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Position Tracking of Ball and Beam System using Fractional Order Controller

Sameera R
Department of Electrical and ElectronicsEngineering
Mar Baselios College of Engineering &Technology
Kerala, India
sameerar80@gmail.com

Arun.S.Mathew
Department of Electrical and Electronics Engineering
Mar Baselios College of Engineering &Technology
Kerala, India
arun.s.mathew83@gmail.com

***Abstract*—The ball and beam system is the classical mechanical system having unstable dynamics and strong nonlinear characteristics which makes the control a challenging task. In this paper a fractional order controller is used to enhance a better position control. The fractional order PID controller is an improved form of normal PID controller with more number of controlling parameters. The ball and beam system is a cascaded system with inner loop servo motor angle control and outer loop with ball position control. The inner loop is a Proportional Derivative controller designed based on time domain specifications and the outer loop is a fractional PID controller. The controlling parameters of fractional PID are designed based on Particle Swarm Optimization (PSO) algorithm. The simulation of the proposed controller is analyzed using Matlab Simulink and the performance of the system is analyzed.**

***Index terms*—nonlinear, position control, cascaded system, fractional order PID, Particle Swarm Optimization (PSO)**

I. INTRODUCTION

Ball and beam system is the most popular and important laboratory model used for testing control techniques. Due to its high level of nonlinearity and instability, the controlling of ball and beam system is considered as a difficult problem for control systems. It is generally linked to real time control problems such as roll control of aircraft during landing. The aim of the system is to control the ball position to a predefined position and reject the external disturbances. The control signal can be obtained by feeding back the information of the ball position. The control voltage signal operates the DC motor and the torque generated drives the beam to a desired angle. The ball and beam system is an inherent open loop unstable system as the ball position changes for a fixed beam angle without any limit. So different control strategies can be used to make the system stable.

There are many research works done on ball and beam system for achieving desired position control. A modified form of feedback linearization is used in [1] where the ball starts from the center of the beam and the model have singularities for standard techniques .For a well-defined set of initial conditions, an asymptotically stabilizing PD controller is proposed [2].Some researchers used non model based control strategies such as Neural Network [3], Fuzzy Logic to control the ball position and beam angle, but these methods do not guarantee the stability of the system. In [5],in order to obtain optimal PID values, meta heuristic technique current search method is used. In [6] Sliding Mode Controller is proposed and it overcome the problem associated with singular states and experimentally validated the results. A strict feedback form with modeling errors is formulated for the dynamic model of ball and beam system. Observer based nonlinear control uses the same coordinate transformation to design a nonlinear observer for the ball velocities[7]

The tuning of PID controller parameters is made easily with the advent of heuristic optimization techniques such as Genetic Algorithm [8] and Particle Swarm Optimization (PSO) [9].These techniques helps to achieve optimum solution. The fractional order PID controllers has proved superior performance than traditional PID controller[10].The fractional order PID controller need five parameters to be tuned. The paper presents PSO as an effective algorithm to tune fractional order parameters.

The rest of the paper is organized as follows. Section

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II covers the mathematical modeling of ball and beam and servomotor .Section III presents a review of fractional PID controller .Section IV describes an overview of steps involved in optimizing the parameters using Particle Swarm Algorithm. Section V describes the controller design and simulation results. Section VI concludes the paper.

II. MATHEMATICAL MODELING

The Ball and Beam module is pictured in Fig 1.It consists of a track on which the metal ball is allowed to roll on it. A linear transducer is fitted on the track to measure the position of the ball. The transducer outputs a voltage signal proportional to the ball position. One side of the beam is attached to a lever arm which is coupled to the load gear of the servomotor. The beam angle can be adjusted to balance the ball to a desired position by controlling the servomotor position. Balance control for the ball and beam system is considered as a difficult control problem. Hence a suitable controller is to be designed for the system.

Fig 1: Quanser Ball and Beam System

To understand the complete dynamics of the system accurately, the analysis of the ball and beam system and the dc servomotor should be done separately. The parameter to develop the dynamics of the system is shown in Table 1.

