6) If the impedance matrix of a simple device $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ Find its [S] matrix.

Solution:

$Z_{11} = 4$, $Z_{12} = 2$, $Z_{21} = 2$, $Z_{22} = 4$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{4}{2} = 2, \quad B = \frac{-(Z_{11}Z_{22} - Z_{12}Z_{21})}{Z_{21}},$$

$$B = \frac{-(4 \times 4 - 2 \times 2)}{2} = -6$$

$$C = \frac{1}{Z_{21}} = \frac{1}{2} \quad , \qquad D = \frac{-Z_{22}}{Z_{21}} = \frac{-4}{2} = -2$$

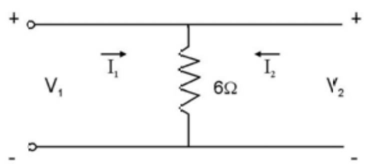
$$S_{11} = \frac{\text{Æ}-B-C+D}{\text{Æ}-B+C-D} = \frac{2+6-\frac{1}{2}-2}{2+6+\frac{1}{2}+2} = \frac{\cancel{11}}{21}$$

$$S_{22} = \frac{-\text{Æ}-B-C-D}{\text{Æ}-B+C-D} = \frac{-2+6-\frac{1}{2}+2}{2+6-\frac{1}{2}+2} = \frac{11}{21}$$

Similarly**,**

$$S_{12} = \frac{28}{21} \; ; \qquad S_{21} = \frac{4}{21}$$

7) Find the Impedance Parameter for the given circuit.



Solution:

Consider $V_1 \rightarrow$ source

$\therefore$ V_2 is open then $I_2 = 0$

At port 1 $\quad V_1 = 6I_1$

At port 2 $\quad V_2 = 6(I_1 + I_2)$, $V_2 = 6I_1$ ($\therefore$ $I_2 = 0$)

$$Z_{11} = \frac{\cancel{V_1}}{I_1} - \frac{6I_1}{I_1} = 6\Omega \;, \quad Z_{21} = \frac{V_2}{I_1} = \frac{6I_1}{I_1} = 6\Omega$$

Consider V_2 as source

Let open circuited V_1 , $I_1 = 0$

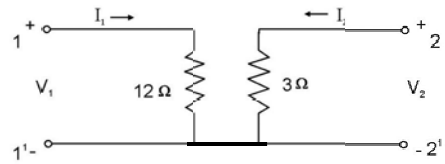
At port 1, $\quad V_1 = 6(I_1 + I_2) = 6I_2 \qquad (\therefore \; I_1 = 0)$

At port 2 , $V_2 = 6I_2$

$Z_{12} = \dfrac{V_1}{I_2} = \dfrac{6I_2}{I_2} = 6\Omega$, $Z_{22} = \dfrac{V_2}{I_2} = 6\Omega$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$$

8) Find the impedance parameter for the given circuit:



Solution:

i) Consider V_1 as source , V_2 is open, $I_2 = 0$

At port 1 , $V_1 = 1_2 I_1$, $Z_{11} = \dfrac{V_1}{I_1} = 12\ \Omega$

At port 2, $V_2 = 0$, $Z_{21} = \dfrac{V_2}{I_1} = \dfrac{0}{I_1} = 0$

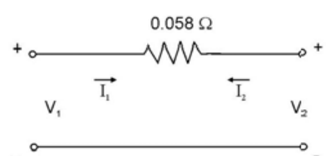
ii) Consider V_2 as source , V_1 is open $I_1 = 0$

At port 1, $V_1 = 0$, $Z_{12} = 0$

At port 2, $V_2 = 3I_2$, $Z_{22} = \dfrac{V_2}{I_2} = \dfrac{3I_2}{I_2} = 3\Omega$

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 12 & 0 \\ 0 & 3 \end{bmatrix}$$

9) Find the admittance parameter of the circuit.



Solution:

i) Consider I_1 as source current, I_2 shunt, $V_2 = 0$

At port 1, $I_1 = 0.05\,(V_1 - V_2) = 0.05\,V_1$

At port 2, $I_2 = 0.05\,(V_2 - V_1) = -0.05\,V_1$

$Y_{11} = \frac{I_1}{V_1} = 0.05$, $Y_{21} = \frac{I_2}{V_1} = -0.05$

ii) Consider I_2 as source current, I_1 shunt, $V_1 = 0$

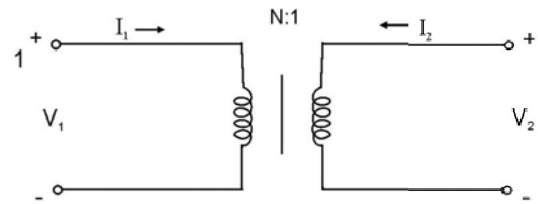
$I_2 = 0.05\,(V_2 - V_1) = 0.05\,V_2$

$I_1 = 0.05\,(V_1 - V_2) = 0.05\,V_2$

$Y_{12} = \frac{I_1}{V_2} = -0.05$, $Y_{22} = \frac{I_2}{V_2} = 0.05$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.05 & -0.05 \\ -0.05 & 0.05 \end{bmatrix}$$

10. Find the ABCD matrix representation of transformer (or) Compute the ABCD Parameter for an RF transformer with his ratio $N = N_1/N_2$ where N_1 is the no. of turn in 1^0 (primary) winding & N_2 is 2^0 (secondary) winding.



