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Relative Controllability of Fractional Dynamical Systems with Distributed Delays and Impulses

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Abstract— **In this paper, we establish sufficient conditions for the global relative controllability of linear and nonlinear fractional dynamical systems with distributed delays and …… impulses in control for finite dimensional spaces. The results are obtained by using the Mittag-Leffler functions and Schauder –fixed point theorem. A numerical example is given to illustrate the obtained main results.**

Keywords— Fractional derivative; Relative Controllability; Mittag-Leffler function; Distributed delays; Fixed-point theorem.

I. INTRODUCTION

In recent years, development of adequate techniques for fractional systems has been in focus of scientific attention because of its outstanding importance for a number of physical applications such as physics, mechanics, chemistry, engineering,etc. Controllability is one of the fundamental concepts in mathematical control theory. It means that it is possible to steer a dynamical system from an arbitrary initial state to an arbitrary final state by using a set of admissible controls.

Time delays are often present in various engineering systems such as biological, economical systems, chemical processes. For instance, they appear as transportation and communication lags and also arise as feedback delay in measurement and closed loop systems. Due to the transmission of signal, the mechanical transmission needs a length of time. Klamka [4,7] studied the controllability of nonlinear dynamical systems with distributed delays in control with the Schauder's fixed point theorem, whereas Balachandran and Somasundaram [8]obtained the relative controllability of nonlinear systems with distributed delays in control and implicit derivative using Darbo fixed point theorem.Impulsive control systems with integer derivative have been investigated in[6,8-10].

However, to the best of our knowledge, the relative controllability fractional dynamical systems with distributed delays in control with impulses has not been established yet. In order to fill this gap, In this paper, we study the relative controllability for both linear and nonlinear fractional dynamical systems with distributed delays in control with impulses.

II. PRELIMINARIES

Let $\alpha,\beta > 0, with\, n-1<\alpha<n, n-1<\beta<n$ and $n\in\mathbb{N}$, D be the usual differential operator. Let $\mathbb{R}^m$ be the m-dimensional Euclidean space, $\mathbb{R}_+=[0,\infty)$, and suppose $f\in L_1(\mathbb{R}_+)$.

The Riemann-Liouville fractional operator is defined as follows:

$$(I_{0+}^{\alpha}f)(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)ds,$$

The Caputo fractional derivative is defined as follows

$$({C_D}_{0+}^{\alpha}f)(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1}f^n(s)ds,$$ where the

Function f(t) has absolutely continuous derivatives upto order (n-1).

The Mittag-Leffler function is defined as follows:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}, \text{ for } \alpha,\beta>0$$

The following properties of the previously mentioned operators are specially interesting:

(i) $\left(c_{D_{0^+}}^{\alpha}1\right)=0.$
(ii) $\left(c_{D_{0^+}}^{\alpha}1\right)=\frac{t^{-\alpha}}{\Gamma(1-\alpha)}.$
(iii) $I_{0+}^{\alpha}\left(f(t)+g(t)\right)=I_{0+}^{\alpha}\left(f(t)+I_{0+}^{\alpha}g(t)\right).$
(iv) $I_{0+}^{\alpha}I_{0+}^{\beta}f(t)=I_{0+}^{\alpha+\beta}f(t)=I_{0+}^{\beta}I_{0+}^{\alpha}f(t).$
(v) $D_{0+}^{\alpha}I_{0+}^{\alpha}=f(t).$
(vi) $I_{0+}^{\alpha}c_{D_{0^+}}^{\alpha}f(t)=f(t)-f(0), 0<\alpha<1.$

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$(vii) D_{0+}^{\beta} D_{0+}^{\alpha} f(t) \neq D_{0+}^{\alpha+\beta} f(t)$, and

$D_{0+}^{\beta} D_{0+}^{\alpha} f(t) \neq D_{0+}^{\alpha} D_{0+}^{\beta} f(t)$.

$(viii)$ The Laplace Transform of the Caputo fractional derivative is

$\mathcal{L}\{ D_{0+}^{\alpha} f(t)\}(s) = s^{\alpha} F(s) - \sum_{k=0}^{n-1} f^{k}(0^{+}) s^{\alpha-k-1}$.

From the aforementioned list, we notice that, in general, both the Riemann-Liouville and the Caputo fractional operators possess neither semigroup nor commutative properties, which are inherent to the derivatives of integer order. However, with some restrictions, for example with $0 < \alpha < 1$ and f is a continuous function in $[a, b]$, both the properties hold true for both of the previously mentioned operators.