Table 1: PARAMETERS OF BALL AND BEAM SYSTEM

Sl No	Parameter	Value
1	Mass of ball	0.064kg
2	Radius of ball	0.0127m
3	Beam length	0.4255m
4	Distance between servo gear shaft and coupled joint	0.0254 m
5	Acceleration due to gravity	$-9.8 m/s^2$

A.Model for Ball and Beam system

Lagrange approach is used for the modeling of ball and beam system. It is based on energy balance of the system. Assume that the ball rolls without slipping. The Euler-Lagrange is used to define the kinetic and potential energy for the system is shown in equations (1) and (2).

$$T = \frac{1}{2}\left[\left(J_b + m_b x^2\right)\dot{\alpha}^2 + \frac{7}{5} m_b \dot{x}^2 \right] \quad (1)$$

J_b -Moment of inertia of beam

m_b - Mass of the ball

x - Position of the ball

α - Beam angle

$$V = m_b g x \sin\alpha \quad (2)$$

g -acceleration due to gravity

The Lagrange function is the difference between kinetic energy and potential energy of the system.

$$L = T - V \quad (3)$$

Lagrange equations are obtained as follows.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = \tau \quad (4)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \quad (5)$$

Where x -ball position on beam

α - beam angle

$$\left(J_b + m_b \dot{x}^2\right)\ddot{\alpha} + 2m_b x\dot{x}\dot{\alpha} + m_b g x\cos\alpha = \tau \quad (6)$$

$$\ddot{x} + \frac{5}{7}\left(g\sin\alpha - x\dot{\alpha}^2\right) = 0 \quad (7)$$

Linearizing equation (7) around the operating points, the equation becomes as follows

$$\ddot{x} = \frac{-5}{7} g \sin\alpha \quad (8)$$

The beam angle and the angle of the gear can be related by equating the arc distance is specified as follows

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$$\alpha = \frac{r_{arm}}{L_{beam}}\theta$$

(9)

On substituting equation (9) in (8) and taking Laplace transforms, the transfer function of the ball and beam transfer function with system parameters can be represented as

$$\frac{X(s)}{\theta(s)} = \frac{0.4182}{s^2} \qquad (10)$$

B. Model for D C Servomotor

Fig 2 illustrates the schematic of DC armature circuit and gear train. The input of the system is the voltage applied to the armature of motor, while the output is the position of the shaft. The rotor and motor shaft are assumed to be rigid. The torque generated by the motor is proportional to armature current

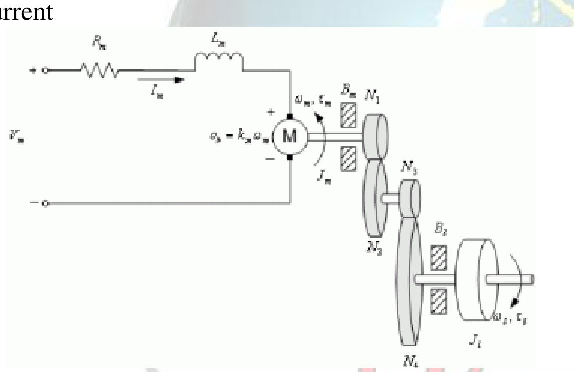

Fig 2: DC armature circuit and gear train

The specifications of dc servomotor to develop the system dynamics is shown in Table 2.

