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Stabilization of Position and Orientation of Quadcopter System

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***Abstract*—This paper describes the modeling and control of unmanned aerial vehicle type quadrotor. A PD attitude and position controller is proposed for trajectory tracking control of quadrotors. Quadrotor was modeled using Euler-Lagrange method considering the parameter variations and external disturbances. Firstly a nominal PD attitude controller was designed which forms the inner loop followed by the designing of nominal PD position controller. The nominal position controller forms the outer loop. Simulation of the proposed controller was carried out in Matlab and performance of the closed loop system was analyzed for the translational and rotational dynamics.**

***Index Terms*—Nominal controller, parameter variations, uncertainties, unmanned aerial vehicle (UAVs).**

I. INTRODUCTION

UNMANNED aerial vehicles are suitable for applications such as remote sensing, surveillance, transport explorations in dangerous and inaccessible environments etc.[1]-[2]. The unmanned aerial vehicle used in this paper is quadrotor. Unlike conventional helicopters, quadrotors have four independent rotors. The four rotors can be controlled separately, each producing a torque that makes rotors spin producing an upward thrust. Just like single rotor helicopters, the torque created would make body of the quadrotor spin in the opposite direction of the rotors. To avoid this phenomenon two opposite rotors spin clockwise and the other two counterclockwise so that the effect is balanced.

Many studies have been done on the design of attitude and position controllers for quadrotors [3]. The quadrotor dynamics involves uncertainties such as parameter variations, non- linearities, coupling and external disturbances which make its controller design a challenging task. Conventional feedback controllers like proportional-derivative (PD) controllers and proportional-integral-derivative (PID) controllers were discussed by [4], [5] to achieve trajectory tracking control. Nonlinear control methods such as backstepping control, nested saturation control, quaternion based feedback control were used in [6], [7], [8] for attitude and position control. The above mentioned techniques require accurate nonlinear model for better tracking. Quadrotors are often used in environments that are constantly subjected to

varying payload conditions, civil and military tasks. Thus designing of a robust controller that can withstand such disturbances and parameter variations has gained utmost importance. A Takagi-Sugeno fuzzy model was developed representing the nonlinear model and a stabilizing fuzzy controller was designed to achieve good dynamical response. [9] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. In [10], a switching model predictive controller was presented to achieve attitude control against atmospheric disturbances.

In [11], a robust controller based sliding mode approach was proposed to restrain the influence of uncertainties involved in the rotational and translational dynamics. The external disturbances existing in both rotational and translational dynamics were ignored in stability analysis of closed loop control system by [12] and [13]. In [14], the robustness of the quadrotor system against external aerodynamics disturbances was demonstrated in simulation. But the effect of uncertainties in the rotational and translational dynamics on the closed loop system was not completely discussed.

This paper is aimed at designing a trajectory tracking controller consisting of an attitude loop for controlling the three attitude angles and position loop for controlling the translational trajectory. Here a simplified linear model still valid for quadrotors under hovering conditions is used and the unmodeled dynamics is assumed to be of multiplicative form.

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The plant is assumed to be of minimum phase.

The attitude and position controllers were designed based on proportional –derivative (PD) controller. Here an important signal, equivalent disturbance, is introduced to represent the perturbations involved in the plant on the tracking properties. The proportional and derivative gains of the controller were chosen by trial and error method. The virtual control inputs from the longitudinal and latitudinal PD controllers were used as the desired references for the pitch and roll dynamics.

The outline of this paper is as follows. Section II is the model description of quadrotor; Section III involves design of nominal attitude and position controller and its application on to the quadrotor model. Simulation results are described in Section IV. Concluding remarks are given in Section V.