III. MAIN RESULTS

Linear Systems

Consider the linear fractional dynamical system with distributed delays in control and impulses are represented by the fractional differential equation of the form

$$^{C}D_{0+}^{\alpha} y(t) = Ay(t) + \int_{-h}^{0} d_{\theta} B(t,\theta) u(t+\theta), t \in [0,T] := J, \quad 0 < \alpha < 1$$

$$\Delta y(t_i) = y(t_i^{+}) - y(t_i^{-}) = I_i(y(t_i^{+})), i = 1,2,\ldots\ldots,k$$

$$y(0) = y_0 \qquad (1)$$

Where $y \in \mathbb{R}^{n}$ and the integral is in the Lebesgue-Stieltjes sense with respect to θ. Let $h > 0$ be given. For function $u: [-h, T] \to \mathbb{R}^{m}$ and $t \in J$,we use the symbol u_t to denote the function on [-h,0] ,defined by $u_t(s) = u(t+s)$ for $s \in [-h, 0)$. A is a $n \times n$ matrix,$B(t,\theta)$, is an $n \times m$ dimensional matrix continuous in t for fixed θ and is of bounded variation in θ on [-h,0] for each $t \in J$ and continuous from left in θ on the interval (-h,0).$I_i: \mathbb{R}^{n} \to \mathbb{R}^{n}$ is continuous for i =1,2,….,k.

The following definitions of complete state and relative controllability f system (1) are assumed [8,10].

Definition 3.1.

The set $y(t) = \{y(t), u_t\}$ is the complete state of the system (1) at time t.

Definition 3.2.

System (1) is said to be globally relatively controllable on J if for every complete state x(0) and every vector $y_1 \in \mathbb{R}^{n}$ there exists a control u(t) defined on J such that the corresponding trajectory of the system (1)satisfies $y(T) = y_1$.

The solution of the system (1) is given by the following expression [29,30]

$$y(t) = E_{\alpha}(At^{\alpha}) y_0 + \int_{0}^{t} (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^{\alpha}) \left[\int_{-h}^{0} d_{\theta} B(s,\theta) u(s+\theta)\right] ds + \sum_{j=1}^{i} I_j(y(t_j^{-})) \qquad (2)$$

Where $E_{\alpha}(At^{\alpha})$ is the Mittag-Leffler matrix function. Now using the well known result of unsymmetric Fubini theorem [19] and change of order of integration to before the last term ,we have

$$y(t) = E_{\alpha}(At^{\alpha}) y_0 + \sum_{j=1}^{i} I_j(y(t_j^{-})) + \int_{-h}^{0} dB_{\theta} [\int_{0}^{t} (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^{\alpha}) u(s+\theta) B(s,\theta)] ds$$

$$= E_{\alpha}(At^{\alpha}) y_0 + \sum_{j=1}^{i} I_j\left(y(t_j^{-})\right) + \int_{-h}^{0} dB_{\theta} \left[\int_{\theta}^{0} \left(t-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(t-(s-\theta)\right)^{\alpha}\right) B(s-\theta,\theta) u_0(s) ds\right] + \int_{0}^{t} [\int_{-h}^{0} \left(t-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(t-(s-\theta)\right)^{\alpha}\right) d_{\theta} B_t(s-\theta,\theta)] u(s) ds \qquad (3)$$

Where $B_t(s,\theta) = \begin{cases} B(s,\theta), & s \le t \\ 0, & s > t \end{cases}$

and dB_{θ} denotes the integration of Lebesgue Stieltjes sense with respect t the variable θ in the function $B(t,\theta)$.

Define $M(t,s) = \int_{0}^{t} [\int_{-h}^{0} \left(t-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(t-(s-\theta)\right)^{\alpha}\right) d_{\theta} B_t(s-\theta,\theta) \qquad (4)$

and the controllability Grammian matrix

$W(0,T) = \int_{0}^{T} M(T,s) M^{*}(T,s) ds$

where the * indicates the matrix transpose.

Theorem 3.1. The linear control system (1) is relatively controllable on [0,T] if and only if the controllability Grammian matrix $W = \int_{0}^{T} M(T,s) M^{*}(T,s) ds \qquad (5)$

is positive definite, for some $T > 0$.

Proof.

