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Fractional Order System Identification of Magnetic Levitation System

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***Abstract*—Magnetic Levitationsystems have always been an area of interest with the development of MAGLEV trains around the world, its control and study became momentous. They have wide application in in high precision positioning platforms, high speed trains, Magnetic bearings, vibration isolation systems and wind tunnel levitation systems. Fractional order (FO) controllers are among the nascent solutions for increasing closed-loop performance, robustness and stability assurance. Hence they can be used in Magnetic levitation systems. The tuning of the Fractional order Proportional Integral Derivative (FOPID) controller parameters are done using Particle swarm optimization and Genetic algorithm. The integer order system model is identified to Fractional order using FOMCON toolbox by using an initial guess Fractional Mag-Lev model. Among the frequency domain identification methods Hartley, Levy& Vinagre method is used in this brief. The stability of the Mag-Lev system with and without the FOPID controllers are analyzed.**

***Index Terms*— Magnetic Bearings, robustness, Fractional order controllers, FOMCON, Frequency domain identification, Vinagre Method**

I. Introduction

In recent years Magnetic levitation systems has been gaining scientific interest due to its ability to eliminate friction as well as high non linearity and instability. They have enormous applications such as inhigh speed trains, magnetic bearings, vibration systems and wind tunnel levitation[3].

Various control solutions have been proposed for such systems, such as back stepping and feedback linearization techniques [4], whichrequire a very accurate model for the magnetic levitation system. This may represent a major problem, because it may be difficult to obtain a precise model, because of the existing non linearities that characterize the system, including the variation of the gain of the magnetic Levitation system[5].Other control approaches are based on designing the controllers for the dynamical model which is linearized at nominal operating points. Nonetheless, the tracking performance of the controllers for the dynamical model deterioratesdrastically with increasing deviation from nominal operating points.

The various controlling techniques used earlier were sliding mode control, nonlinear control,µ-synthesis, PIDs combined with notch filters, gain scheduling, fuzzy neural network based controllers have been proposed,[3],[4].Lepatic et al applied a method of fuzzy predictive functional control to Magnetic levitation system.First,a lead compensator was designed to stabilize the system then fuzzy controller is applied based on TS rule.Fractional controlhas emerged as a modern trend in control engineering, withmany applications in both modeling [10], [12] and control design. The main advantage of fractional order controllers is that fractional order models of real systems are more adequate than usually used integer order models. They also have the ability to intensify the performance of closed loop systems and they also increase the robustness. The tuning approach for FOPID controllers consists of specifying a set of performance criterion and making them into equations. Using optimization routines or graphical methods the controller parameters are determined. [7, [8].]For unstable systems few papers dealt with the design of FO controllers. ForMagnetic levitations FO controllers have been previously designed. However all these papers propose the general form of FO PI or PD controllers and tuning techniques uses the frequency domain procedures. In this brief an FOPID controller is proposed for the magnetic levitation system. The tuning procedure involves optimization routines.

The controller parameters are determined using Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) and the results were compared.As the tuning by PSO gave the better performance of the system.i.e reduced overshoot, reduced settling time, and reduced steady state error it is used for tuning of FOPID

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controller when applied to the Magnetic Levitation system. The system response withFOPID controller tuned using PSO and GA is evaluated.The integer order

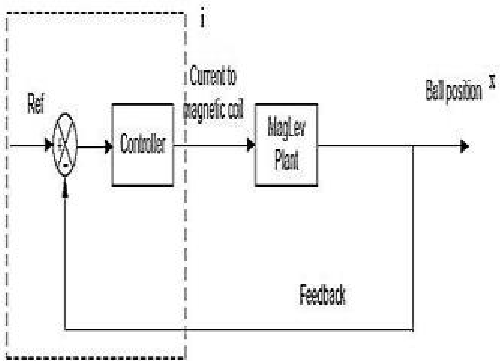

Fig.1 Block diagram of Maglev system with controller

System model is identified to Fractional order using FOMCON toolbox by using an initial guess Fractional Mag-Lev model. Among the frequency domain identification methods, Hartley, Levy& Vinagre method is used in this brief [9]. The fractional order system response is studied using the FOPID controller implemented for the Fractional order system.

The brief is structured into Five main parts. SectionII presentsthe modeling of magnetic levitation considering two operating points as well as the system response without controller. Section IIIdetails the tuning procedure for FOPID controller and the simulation results. Section IV presents the system identification of Integer Order model to Fractional order model using FOMCON toolbox. Section V presents the stability analysis of Mag-Lev system with and without controller. The final section summarizes the main outcome of this brief.