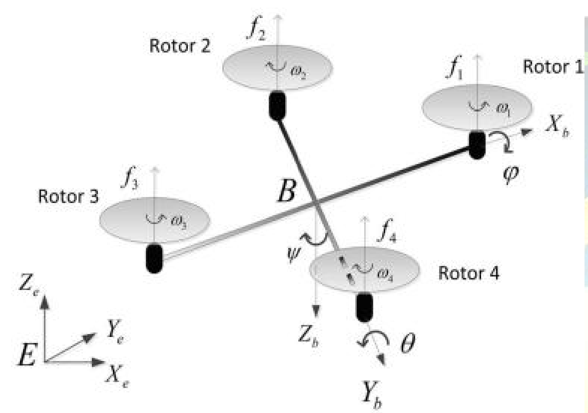

Fig. 1. Schematic of the quadrotor

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II. Model Description

Movements of quadrotor are obtained by the changes in the thrust forces generated by the rate of change of rotational speed of four rotors of the quadrotor as shown in Fig. 1. The upward movement is obtained when all the rotors spin at the same rate. In this case total thrust vector acts perpendicular to the ground and when the absolute force becomes exactly equal to the gravitational force on the quadrotor, the vehicle will hover at a particular height. The lateral and longitudinal movement of the quadrotor is obtained by decreasing the rate of rotation of one rotor and increasing the rate of rotational of the opposite one by the same amount. As a result the qaudrotor will roll or pitch in the direction of slow propeller and lateral and longitudinal movement is achieved. Finally the yaw movement can be obtained by slowing down the spinning rate of two opposite propellers and increasing the spinning rate of others by the same amount so that the total thrust vector is not influenced.

To develop the model of quadrotor we assume two frames of references, one inertial frame and the body frame. The inertial frame is defined by the ground, with gravity pointing in the negative z direction. The body frame is defined by the orientation of the quadrotor, with the rotor axes pointing in the positive z direction and the arms pointing in the x and y directions. The orientation of the quadrotor from body frame to the inertial frame is given by a rotation matrix, R. This is obtained by three successive rotations about z-axis followed by a rotation about the newly obtained y-axis and ending with a rotation about the newly obtained x-axis. x, y, z form the longitudinal, latitudinal and vertical positions of the quadrotor. The Euler angles, i.e., the roll angle ϕ, the pitch angle θ, and the yaw angle ψ are used to describe the rotational motions. To avoid the singularities the roll and pitch angles are bounded as $|\phi| \le \pi/2$ and $|\theta| \le \pi/2$. The rotational matrix is obtained as follows:

$$R = \begin{bmatrix} C_\theta C_\psi & C_\psi S_\phi S_\theta - C_\phi S_\psi & S_\phi S_\psi + C_\phi C_\psi S_\theta \\ C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & C_\phi S_\theta S_\psi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\phi C_\theta \end{bmatrix}$$

where C_i and S_i indicate $cos(i)$ and $sin(i)$ respectively.

The equations of motion describing the rotational and translational dynamics of quadrotor are as follows:

$$\begin{aligned}
\ddot{x} &= (\cos\varphi \sin\theta \cos\psi + \sin\varphi \sin\psi)\frac{f}{m} \\
\ddot{\varphi} &= \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) + \frac{J_R}{I_x}\dot{\theta}\,\Omega_R + \frac{L}{I_x}\tau_\varphi \\
\ddot{y} &= (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)\frac{f}{m} \\
\ddot{\theta} &= \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + \frac{J_R}{I_y}\dot{\phi}\,\Omega_R + \frac{L}{I_y}\tau_\theta \\
\ddot{z} &= -g + (\cos\phi \cos\theta)\frac{f}{m} \\
\ddot{\psi} &= \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}\tau_\psi
\end{aligned} \tag{1}$$

where m denotes the mass of the quadrotor, f and $\tau = [\tau_\phi \quad \tau_\theta \quad \tau_\psi]$ are the aerodynamic lift forces and torques with respect to the body fixed frame. I_x, I_y and I_z are the moment of inertia. J_R and Ω_R are the moment of inertia and angular velocities of the propeller blades. The lift forces are given as

$$f = k_f \omega_i^2, \quad i = 1,2,3,4$$

and the reactive torques generated by the four rotors are given as

$$\tau_i = k_\tau \omega_i^2, \quad i = 1,2,3,4$$

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where k_f and k_τ are positive parameters depending on the factors such as the air density and the type of the blades. Then, f and τ can be given as

$$\begin{aligned} f &= k_f(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2) = k_f u_z \\ \tau_\varphi &= k_f(\omega_2^2-\omega_4^2) = k_f u_\varphi \\ \tau_\theta &= k_f(\omega_1^2-\omega_3^2) = k_f u_\theta \\ \tau_\psi &= k_\tau(\omega_1^2-\omega_2^2+\omega_3^2-\omega_4^2) = k_\tau u_\psi \end{aligned} \qquad (2)$$

where u_z, u_ϕ, u_θ and u_ψ are the collective, roll, pitch and yaw control inputs to the quadrotor plant.

