



A Mathematical Model to Find the Effect of Indoor Allergens in Allergic Asthma

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Abstract: This study deals with minimum dynamic discrimination information approach. Asthmatic patients with responses to inhaled house dust mite extract were taken three oral doses of montelukast (10mg) or matching placebo. This study investigates the changes in baseline forced expiratory volume in one second FEV1 by the minimum dynamic discrimination information relative to the exponential distribution. The result leads to minimum dynamic discrimination information characterizations of many life time models.

Keywords: Asthma, House dust mite, FEV1, minimum dynamic discrimination information

I. INTRODUCTION

Asthma is a chronic disease of the lungs. It may cause uncomfortable respiratory symptoms such as wheezing, shortness of breath, coughing or a 'tight' chest. Internationally, the rate of increase has been estimated to be as much as 50% every 10 to 15 years. It is influenced by many lifestyles, socio-economic, educational, cultural, healthcare and genetic factors. The causes of asthma remain uncertain. It is not yet understood why symptoms develop or disappear [1]. The central feature of asthma is that the lining of the air passages in the lungs is persistently inflamed and sensitive - even if there are no symptoms at the time. The treatment of asthma concentrates on trying to suppress this inflammation. It is a condition that usually responds to appropriate medication and in the majority, symptoms are easily controlled allowing the person with asthma to live a normal life [3]. An important aspect of care is that the person with asthma can be in control of their own management essential to achieving their goal. It is routinely described as an atopic disease. The cases of asthma that are not demonstrably related to atopy (so-called "intrinsic" asthma) are considered minor anomalies that do not challenge the atopy/ asthma concept. Things that cause asthma episodes are called triggers [2]. Dust mite allergen is a very potent allergic trigger for some asthmatics. Each

person may have different triggers. Triggers can be colds, smoke, allergies, or exercise and other non-specific irritants.

II. TREATMENT METHODS

A. FEV1

The FEV1 is a calculated ratio used in the diagnosis of lung disease. It represents the proportion of a person's vital capacity that they are able to expire in the first second of forced expiration. Normal values are approximately 80%. Predicted normal values can be depend on age, sex, height, mass and ethnicity as well as the research study that they are based upon. A derived value of FEV1% is FEV1% predicted.

B. Atopy to house dust mite

House dust mites (HDM) are one of the most common sources of indoor allergens and are a cause of symptoms in allergic asthma [8]. Dust mites are tiny, invisible insects that live in fibers on carpeting, plush furniture, curtains, mattresses, pillows and bedding. Unlike spiders or ticks, dust mites are not visible to the naked eye and do not bite or transmit disease. The average adult sheds two pounds of dead skin per year much of it while sleeping [5]. Dust mites live in bedding and mattresses and eat these flakes of skin. In people allergic to dust mite, it is often not the mite itself but proteins in their droppings which cause the allergy. Each mite produces about 20 of these waste droppings every day and the droppings continue to cause



allergic symptoms even after the mite has died. However, remember that dust mite allergen is found in all rooms of the house, on the floor and in soft furnishings, not just in the bedroom [6].

C. Preliminary Design: (concepts)

The study had a double-blind, placebo-controlled design. Asthmatic volunteers with dual responses to inhaled dust mite extract participated in the study. They have atopy to house dust mite. At the time of random experiment the skin sensitivity was observed by the house dust mite extract. Immediately, they inhaled the allergen diluents. Each subject inhaled three doses of allergen containing the same cumulative concentration. Allergen challenges were also performed according to a standardized protocol [4].

Eligible patients were randomly assigned to receive three oral doses of montelukast as 10mg tablets or matching placebo. Montelukast was administered in three oral doses of 10 mg each at 36 and 12 h before, and at 12 h post allergen challenge. The airway response to allergen was measured by FEV1 in 10 min until 8 h after the last inhalation. The activity of montelukast on the allergen-induced airway responses was determined by comparing the maximum percentage fall in FEV1 between placebo and montelukast in the treatment periods. Treatment –related adverse effects were monitored at each visit. For the assessment of treatment differences (montelukast vs. placebo) in the parameters, Mathematical model was applied. The model contained factors for sequence, patient within sequence, period and pretreatment [4].

III. INDENTATIONS AND EQUATIONS

A. FEV1 (% change from baseline) values

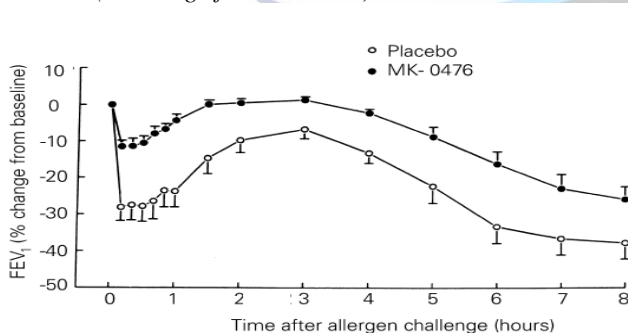


Fig. 1. FEV1in0–8 h after allergen challenge during placebo and montelukast

B. Characterization of the Model

Consider a set of distribution where has PDF and is absolutely continuous with respect to reference distribution that has PDF. The minimum dynamic discrimination information model in relative to is with PDF such that for all. That is, among all the residual PDFs induced by all member of the minimum dynamic discrimination information model is that whose residual PDF retains its minimum dynamic discrimination information property for all. The following theorem gives the properties of the minimum dynamic discrimination information distributions in classes of distributions with mean residual life inequality constraints[7,9,10].