II SYSTEM DESCRIPTION MODELING OF MAGNETIC LEVITATION SYSTEM

The system with controller is shown in Fig.1. For the modeling of Mag-Lev system, the free body diagram given in Fig.2 is used, as well as electromagnetic and mechanical equations. The resulting total force acting on the permanent disk magnet, denotedF_{total},is computed using Newton's law of motion while neglecting friction

$$F_{Total} = F_g - F_l = mg - c\frac{i^2}{x^2} \quad (1)$$

Where F_l is the levitation force [7] and the F_g is the gravitational force, m is the mass of the permanent magnet[kg],g=9.81is the gravitational speed constant[m/s^2],x is the distance [m],

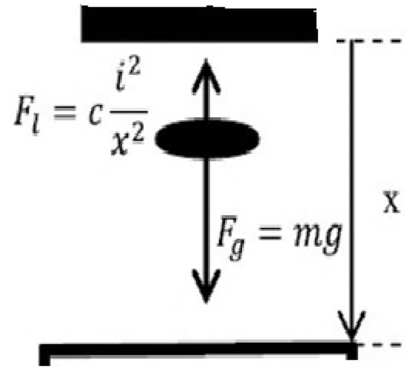

Fig.2 free body diagram of Magnetic Levitation system

$\ddot{x}$ is the acceleration of the permanent magnet [m/s^2], and c is the magnetic force constant. The net force is equal to

$$F_{Total} = m\ddot{x} \quad (2)$$

Equating (2) to (1) it is obtained

$$m\ddot{x} - mg + c\frac{i^2}{x^2} = 0 \quad (3)$$

Denoting the equation in (3) $f(x,\ddot{x},i) = m\ddot{x} - mg + c\, i^2/x^2$ is then linearized with respect to equilibrium point (x_0,i_0) by applying the Taylor series expansionas

$$f(x,\ddot{x},i) = f(x_0,\ddot{x}_0,i_0) + \left.\frac{\partial f}{\partial x}\right|_{(x_0,\ddot{x}_0,i_0)}(x-x_0) + \left.\frac{\partial f}{\partial \ddot{x}}\right|_{(x_0,\ddot{x}_0,i_0)}(\ddot{x}-\ddot{x}_0) + \left.\frac{\partial f}{\partial i}\right|_{(x_0,\ddot{x}_0,i_0)}(i-i_0) \quad (4)$$

Considering now the linearization point (x_0, i_0), and denoting $\Delta i = i - i_0$ and $\Delta x = x - x_0$, (4)may be rewritten

$$f(x,\ddot{x},i) = -\frac{2c\, i_0^2}{x_0^3}\Delta x + m\Delta\ddot{x} + \frac{2c\, i_0}{x_0^2}\Delta i = 0 \quad (5)$$

Application of Taylor's series leads to

$$-\frac{2ci_0^2}{x_0^3}\Delta x(s) + ms^2\Delta x(s) + \frac{2ci_0}{x_0^2}\Delta i(s) = 0 \quad (6)$$

This leads to the final transfer function

$$\frac{\Delta x(s)}{\Delta i(s)} = -\frac{\frac{2ci_0}{x_0^2}}{ms^2 - \frac{2ci_0}{x_0^3}} \quad (7)$$

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This leads to the final transfer function of the mechanical sub system as

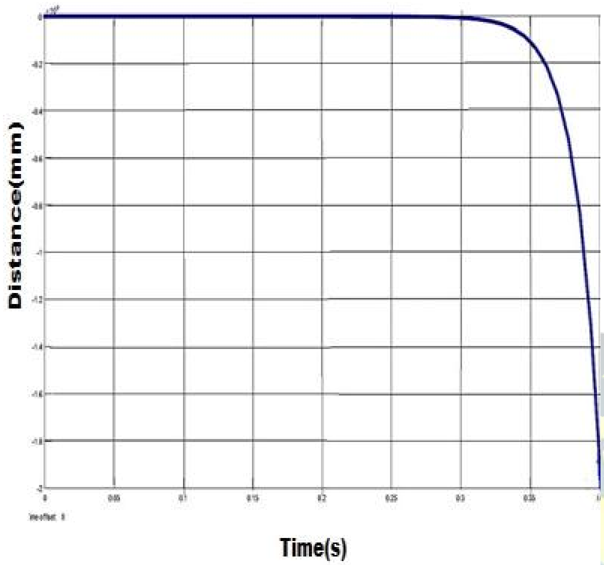

Fig 3 Response of Mag-Lev system without controller

$$\frac{\Delta x(s)}{\Delta i(s)} = \mathrm{P(s)} = -\frac{253.9}{s^2 - 3011} \quad (8)$$