Since the quadrotor typically operates in the hovering condition we can make small angular approximations and neglect higher order terms. Thus the linear dynamics get reduced to the following form:

$$\ddot{x} = g\theta, \qquad \ddot{y} = -g\varphi \qquad \ddot{z} = \frac{u_z}{m} - g$$
$$\ddot{\varphi} = \frac{L}{I_x}u_\varphi, \quad \ddot{\theta} = \frac{L}{I_y}u_\theta, \quad \ddot{\psi} = \frac{1}{I_z}u_\psi \qquad (3)$$

But there are perturbations, parameter variations and external disturbances in a practical system. Hence we define the term q_i, equivalent disturbance [15] which contains all the uncertainties and external disturbances. Therefore the model of practical system can be written as

$$\begin{aligned} \ddot{x} &= g(\theta + q_x) \\ \ddot{y} &= -g(\varphi + q_y) \\ \ddot{z} &= \frac{(u_z + q_z)}{m} - g \\ \ddot{\varphi} &= \frac{L}{I_x}(u_\varphi + q_\phi) \\ \ddot{\theta} &= \frac{L}{I_y}(u_\theta + q_\theta) \\ \ddot{\psi} &= \frac{1}{I_z}(u_\psi + q_\psi) \end{aligned}$$

From (4), we can see that uncertainties exist in every equation and it involves both parameter variations and external disturbances entering through the input side.

III. PD OUTPUT TRACKING CONTROLLER

The nominal attitude and position PD controller is designed based on the model given in (4). The main function of the nominal position controller is to guide the longitudinal, latitudinal and height dynamics along the desired reference paths. The virtual control inputs from the nominal longitudinal and latitudinal controller is also used as the required references for the pitch and roll dynamics. The nominal attitude PD controller is used to track the required roll, pitch and yaw angles.

A. Nominal Position Controller

The position controller keeps the quadrotor at the desired point in the inertial frame. It consists of longitude control, latitude control and altitude control. The horizontal movement is brought about by orienting the thrust vector in the desired direction. This is done by pitching or rolling the quadrotor to the desired reference angles. These reference angles are provided by the position controller. The virtual control inputs for the longitudinal, latitudinal and height channels are as follows:

$$\begin{aligned} \theta_r &= -(k_x^p e_x + k_x^d \dot{e}_x) = r_\theta \\ \phi_r &= k_y^p e_y + k_y^d \dot{e}_y = r_\phi \\ u_z^N &= -(k_z^p e_z + k_z^d \dot{e}_z) \end{aligned} \qquad (6)$$

TABLE I
NOMINAL PD CONTROLLER PARAMETERS

Parameter	Value	Parameter	Value	Parameter	Value
k_x^p	13	k_y^p	8	k_z^p	13
k_ϕ^p	7.5	k_θ^p	8	k_ψ^p	13
k_x^d	3	k_x^d	2.5	k_x^d	3
k_ϕ^d	1	k_θ^d	2.5	k_ψ^d	2

where $e_x = x - r_x$, $e_y = y - r_y$, $e_z = z - r_z$ and $k_i^j\,(i = x, y, z;\, j = p, d)$ are the nominal position controller parameters. r_θ and r_ϕ are the desired references for the pitch and roll angles.

B. Nominal attitude controller

Attitude controller is an integral part of the whole control system and a precondition for the position control. The virtual control inputs for the roll, pitch and yaw channels are

$$u_i^N = -(K_i^p e_i + K_i^d \dot{e}_i); \quad i = \phi, \theta, \psi \qquad (7)$$

where $e_i = i - r_i\,(i = \theta, \phi, \psi)$ and $k_i^j\,(i = \theta, \phi, \psi;\, j = p, d)$ are the positive control parameters.

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IV. SIMULATION RESULTS

A quadrotor system was modeled and the nominal PD controller was designed and applied to the Simulink model of the system. All the simulations were carried out in Matlab/ Simulink. Step input was given as the references for both rotational and translational dynamics to study the response of the quadrotor model.