Define the control function $u(t) = M^{*}(T,t)$

$$W^{-1}(y_1 - E_{\alpha}(At^{\alpha}) y_0 - \sum_{j=1}^{i} I_j(y(t_j^{-})) - \int_{-h}^{0} dB_{\theta} [\int_{\theta}^{0} \left(T-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(T-(s-\theta)\right)^{\alpha}\right) B(s-\theta,\theta) u_0(s) ds]) \qquad (6)$$

Since W is positive definite,that is ,it is non-singular and so its inverse is well-defined.

Where the complete state x(0) and the vector $y_1 \in \mathbb{R}^{n}$ are chosen arbitrarily. Inserting (6) in (3) and using (4), we have

$$y(T) = E_{\alpha}(AT^{\alpha}) y_0 + \sum_{j=1}^{i} I_j(y(t_j^{-})) + \int_{-h}^{0} dB_{\theta} [\int_{\theta}^{0} \left(T-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(T-(s-\theta)\right)^{\alpha}\right) B(s-\theta,\theta) u_0(s) ds]$$
$$+ \int_{0}^{T} [\int_{-h}^{0} \left(T-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(T-(s-\theta)\right)^{\alpha}\right) d_{\theta} B_T(s-\theta,\theta)]$$
$$\times \; [\int_{-h}^{0} \left(T-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(T-(s-\theta)\right)^{\alpha}\right) d_{\theta} B_T(s-\theta,\theta)]^{*}$$
$$W^{-1}(x_1 - E_{\alpha}(AT^{\alpha}) x_0 - \sum_{j=1}^{i} I_j(y(t_j^{-})) - \int_{-h}^{0} dB_{\theta} [\int_{\theta}^{0} \left(T-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(T-(s-\theta)\right)^{\alpha}\right) B(s-\theta,\theta) u_0(s) ds])\, d\theta$$
$$= y_1 .$$

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Thus the control u(t) transfers the initial state x(0) to the desired vector $y_1 \in \mathbb{R}^n$ at time T .Hence the system (1) is controllable .

Suppose that the system (1) is controllable on J, but W is not positive definite, then there exists a nonzero x such that $x^*Wx = 0$, that is $x^*\int_0^T M(T,s)M^*(T,s)x ds = 0$, $x^*M(T,s) = 0$, on [0,T].

Let $y_0 = [E_\alpha(AT^\alpha)]^{-1}y$. By the assumption, there exists an input u such that it steers the complete initial state $x(0) = \{y_0, u_0(s)\}$ to the origin to the interval [0,T].It follows that

$$y(T) = E_\alpha(AT^\alpha)y_0 + \sum_{j=1}^{i} I_j\left(y(t_j^-)\right) + \int_{-h}^{0} dB_\theta \left[\int_\theta^0 (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(T-(s-\theta))^\alpha\right) B(s-\theta,\theta)u_0(s)ds\right] + \int_0^T \left[\int_{-h}^{0} (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(T-(s-\theta))^\alpha\right) d_\theta B_T(s-\theta,\theta)\right]u(s)ds$$

$$= x + \int_{-h}^{0} dB_\theta \left[\int_\theta^0 (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(-(s-\theta))^\alpha\right)B(s-\theta,\theta)u_0(s)ds\right] + \sum_{j=1}^{i} I_j\left(y(t_j^-)\right) + \int_0^T \left[\int_{-h}^{0} (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(T-(s-\theta))^\alpha\right) d_\theta B_T(s-\theta,\theta)\right]u(s)ds = 0$$

Thus,

$$0 = x^*x + \int_0^T x^* M(T,s)u(s)ds + x^*\sum_{j=1}^{i} I_j\left(y(t_j^-)\right) + x^*\int_{-h}^{0} dB_\theta \left[\int_\theta^0 (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(-(s-\theta))^\alpha\right) B(s-\theta,\theta)u_0(s)ds\right]$$

But the second, third and fourth term are leading to the conclusion $x^*x = 0$. This is a contradiction to $\neq 0$. Thus W is positive definite. Hence the proof.