The value of c is computed as the average value over the interval for x_0 between 3 and 6.5 mm. For an equilibrium point chosen as (x_0, i_0)= $(4.2 \times 10^{-3}$ m, 49.8×10^{-3} A), and m=1.79g.
The state space model of the system can be represented as

$$\left.\begin{array}{l} A = \begin{bmatrix} 0 & 3011 \\ 1 & 0 \end{bmatrix} \\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ C = \begin{bmatrix} 0 & -253.9 \end{bmatrix} \\ D = 0 \end{array}\right\} \quad (9)$$

Mathematical model of nonlinear magnetic levitation system was developed considering different forces acting on the payload. System's state space model was also found from the transfer function model. In order to understand the importance and necessity of a controller in the nonlinear Maglev system, the system is simulated without any controller. The simulation results can be seen in Fig.3.It is clear from the response that the system is unstable without controller; hence a controller is needed in order to stabilize the system. Thus a Fractional OrderPID controller (FOPID) was implemented for the system. But to tune the controller parameters, some calculation criterion are required.

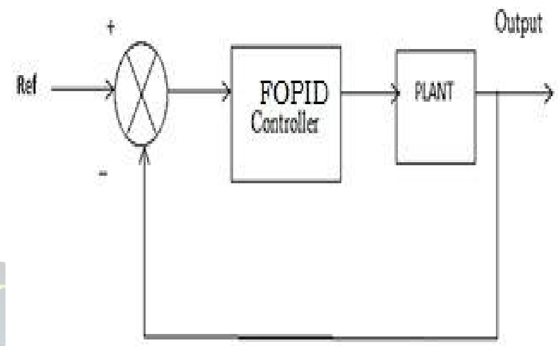

Fig 4 Schematic diagram of controller with the plant

III TUNING OF FOPID CONTROLLER USING PARTICLE SWARM OPTIMIZATION AND GENETIC ALGORITHM

Fractional order PID controller has five design parameters such as Kp, Ki, Kd, μ & λ which provides accurate closed loop performance.
The transfer function of FOPID controller is

$$G_{FOPID} = K_p + \frac{K_i}{S^{\lambda}} + K_d S^{\mu} \; (\lambda, \mu \geq 0) \quad (11)$$

The schematic diagram of plant with FOPID controller isshown in the fig.4.Objective functions were developed to find FOPID controller parameter that results in reasonably small overshoot, fastest rise time or quickest settling time.
For this purpose four objective functions were tested and these are Integral time square Error (ITSE), Integral of Absolute Error (IAE), Integral of Squared Error (ISE), and Integral of Time Absolute Error (ITAE), where they are given by Eqn. (10)

$$\left.\begin{array}{l} \mathrm{IAE} = \int_0^T |e(t)| dt \\ \mathrm{ISE} = \int_0^T e^2(t) dt \\ \mathrm{ITAE} = \int_0^T t|e(t)| dt \\ \mathrm{ITSE} = \int_0^T te^2(t) dt \end{array}\right\} \quad (10)$$

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In the end an objective function was chosen with which the error can be minimized and the error criterion chosen was ITAE.Overall objective of FOPID controller was to minimize the error, thus fitness value are measured using the error. Fitness value was defined as Fitness value $=\frac{1}{ITAE}$.

A.Tuning of FOPID by Particle Swarm Optimization(PSO)

FOPID controllers can be used to control any system owing to their robustness and faster response.The crucial requirement for a closed loop control system including the controller is to sustain the stability and robustness throughthe rejection of the disturbance and exclusion of noise.

In this brief, it is decided to tune the parameters lambda and mu besides setting the proportional, integral and derivative constants. For tuning of the FOPID controller, a set of performance indices are chosen. The performance criterion chosen here is ITAE. Performance index is defined as a quantitative measure to depict the system performance when used the FOPID controller along with the system. Using this technique an 'optimum system' can be designed and a set of FOPID parameters in the system can be manipulated to meet the required specifications. The aim of the PSO-based optimization is to obtain a set of FOPID parameters suchthat the feedback control system has minimum performance index.

B.Tuning of FOPID controller by Genetic Algorithm

In this brief, Genetic Algorithm is used to find the optimum tuning of the FOPID controller, by forming double precision chromosomes is created representing the solution space for the PID controller parameter (kp,ki,kd, λ and μ), which represent the genes of chromosomes.The GA proceeds to find the optimal solutionthrough several generations [8].The table I shows the parameters used in genetic algorithm..

From the system response, it is clear that PSO tuning is better for tuning parameters of FOPID controllers.Hence we used it for tuning of FOPID controller.. The Simulink model for comparing GA and PSO tuned values with the Mag-Lev system is shown in fig 5 and the simulation results are shown in fig 6 .The Table IIshows the values obtained after PSO and GA method of tuning the parameters.