Response of nominal controller

The nominal controller is the PD controller which involves three PD controllers for rotational dynamics and another three for translational dynamics. The nominal controller parameters are chosen such that it guarantees required tracking and also maintains the stability of the overall system. Trial and error method is used to choose the controller parameters based on steady state error, overshoot and settling time requirements. The designed controller parameters are given in Table I for trajectory tracking performance. The response of the nominal controller without considering the equivalent disturbances is shown in Fig. 3 and Fig. 4, respectively. In the absence of perturbations, the nominal PD controller gives good performance. Now a random disturbance of magnitude 250 and frequency 10Hz is applied to the quadrotor model. The response of the controller in the presence of disturbance without changing the control parameters is shown in Fig.5.

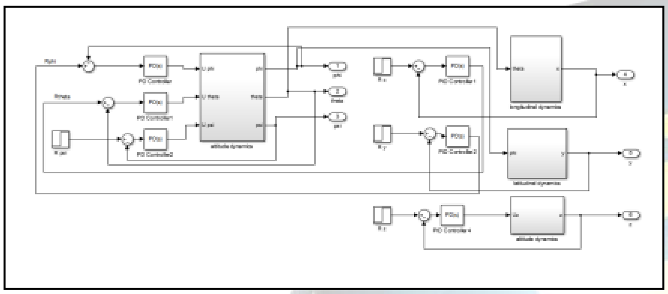

Fig.2. Simulink diagram of nominal controller

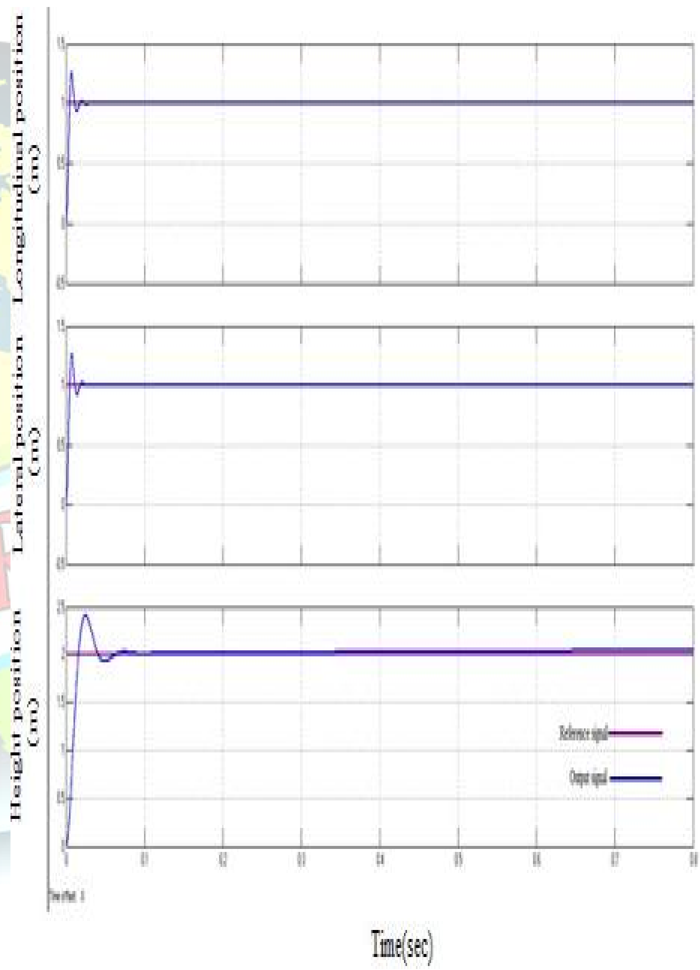

Fig. 3. Responses of nominal position controller without considering equivalent disturbances.