Table 2: SPECIFICATIONS OF DC SERVOMOTOR

Symbol	Description	Value
k_m	Back emf constant of motor	7.68*10^-3 Vs/rad
k_t	Current torque constant	7.68*10^-3 Nm/A
K_g	Gear ratio	70
w_m	Motor shaft speed	628.3 rad/s
R_a	Motor armature resistance	2.6 ohm
η_g	Gearbox efficiency	0.90
η_m	Motor efficiency	0.69
J_m	Motor shaft moment of inertia	3.9*10^-7 kg/m^2
J_l	Load shaft moment of inertia	7.06*e^-8
B_{eq}	Equivalent viscous friction on shaft	0.015 Nms/rad

Using Kirchhoff's law to the armature circuit and neglecting the armature winding inductance

$$V_m(t) - I_a R_a - L_a\frac{dI_a}{dt} - k_m w_m(t) = 0 \qquad (11)$$

V_m -input motor voltage

k_m -back emf constant of motor

w_m -motor shaft speed

$$w_m(t) = K_g w_l(t) \qquad (12)$$

The torque at load shaft due to applied motor torque is given by

$$\tau_l(t) = \eta_g K_g \tau_{ml}(t)$$

$$\tau_{ml}(t) = \frac{\tau_l(t)}{\eta_g K_g} \qquad (13)$$

η_g -gearbox efficiency

Equation for motion of load can be expressed as follows

$$J_l\frac{dw_l(t)}{dt} + B_l w_l(t) = \tau_l(t) \qquad (14)$$

J_l -moment of inertia of load

B_l - inertia coefficient of load

τ_l -total torque applied on load

Equation for motor shaft can be obtained as follows

$$J_m\frac{dw_m(t)}{dt} + B_m w_m(t) + \tau_{ml}(t) = \tau_m(t) \qquad (15)$$

J_m -moment of inertia of motor shaft

τ_{ml} -resulting torque acting on the motor shaft

Substitute (12),(13),(14) in (15) gives

$$J_{eq}\frac{dw_l(t)}{dt} + B_{eq} w_l(t) = \tau_m(t)\eta_g K_g \qquad (16)$$

Where

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$$J_{eq} = K_g{}^2\eta_g J_m + J_l$$
$$B_{eq} = Kg^2\eta_g B_m + B_l$$
(17)

Combining the dynamics of load and motor, the equation becomes as

$$J_{eq}\frac{dw_l(t)}{dt} + B_m w_l(t) = A_m V_m(t) \quad (18)$$

Let

$$B_m = B_{eq} + \frac{\eta_g \eta_m K_g{}^2 k_m k_t}{R_a}$$
(19)
$$A_m = \frac{\eta_g \eta_m K_g k_t}{R_a}$$

Taking the Laplace transform, the transfer function of the motor becomes as

$$\frac{\theta_1(s)}{V_m(s)} = \frac{A_m / B_m}{s(1 + sJ_{eq} / B_m)} = \frac{K}{s(\tau s + 1)} \quad (20)$$

Substituting the nominal parameters K and τ from the motor specifications

$$\frac{\theta_l(s)}{V_m(s)} = \frac{1.53}{s(1 + .0211\ s)} \quad (21)$$

III. FRACTIONAL ORDER PID [FOPID] CONTROLLER

Fractional order control is the non-conventional way of robust control which is based on fractional order derivative. Fractional calculus is the generalization of integration and differentiation. It is represented by a fractional order fundamental operator ${}_\alpha D^\beta{}_t f(t)$ where α and t are limits, $\beta\ \varepsilon\ R$ is the order.

The operator is defined as

$${}_\alpha D^\beta{}_t f(t) = \begin{cases} \frac{d^\beta}{dt^\beta} : \beta \rangle 0 \\ 1 : \beta \approx 0 \\ \int_\alpha^t (d\tau)^{-\beta} : \beta \langle 0 \end{cases} \quad (22)$$

The control action of Fractional order PID control using integral differential is given by

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^\delta e(t) \quad (23)$$

Applying Laplace transform, the transfer function of the controller can be expressed by

$$C(s) = K_p + K_i s^{-\lambda} + K_d s^\delta \quad (\lambda, \delta > 0) \quad (24)$$

The FOPID controller not only needs three parameters K_p, K_i and K_D but also need to design two orders λ and δ of integral and derivative controllers. The FOPID expands the integrator of real order λ and the differentiator of real order δ from point to plane. This expansion adds more flexibility to controller design and gives an opportunity to adjust the dynamical properties of the fractional order control systems.