IV FRACTIONAL ORDER FREQUENCY DOMAIN SYSTEM IDENTIFICATION USING FOMCON TOOLBOX

Fractional order system can be denoted as

$$a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \ldots\ldots + a_0 y(t) = u(t) \quad (11)$$

Where y (t) is system output and u (t) is step input. It should be noted that the differential order, $\alpha_n, \alpha_{n-1}, \ldots\ldots \alpha_0$ are of fractional order. The objective of system Identification is to estimate the model parameters and

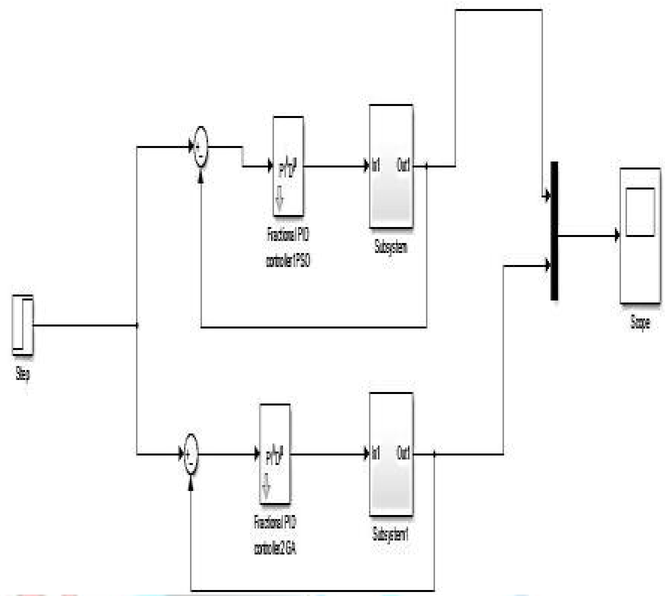

Fig 5 Simulink model of GA tuned and PSO tuned FOPID controller

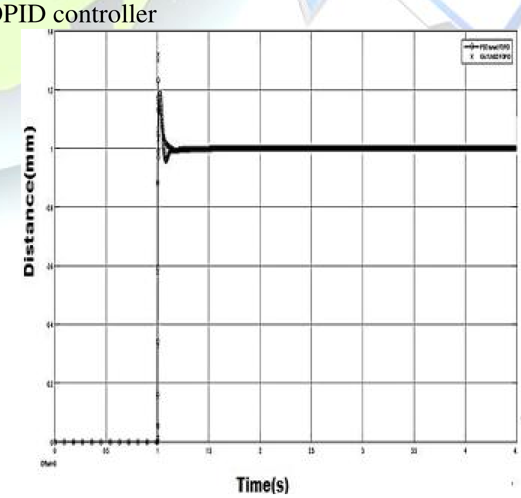

Fig 6 simulation results of GA tuned and PSO tuned FOPID controller

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fractional order vector $[\alpha]=[\alpha_1, \alpha_2 \ldots \alpha_n]$ Fractional order identification is performed with FOMCON toolbox implementing the Grunwald-Letnikov differ Integral method, Oustaloup filter method and Refined Oustaloup filter in time domain.
whereas in frequency domain Hartley, Levy and Vinagre methods are used [9].
The integer order system is identified to fractional order system by FOMCON Toolbox. FOMCON provides time-domain and frequency-domain fractional-order system analysis, as well as verifying system stability. The toolbox is comprised of the following modules: Main module (fractional system analysis); Identification module (system identification intime and frequency domains);
Control module (fractional PID controller design, tuning and optimization tools as well as some additional features) [9].Fig 7 shows the FOMCON main module relations of fractional order transfer function analysis.
Fig 8 shows the fractional order system identification tool.
The integer order system is identified to fractional order system by using an initial guess fractional order model, which is a fractional order Mag-Lev system. This is given by

$$\frac{\Delta x(s)}{\Delta i(s)} = \frac{53.012 s^{0.17885}}{23.62 s^{0.81016} + 27.575 s^{0.01899} + 28.663} \quad (12)$$

Among the models identified, Vinagre method gave the better performance as the error between identified model and initial guess model is less.
The bode plot of the systems identified using the Hartley frequency domain method are shown in fig 9. The error betweeninitial guess model and obtained model using Hartley method were found to be 0.0340.The identified model is given by

$$G(s) = \frac{1}{0.064883 s^{0.8} + 0.92371 s^{0.4} + 1.8947} \quad (13)$$

The identified model using levy and vinagre method is given as

$$G(S) = \frac{0.0067994 s^{0.8} + 0.06539 s^{0.4} + 0.3981}{0.2054 s^{0.8} + 0.3704 s^{0.4} + 1} \quad (14)$$

$$G(S) = \frac{-0.097015 s^{0.8} + 26699 s^{0.4} + 0.1457}{01.9038 s^{0.8} + 1.4074 s^{0.4} + 1} \quad (15)$$

The errors were found to be as 0.0073 and 0.0042 in Levy and Vinagre method respectively. The bode plot obtained by levy method is shown in fig 10 and that of vinagre is shown in fig 11. [2] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction.
The results prove that the response given by Vinagre method was better and also the stability results obtained were given that the system was stable with an order of 0.4.This is shown in fig 17.The step response of Model identified using Vinagre method is shown in fig 12.The step response is found to be better in Vinagre method.