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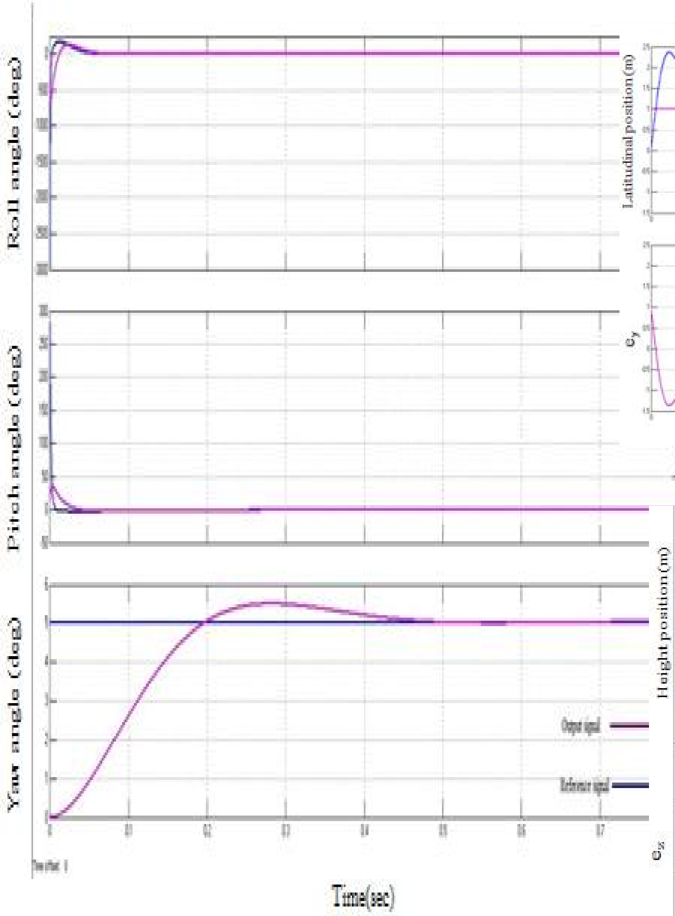
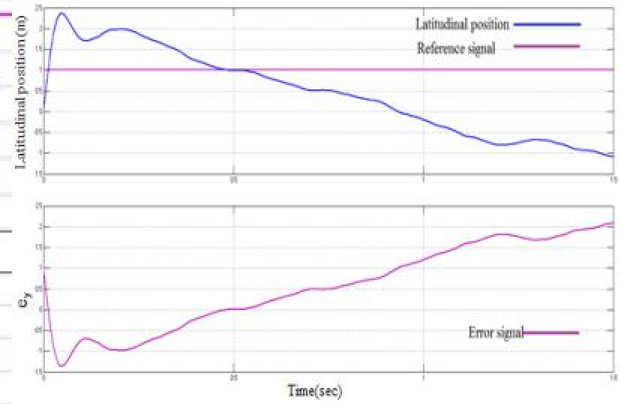
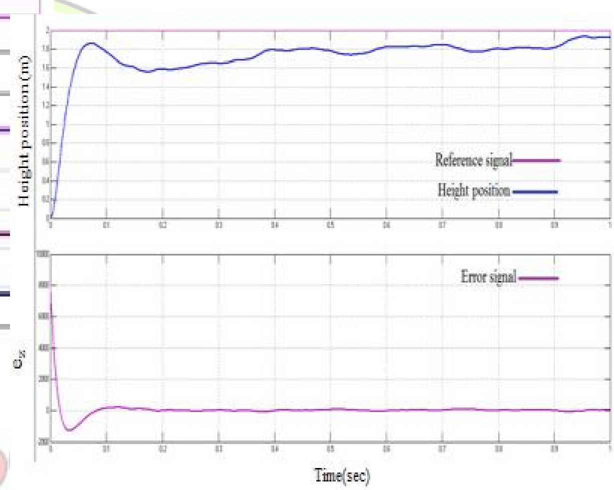


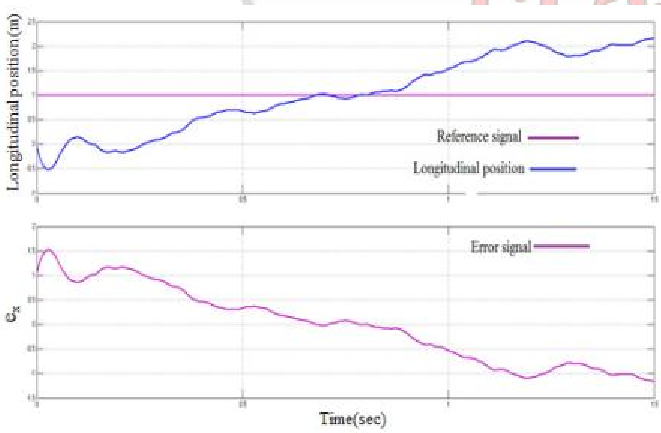
Fig. 4. Responses of nominal attitude controller without considering equivalent disturbances



(b)



(c)



(a)

Fig. 5. Responses of three positions in the presence of random disturbance without changing the control parameters (a) Longitudinal response (b) Latitudinal response (c) Height response.