Theorem 3.2.1

Let $\Omega_F = \{F : \mu_F(t) < q(t)\}$ be a compact set of distributions, where F is absolutely continuous with respect to a reference distribution G . Let $F^* \in \Omega_F$ be such that $\mu_F(t) = q(t)$. If $\log(f^*(y)/g(y))$ is decreasing and concave then F^* is the minimum dynamic discrimination information distribution relative to G . The same result holds, with

$$\Omega_F = \{F : \mu_F(t) \geq q(t)\},$$

if $\log(f^*(y)/g(y))$ is increasing and convex.

Proof

We will prove the case in which $\mu_F(t) < q(t)$.

First note that

$$\begin{aligned} K(f : g; t) &= \int_t^\alpha (f(y; t) \log f(y; t)) dy / g(y; t) \\ &= \int_t^\alpha (f(y; t) \log f^*(y; t)) dy / g(y; t) + \\ &\quad \int_t^\alpha (f(y; t) \log f(y; t)) dy / f^*(y; t) \\ &\geq \int_t^\alpha (f(y; t) \log f^*(y; t)) dy / g(y; t). \end{aligned} \quad \dots (1)$$

Where the inequality is due to the fact that the second integral in (1) equals $K(f : f^*; t) \geq t$. This gives

$$K(f : g; t) - K(f^* : g; t)$$



$$\geq \int_t^\alpha (f(y;t) \log f^*(y;t)) dy / g(y;t) - \int_t^\alpha (f^*(y;t) \log f^*(y;t)) dy / g(y;t)$$

$$= \int_t^\alpha (f(y;t) \log f^*(y)) dy / g(y) - \int_t^\alpha (f^*(y;t) \log f^*(y)) dy / g(y)$$

Since $\log(f^*(y)/g(y))$ is decreasing and concave and $\mu_F(t) \leq q(t) = \mu_{F^*}(t)$, the last inequality follows from Theorem 3.A.13 of Shaked and Shantikumar (1994). The proof for $\mu_F(t) \geq q(t)$ is similar. When G has a uniform PDF over $\{y: 0 < y < b\}$, the residual model life PDF $g(y;t)$ is also uniform over $\{y: t < y < b\}$ and the minimum dynamic discrimination information model reduces to the MDE model. The MDE model in a set of distributions $\mu_F = \{F\}$ is the distribution with PDF $f^*(y)$ such that $H(f;t) \leq H(f^*:t)$, for all $t \geq 0$.

Corollary 3.2.1.1

Let $\Omega_F = \{F: \mu_F(t) \leq q(t)\}$ be a compact set of absolutely continuous distributions. Let $F^* \in \Omega_F$ be such that $\mu_{F^*}(t) = q(t)$ if $f^*(y)$ is increasing and log convex. After that F^* is used as the MDE distribution. The same results holds, with $\Omega_F = \{F: \mu_F(t) \geq q(t)\}$. If $f^*(y)$ is decreasing and log-concave.

Here minimum dynamic discrimination information model is related to the exponential distribution, with mean residual inequality constraint [11, 12, and 13]. For generalized normal combination

$$f^*(y) = e^{-(\beta^2/2\alpha)} / \beta \{ (y + \beta)^2 / \alpha - 1 \} e^{-(1/2\alpha)} (y + \beta)$$

With $0 < \alpha < \beta^2$.