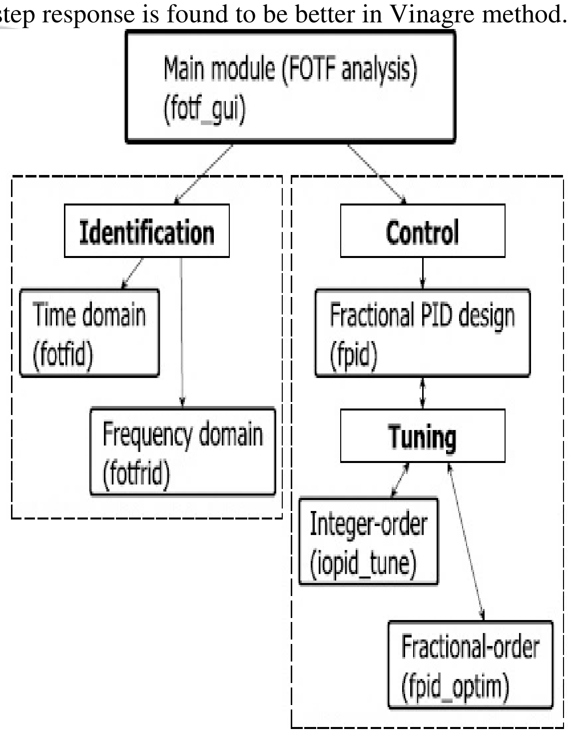

Fig 7 FOMCON module relations

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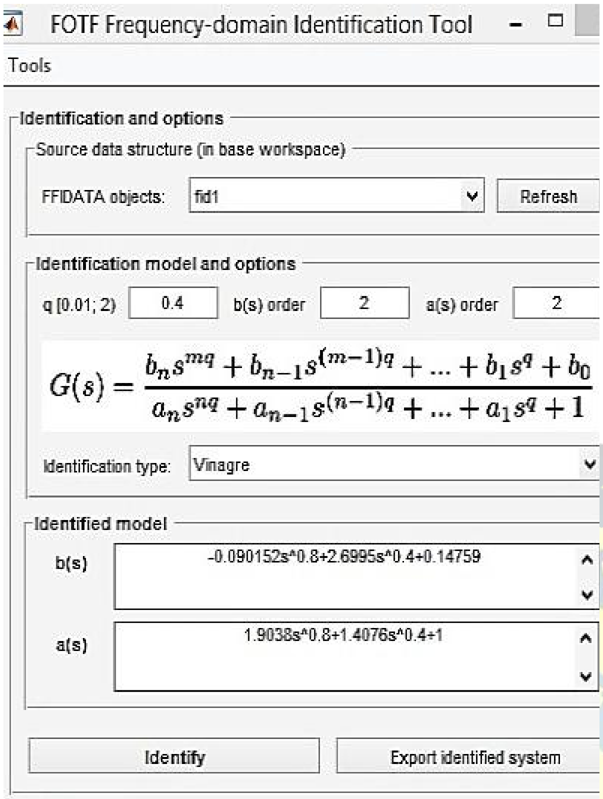