Now the proportional and derivative parameters are tuned to obtain the desired performance specifications as given in Table II.

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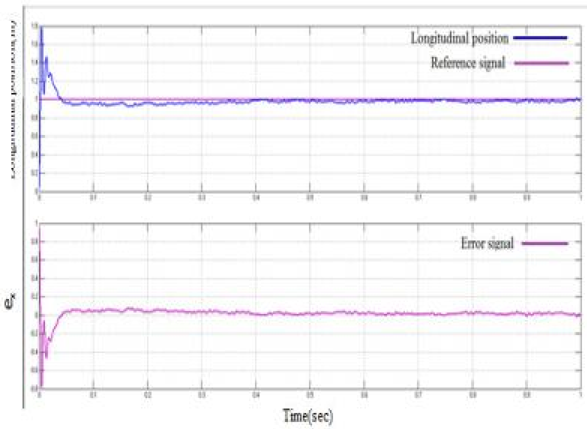
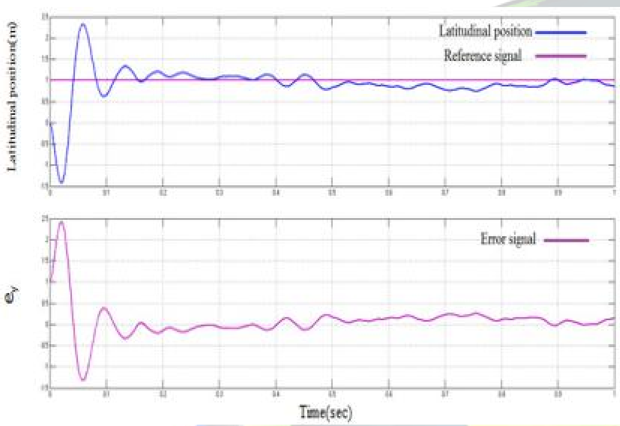
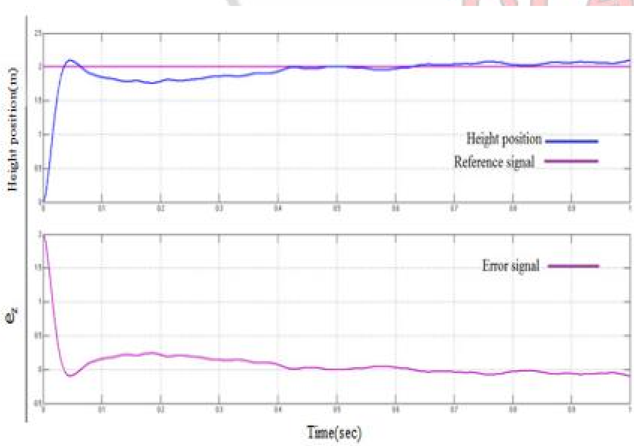


(a)



(b)



(c)

Fig.6. Response of the nominal controller for the new control parameters (a) Longitudinal response (b) Latitudinal response (c) Height response.

V. CONCLUSIONS

A PD output tracking controller was proposed to achieve the trajectory tracking control of quadrotor with uncertain parameters. The controller includes nominal position controller and nominal attitude controller. The controller was designed, simulated and the robustness properties were studied. For perturbations of high magnitude the controller could not provide considerable reduction in steady state error and settling time. The main disadvantage of this method is that PD controller has to be tuned according to variations and disturbances entering the system. Hence this PD output tracking controller should be made robust by introducing a low pass filter to eliminate the parameter variations and guide the PD controller to track the desired reference paths.

TABLE II
PERFORMANCE SPECIFICATIONS

	Control parameters k_i^P	k_i^d	Percent Error, e	Settling time, t_s
Longitudinal dynamics, x	30	5	1	1
Latitudinal dynamics, y	30	1	3	1
Altitude dynamics, z	35	2	8	1

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