IV. PARTICLE SWARM OPTIMIZATION

The Particle Swarm Optimization is a new intelligent optimization algorithm designed by Kennedy and Eberhert in 1995.The algorithm simulates the migration and aggregation of bird flock when they search for food. The search path determined by the PSO algorithm has better performance, has less parameters and easier to realize than early intelligent algorithm.

The PSO algorithm creates initial particles and assigns initial velocities. It checks the objective function at each particle location and then chooses the best function value and location, the algorithm selects new velocities and locations which is nearer to the current velocity and location. Then the algorithm updates the particle location and velocities iteratively .This continues until a stopping criteria is reached.

V. CONTROLLER DESIGN AND SIMULATION RESULTS

The ball and beam system is an open loop unstable system. The open loop response is shown in fig 3

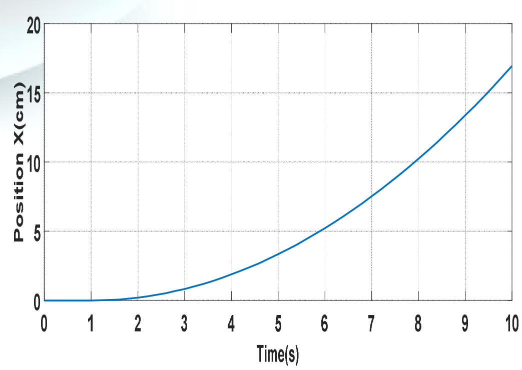

Fig 3: Open loop response of ball and beam system

The response shows that the system is unstable. So

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some controllers are necessary to bring the ball to a desired position. The ball and beam system is having a cascaded control design which is shown in fig 4.

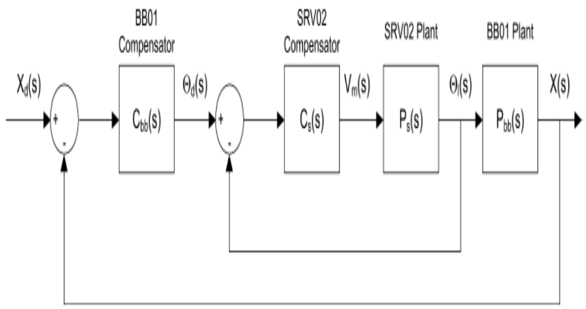

Fig 4:Cascaded control system to control ball position

BB01 compensator represents the controller for the Ball and Beam system(BB01Plant).SRV02 Compensator is the inner loop controller for servomotor (SRV02 Plant) .Based on the measured ball position, the controller in the outer loop computes the servo load shaft angle to attain the desired ball position. The inner loop is a servo position control and the controller in the inner loop calculates the motor voltage required to track the desired load shaft angle. The paper presents PD controller for inner loop Fractional PID controller for outer loop.

A.Inner loop controller design

The inner loop implements a Proportional Derivative (PD) controller to manage the position of servomotor. The time-domain specifications for controlling the position of servomotor are :

Steady state error=0
Settling time=3.5s
Peak overshoot=10.0%

A high pass filter is used for the inner loop. The ball position is measured using an analog sensor and it has noise associated with it. So whenever this value is fed back to the motor, it produces an amplified noise. High pass filter can prevent this problem. The transfer function of high pass filter is

$$H(s) = \frac{w_f^{\,2} s}{s^2 + 2\xi w_f s + w_f^{\,2}} \qquad (19)$$

Where w_f is the cut off frequency. Choose w_f=1Hz
Based on time domain specifications, the value of PD controller is

K_p=11.6 ;K_d=0.103

Fig 5 and Fig 6 shows the servo voltage response and servo angle response respectively.