Fig 8Frequency domain identification tool

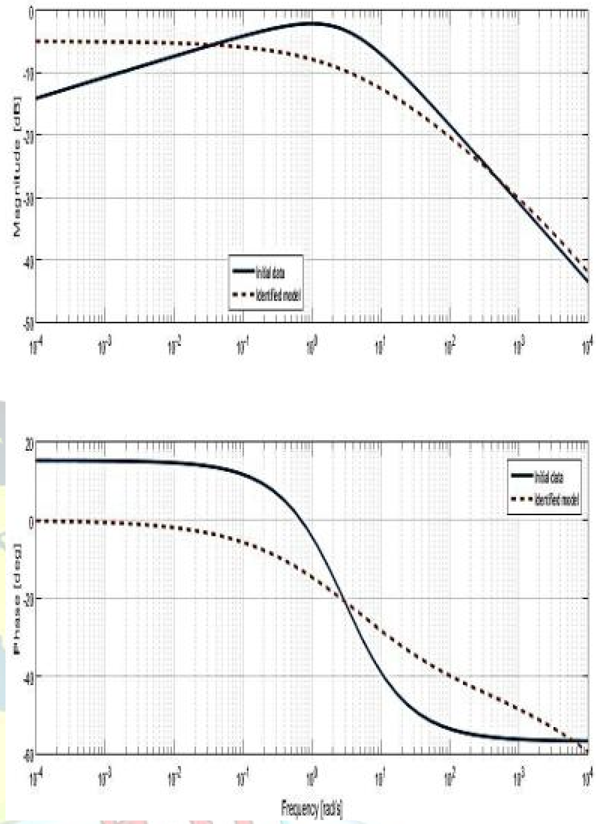

Fig 10Bode plot of identified model using Levy method

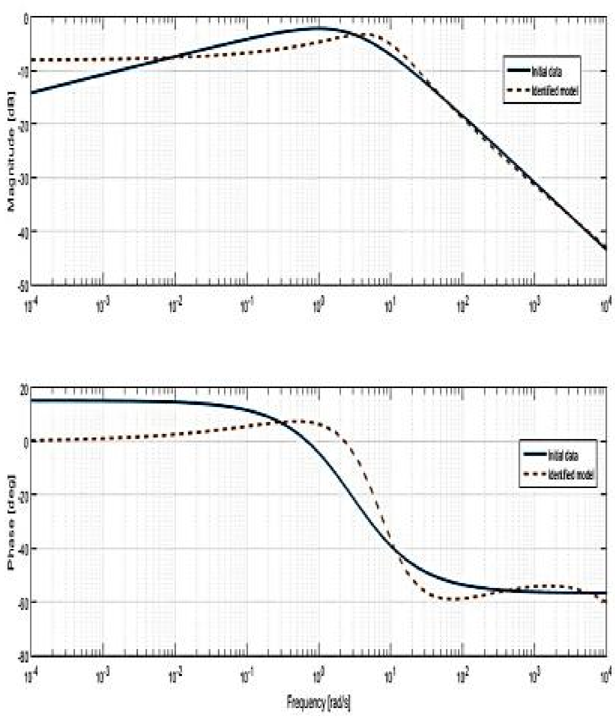

Fig 9 Bode plot of identified model using Hartley method

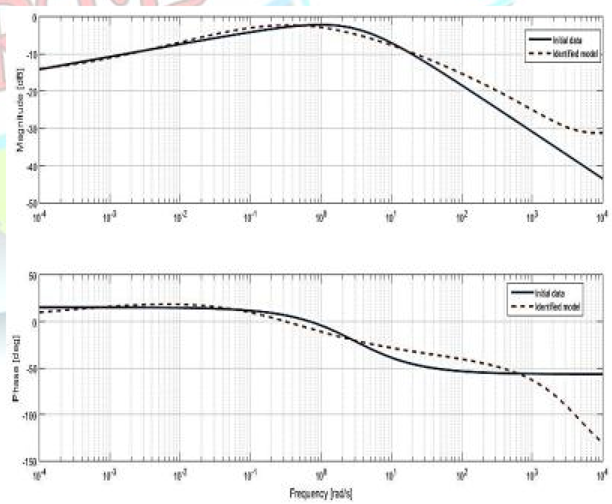

Fig 11Bode plot of identified model using Vinagre method

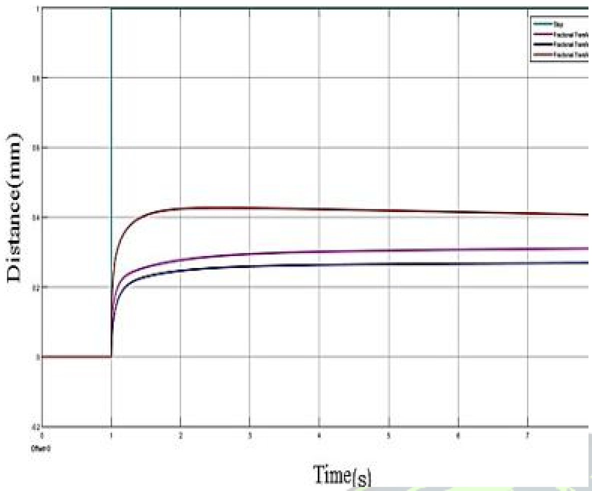

Fig12 comparison of open loop Responses of identified models

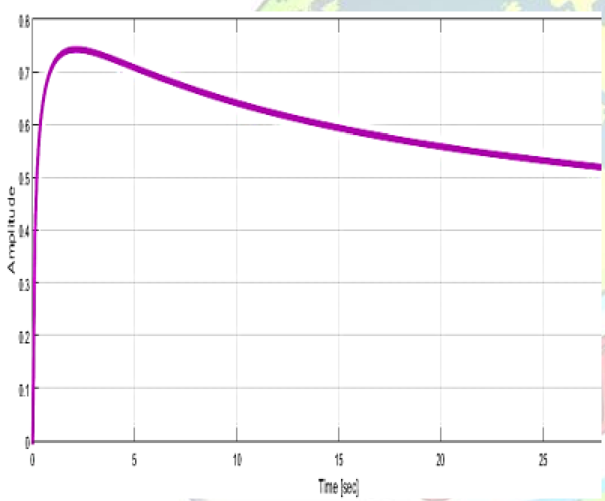

Fig 13 Step response of Vinagre model

V STABILITY ANALYSIS OF PSO TUNED FOPID CONTROLLER USING FOMCON TOOLBOX

After running the PSO code in MATLAB, we obtained the position vector of the best particle (optimized values of controller design parameter) i.e. kp, ki,kd, λ and μ. The values are shown in Table II. On substituting the value of design parameters) into (11), we got the following fractional order PID controller for Magnetic levitation system.

$$G_{FOPID}(S) = 136.21 + \frac{5870}{S^{1.23}} + 0.23S^{1.32} \quad (16)$$

The transfer function of unity feedback control loop withfractional order controller and the Magnetic levitation system is of the following form.

$$G(s)=\frac{Go(s)}{1+Go(s)}=\frac{P(s)G_{FOPID}(s)}{1+P(s)G_{FOPID}(s)}=$$

$$\frac{58.397s^{2.55}+34584s^{1.23}+1.4904e+06}{s^{3.23}+58.397s^{2.55}+37595s^{1.23}+1.4904e+06} \quad (17)$$