Solution:

Since the voltages in the transformer are related as

$\frac{V_1}{N_1} - \frac{V_2}{N_2}$, $\frac{V_1}{V_2} = \frac{N_1}{N_2} = N$

ABCD matrix $V_1 = AV_2 = Bi_2$ ------ (1), $i_1 = CV_2 - Di_2$ ------ (2)

Open circuit $i_2 = 0$ (1) $\Rightarrow$ $V_1 = AV_2$, $A = \frac{V_1}{V_2} = N$

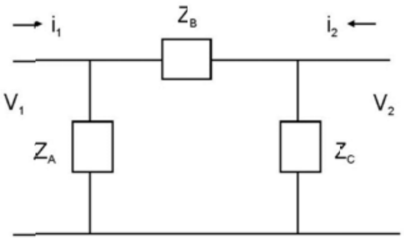
(2) $i_1 = CV_2$, $C = \frac{i_1}{V_2} = 0$

Short circuit $V_2 = 0$ (1) $\Rightarrow V_1 = -Bi_2$, $B = \frac{-V_1}{i_2} = 0$ [$\because V_1 = NV_2$)

(2) $\Rightarrow$ $i_1 = -Di_2$, $D = \frac{-i_1}{i_2} = \frac{1}{N}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} N & 0 \\ 1 & \frac{1}{N} \end{bmatrix}$$

11. ABCD matric computation of a 'π' network:



Solution:

Port 2 is open circuited (i.e.) $i_2 = 0$

$$Z_{11} = \frac{V_1}{i_1}\Big|_{i_2 = 0} = Z_A \parallel (Z_B + Z_C)$$

$$Z_{11} = \frac{Z_A\,(Z_B + Z_C)}{Z_A + Z_B + Z_C}$$

$$Z_{12} = \frac{V_1}{i_2}\Big|_{i_1 = 0}$$

By using voltage divided rule $V_1 = \frac{Z_A}{Z_A + Z_B} V_{AB}$

$$V_{AB} = i_2\,[Z_C \parallel (Z_A + Z_B)] = i_2\left[\frac{Z_C\,(Z_A + Z_B)}{Z_C + Z_A + Z_B}\right]$$

$$Z_{12} = \frac{V_1}{i_2} = \frac{Z_A}{Z_A+Z_B} \frac{i_2}{i_2} \left[\frac{Z_C (Z_A+Z_B)}{Z_C+Z_A+Z_B}\right] = \frac{Z_A Z_C}{Z_C+ Z_A+Z_B}$$

Port 1 is open circuited

$$Z_{21} = \frac{V_2}{i_1}\bigg| i_2 = 0 \quad , \; V_2 = \frac{Z_C}{Z_B+ Z_C} . \; V_{CB}$$

$$V_{CB} = i_1 [Z_A \parallel (Z_B + Z_C)] \; , Z_{21} = \frac{Z_C}{Z_B+ Z_C} \frac{i_1}{i_1} \left[\frac{Z_A (Z_B+Z_C)}{Z_A+Z_A+Z_B}\right]$$

$$Z_{21} = \frac{Z_A Z_C}{Z_A+ Z_B+ Z_C}$$

$$Z_{22} = \frac{V_2}{i_2}\bigg| i_1 = 0 \; , \; = Z_C \parallel (Z_A + Z_B)$$

$$Z_{22} = \frac{Z_C (Z_A+Z_B)}{Z_A+ Z_B+ Z_C}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{pmatrix} \frac{Z_A (Z_B+ Z_C)}{Z_A+ Z_B+ Z_C} & \frac{Z_A Z_C}{Z_A+ Z_B+ Z_C} \\ \frac{Z_A Z_C}{Z_A+ Z_B+ Z_C} & \frac{Z_C (Z_A+Z_B)}{Z_A+ Z_B+ Z_C} \end{pmatrix} \frac{1}{(Z_A+ Z_B + Z_C)} \begin{pmatrix} Z_A(Z_B+Z_C) & Z_A Z_C \\ Z_A Z_C & Z_C (Z_A +Z_B) \end{pmatrix}$$

12. Prove that it is impossible to construct a perfectly matched, lossless, reciprocal three port junction.

Solution:

‘S’ matrix for three port network is $[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$

A perfectly matched three port junction has a ‘S’ matrix.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

By symmetry & unitary property.

$S_{12}\,S^*_{12} + S_{13}\,S^*_{13} = 1$ --------- (1) ; $S_{12}\,S^*_{12} + S_{23}\,S^*_{23} = 1$ ------------- (2)

$S_{13}\,S^*_{13} + S_{23}\,S^*_{23} = 1$ --------- (3) ; $S_{13}\,S^*_{23} = S_{12}\,S^*_{23} = S_{12}\,S^*_{23} = 0$ -------- (4)

If S_{11} is not equal to zero the fourth equation gives $S_{13} = 0$ then $S_{23} = 0$, this inconsistency proves the statement. (i.e.) impossible to construct a perfect matched, lossless & reciprocal three port network.

REFERENCES

[1] Christo Ananth, S.Esakki Rajavel, S.Allwin Devaraj, M.Suresh Chinnathampy. "RF and Microwave Engineering (Microwave Engineering).", ACES Publishers, Tirunelveli, India, ISBN: 978-81-910-747-5-8, Volume 1,June 2014, pp:1-300.

[2] Christo Ananth, Vivek.T, Selvakumar.S., Sakthi Kannan.S., Sankara Narayanan.D, “Impulse Noise Removal using Improved Particle Swarm Optimization”, International Journal of Advanced Research in Electronics and Communication Engineering (IJARECE), Volume 3, Issue 4, April 2014,pp 366-370