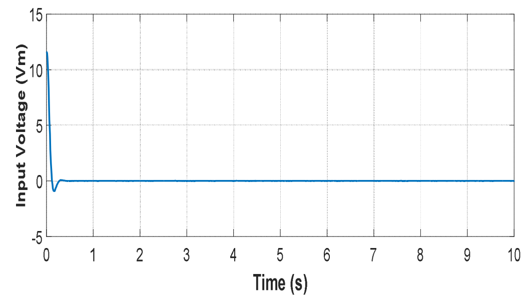

Fig 5:Servo motor input voltage

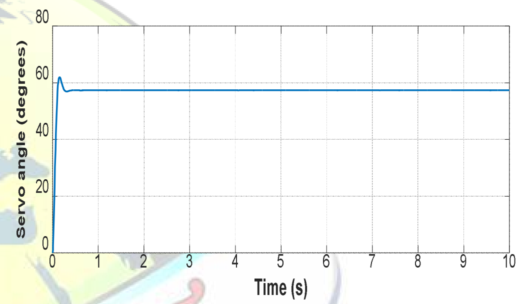

Fig 6:Servo motor angle

B. Outer Loop Controller Design

Tuning of Fractional PID using PSO

First consider the number of variables to be tuned in order to optimize the solution to the problem. Then define the PSO parameters. In our case, the food is analogous to reference position of the ball. Since each iteration requires large time, achieve the global optimum in minimum number iterations The particles are initialized at random positions within the bounds on Fractional PID gains and are not to allowed to trespass during the algorithm. Then find the fitness value needs to be selected. Any performance index can be used as fitness criterion like IAE (Integral Absolute Error), ITAE (Integral Time Absolute Error), ISE (Integral Squared Error).Here we select ITAE as fitness criterion. Then find the most optimum solution within the given number of iterations. [4] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. Utilizing the versatile back venturing method, every one of the six DOF are effectively

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followed utilizing just four control inputs while within the sight of un demonstrated flow and limited unsettling influences.
The tuned controller parameter is shown in Table 3

Table 3: TUNING PARAMETERS OF FOPID CONTROLLER

K_p	K_i	K_d	λ	δ
5	0.1	5	0.9	0.83

The results of simulation with the tuned fractional PID parameter are shown in Fig 7 and Fig 8 .The nonlinear plant model has been used for the simulations. The motor input voltage should not exceed $\pm 10\ V$.

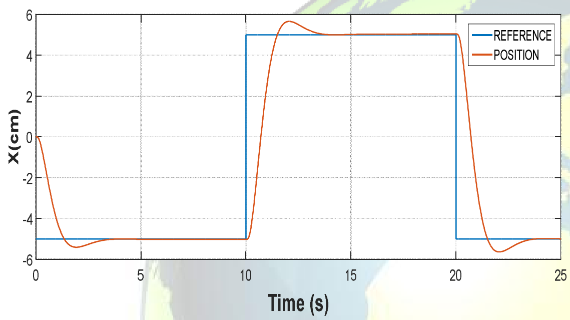

Fig 7: Ball position using fractional PID controller

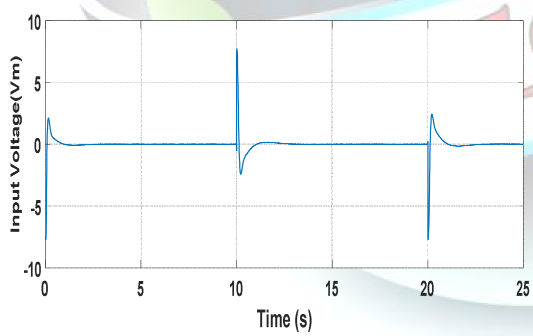

Fig 8: Servomotor input voltage using fractional PID controller.

The ball response is satisfactory with less overshoot and has good settling time. The ball position get tracked with the reference position . Also it shows zero steady state error. The servomotor input voltage is within the saturation limits.

VI. CONCLUSION

The ball and beam system have been widely used for studying new control techniques. A new design method is used to determine fractional order controller parameters using Particle Swarm Optimization method is presented. Simulation results show that the proposed controller can perform well for controlling the ball position. The fractional order controller gives satisfactory performances and possesses minimum settling time with less overshoot.

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