The fractional order Magnetic Levitation system is simulated in MATLAB environment using FOMCON Toolbox.The stability region of Magnetic levitation system without the controller is shown in Fig 14. When fractional order PID controller is implemented with the Mag-Lev system, it became stable. The stability region of Magnetic levitation system controlled by fractional order PID controller is shown in Fig 15 .From the dialog box it is confirmed that the system is stable with the FOPID controller.

VI CONCLUSION

This brief aims to find better tuning method for FOPID controller. The tuning procedure for the controllers is based on optimization techniques called Particle swarm optimization and Genetic Algorithm. The better results were obtained by using Particle Swarm Optimization The brief also focused upon a far better solution for the stabilization of Magnetic Levitation systems. The simulation results show that the designed FOPID controller ensures the stability of the closed loop Magnetic levitation system at the chosen equilibrium point. In order to study the fractional order systems, the integer order is identified to fractional order by fractional order system identification using FOMCON toolbox. The comparison of frequency domain identification methods (Hartley, Levy& Vinagre) is done.The stability analysis using FOMCON toolbox is done. Further developments include implementation of FOPID controller for the identified Fractional order system is identified using Vinagre method.

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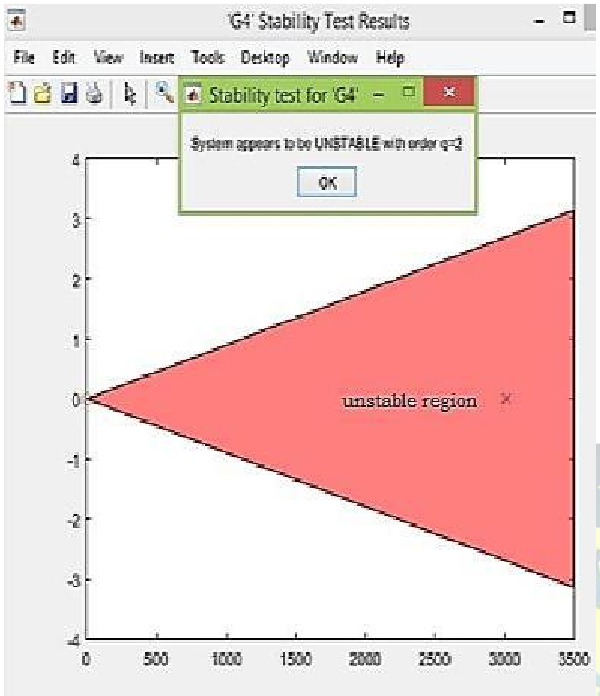

Fig 14 FOTF identification tool showing instability of Mag-Lev system without controller.