IV. MATHEMATICAL INTERPRETATION

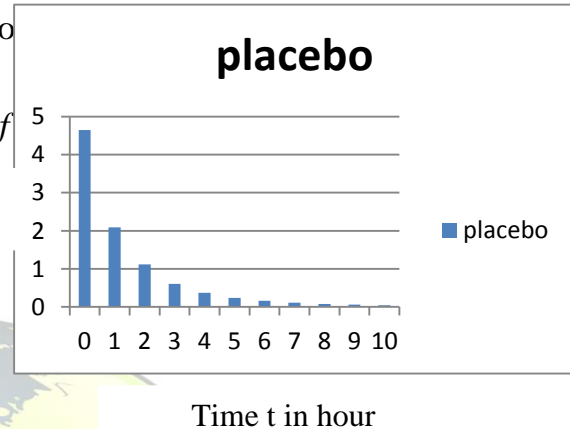


Fig.2 FEV1 in 0-8 h after allergen challenge during placebo

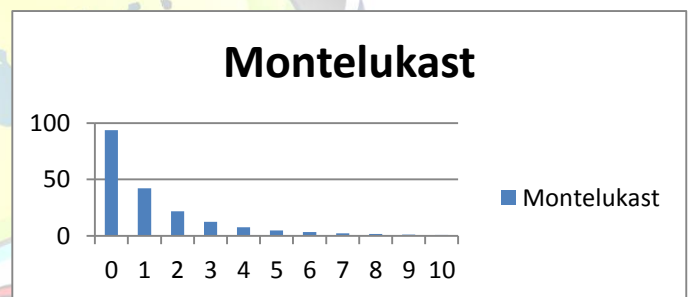


Fig.3 FEV1 in 0-8 h after allergen challenge during montelukast

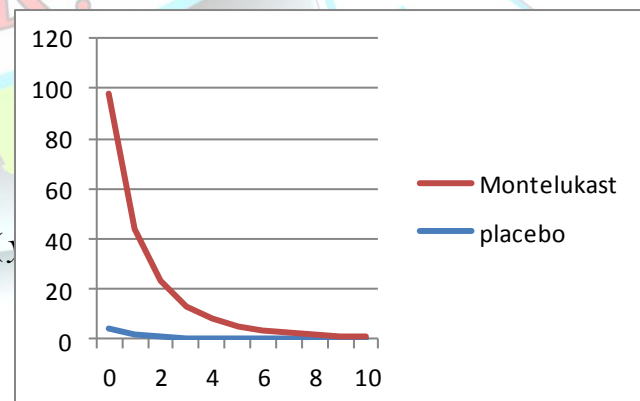


Fig.4 FEV1 in 0-8 h after allergen challenge during placebo and Montelukast



V. CONCLUSION

The minimum dynamic discrimination information MDDI approach to probability modeling is very useful in analyzing different biological processes in the absence of any constraint on the probabilities. This study used MDDI model which is related to the exponential distribution for the generalized normal combination of the functions. That model is used to find the level of FEV1 values for the effect of treatment with oral montelukast on allergen-induced airway responses. Here montelukast group is compared with placebo group. In the montelukast group, the FEV1 values are very high at initial level then it decreases when time value increases. And also the FEV1 values decreases slowly when time value increases for the placebo group. Oral montelukast protects against allergen-induced airway responses in asthma.

REFERENCES

- [1]. Bouchner BS, Udem BJ, Lichtenstrin LM, Immunological aspects of allergic asthma, *Annu Rev Immunol* 1994; 12:295-335.
- [2]. Braback L, Forsberg B. 2009. "Does traffic exhaust contribute to the development of asthma and allergic sensitization in children: findings from recent cohort studies. *Environmental Health* 8:17 DOI 10.1186/1476-069X-8-17.
- [3]. Clark NA, Demers PA, Karr CJ, Koehoorn M, Lencar C, Tamburic L, Brauer M. 2010. Effect of early life exposure to air pollution on development of childhood asthma. *Environmental Health Perspectives* 118(2):284–290 DOI 10.1289/ehp.0900916.
- [4]. Diamant Z., Grootendorst D.C., Veselic-Charvat M., Timmers M.C., De Smet M., Leff A., Seidenberg B.C., Zwinderman A. H., Peszek I., and Sterk P.J., "The effect of montelukast(MK-0476), a cysteinyl leukotriene receptor antagonist, on allergen-induced airway responses and sputum cell counts in asthma", *Clinical and Experimental Allergy*, 1999, Volume 29, pages 42-51.
- [5]. Gøtzsche PC, Johansen HK (2008) House dust mite control measures for asthma.
- [6]. Haahtela T (2014) what is needed for allergic children? *Pediatr Allergy Immunol* 25: 21-24.
- [7]. Jaynes E. T. (1982), On the Rationale of Maximum Entropy Methods *Proc., IEEE*, 70: 939–952.
- [8]. Peat JK, Tovey E, Toelle BG, Haby MM, Gray EJ, et al. (1996) House dust mite allergens. A major risk factor for childhood asthma in Australia. *Am J Respir Crit Care Med* 153: 141-146
- [9]. Pincus S. M. (1991), Approximate Entropy as a Measure of System Complexity, *Proc. Natl.Acad. Sci., USA*, 88: 2297–2301.
- [10]. Pincus S. M. (1992), Approximating Markov Chains, *Proc. Natl. Acad. Sci., USA*, 89: 4432–4436.
- [11]. Pincus S. M., and B. H. Singer (1996), Randomness and degrees of irregularity. *Proc. Natl.Acad. Sci., USA*, 93: 2083–2088.
- [12]. Pincus S. M., and R. E. Kalman (1997), Not All (Possibly) "Random" Sequences are Created Equal, *Proc. Natl. Acad. Sci., USA*, 93:3513–3518.
- [13]. Pincus S. M., and W. M. Huang (1992), Approximate Entropy: Statistical Properties and Applications, *Commun. Stat. Theory Methods*, 21: 3061–3077.



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