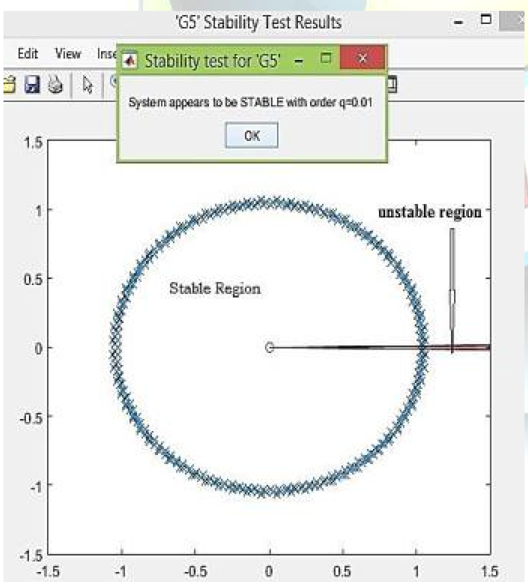

Fig 15 FOTF identification tool showing stability of Mag-Lev system with FOPID controller

Table I

Selection Method	Tournament
Population size	120
Generation Size	40
Cross Over Method	Arithmetic Crossover
Mutation Method	Uniform Mutation
Selection Probability	0.08
Elite count	6

Parameters of Genetic Algorithm

Table II
Performance indices and tuning values

Tuning method and performance indices	K_P	K_I	K_D	λ	μ
PSO-FOPID (ITAE)	136.21	5870	0.23	1.23	1.32
GA-FOPID (ITAE)	150.21	5000	1.325	1.221	1.22

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REFERENCES

1. Silviu Folea, Cristina I. Muresan, Robin De Keyser, and Clara M. Ionescu, "*Theoretical Analysis and Experimental Validation of a Simplified Fractional Order Controller for a Magnetic Levitation System" IEEE transactions on control systems technology* May 21, 2015.
2. Christo Ananth,"A NOVEL NN OUTPUT FEEDBACK CONTROL LAW FOR QUAD ROTOR UAV",International Journal of Advanced Research in Innovative Discoveries in Engineering and Applications[IJARIDEA],Volume 2,Issue 1,February 2017,pp:18-26.
3. K. S. Peterson, J. W. Grizzle, and A. G. Stephanopoulos, "Nonlinear control for magnetic

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ISSN 2394-3785 (Online)
Available online at www.ijartet.com

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levitation of automotive engine vales," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 2, pp. 346–354, Mar. 2006.

4. C. Ionescu, J. T. Machado, and R. De Keyser, "Fractional-order impulse response of the respiratory system," *Comput. Math. Appl.*, vol. 62, no. 3, pp. 845–854, 2011.
5. R.-J. Wai and J.-D. Lee, "*Robust levitation control for linear Maglev rail system using fuzzy neural network*," *IEEE Trans. Control Syst. Technol* vol. 17, no. 1, pp. 4–14, Jan. 2009
6. Suresh Kumar1, Smt.K.Vindhya *"Design of Tuning Methods for Fractional order PIDController using PSO Algorithm"*. Smitha,Volume 2 Issue XII, ISSN: 2321-9653December 2014
7. Long-Yi Chang Hung-Cheng Chen"*Tuning of Fractional PID Controllers Using Adaptive Genetic"Algorithm for Active Magnetic Bearing System* wseas transactions on systems
8. Aleksei Tepljakov, Eduard Petlenkov, and Juri Belikov" *FOMCON: a MATLAB Toolbox for Fractional-order System Identification and Control"* International journal of Microelectronics and computer science vol.2.no.2 2011
9. Ji Zhen,Liu Huilian,Wu Qinghua *"Particle Swarm Optimization And Application,* "Beijing Science press(2009)
10. Mahmud Iwan Solihin, Lee Fook Tack and Moey Leap Kean "*Tuning of PID Controller Using Particle SwarmOptimization (PSO)* "14 - 15 January 2011
11. Zhuo Lia Lu Liuab Sina Dehghana YangQuan Chena∗and Dingy¨u Xueb "*A Review and evaluation of numerical tools for fractional calculus and fractional order controls"* International Journal of ControlVol. 00, No. 00, Month 20XX, 1–2
12. Podlubny I., Dorcak L. and Misanek J., 1995, "Application of fractional-order derivatives to calculation of heat load intensity changein blast furnace walls", Transactions of Tech. Univ. of Kosice, vol. 5, no. 5, pp.1 7-144.
13. Chen Y.Q., Petras I. and Dingyu Xue., 2009, "Fractional Order Control:A Tutorial", American Control conference, June 10-12
14. Wen, Yu. and Floriberto, Ortiz., 2005, "Stability analysis of PD regulation for ball and beam system", Proceedings of the 2005 IEEEConference on Control Applications, pp. 517-522.
15. Monje C. A.,Vinagre B. M., Feliu V., ChenY.Q., Tuning and Auto-Tuning of Fractional Or-der Controllers for Industry Application, *ControlEngineering Practice*, 2008, pp.798-812.
16. Petras I., The fractional-Order Controllers:Methods for their Synthesis and Application,*Journal of Electrical Engineering*, No. 9-10,1999, pp.284-288.