Nonlinear systems

Consider the nonlinear fractional dynamical system with distributed delays in control and impulses are represented by the fractional differential equation of the form

$$^{C}D_{0+}^{\alpha} y(t) = Ay(t) + \int_{-h}^{0} d_\theta B(t,\theta)u(t+\theta) + f(t,y(t),u(t)), t \in [0,T] := J, 0<\alpha<1$$
$$\Delta y(t_i) = y(t_i^+) - y(t_i^-) = I_i\left(y(t_i^+)\right), i = 1,2,\dots\dots,k$$
$$y(0) = y_0 \quad (7)$$

where A and B are as above and $f: J \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a continuous function. Assume the following space,

Denote Q as the Banach space of continuous $\mathbb{R}^n \times \mathbb{R}^m$ valued functions defined on the interval J with the uniform norm $\|(z,v)\| = \|z\| + \|v\|$ where $\|z\| = \sup\{|z(t)|: t \in J\}$. $Q = C_n(J) \times C_m(J)$, where $C_n(J)$ is the Banach space of continuous $\mathbb{R}^n$ valued functions defined n the interval J with the sup norm. For each $(z,v) \in Q$, consider the linear fractional dynamical system

$$^{C}D_{0+}^{\alpha} y(t) = Ay(t) + \int_{-h}^{0} d_\theta B(t,\theta)u(t+\theta) + f(t,z(t),v(t)), t \in [0,T] := J, 0<\alpha<1$$
$$\Delta y(t_i) = y(t_i^+) - y(t_i^-) = I_i\left(y(t_i^+)\right), i = 1,2,\dots\dots,k$$
$$y(0) = y_0 \quad (8)$$

Then the solution of the system (8) is given in the following expression [29,30]

$$y(t) = E_\alpha(At^\alpha)y_0 + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha) \times \left[\int_{-h}^{0} d_\theta B(s,\theta)u(s+\theta)\right]ds + \sum_{j=1}^{i} I_j\left(y(t_j^-)\right) + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha) f(s,z(s),v(s))ds$$

Using the well known result of unsymmetric Fubini theorem [19] and change of order of integration to the second term, we have

$$y(t) = E_\alpha(At^\alpha)y_0 + \sum_{j=1}^{i} I_j\left(y(t_j^-)\right) + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha) f\left(s,z(s),v(s)\right)ds + \int_{-h}^{0} dB_\theta \left[\int_\theta^0 (t-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(t-(s-\theta))^\alpha\right) B(s-\theta,\theta)u_0(s)ds\right] + \int_0^t \left[\int_{-h}^{0} (t-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(t-(s-\theta))^\alpha\right) d_\theta B_t(s-\theta,\theta)\right]u(s)ds \quad (9)$$

Where $B_t(s,\theta) = \begin{cases} B(s,\theta), & s \le t \\ 0, & s > t \end{cases}$

and dB_θ denotes the integration of Lebesgue Stieltjes sense with respect to the variable θ in the function $B(t,\theta)$. Define

$$\psi(x(0),y_1;z,v) = y_1 - E_\alpha(AT^\alpha)y_0 - \sum_{j=1}^{i} I_j\left(y(T_j^-)\right) - \int_{-h}^{0} dB_\theta \left[\int_\theta^0 (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(T-(s-\theta))^\alpha\right)B(s-\theta,\theta)u_0(s)ds\right] - \int_0^T (T-s)^{\alpha-1} E_{\alpha,\alpha}(A(T-s)^\alpha) f\left(s,z(s),v(s)\right)ds$$

Define the control function

$$u(t) = M^*(T,t)W^{-1}\psi(x(0),y_1;z,v)$$

Where the complete state x(0) and the vector $y_1 \in \mathbb{R}^n$ are chosen arbitrarily and * denotes the matrix transpose.

Theorem 3.2. Let the continuous function f satisfies the condition $\lim_{|y,u|\to\infty} \frac{|f(t,y,u)|}{|y,u|} = 0$ uniformly in $t \in J$,and suppose that the linear fractional system (1) is globally relatively controllable. Then the nonlinear system (7) is globally relatively controllable on J.

Proof.

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Define the operator

$\eta: Q \to Q$ by $\eta(z,v) = (y,u)$

Where

$$u(t) = M^*(T,t)W^{-1}(x_1 - E_\alpha(AT^\alpha)y_0 - \sum_{j=1}^{i} I_j\left(y(T_j^-)\right) - \int_{-h}^{0} dB_\theta\left[\int_\theta^0 (T-(s-\theta))^{\alpha-1} E_{\alpha,\alpha}\left(A(T-(s-\theta))^\alpha\right)B(s-\theta,\theta)u_0(s)ds\right] - \int_0^T (T-s)^{\alpha-1} E_{\alpha,\alpha}(A(T-s)^\alpha)f(s,z(s),v(s))ds))$$

and

$$y(t) = E_\alpha(At^\alpha)y_0 + \sum_{j=1}^{i} I_j\left(y(t_j^-)\right) + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha)f(s,z(s),v(s))d$$
$$+ \int_{-h}^{0} dB_\theta\,[\int_\theta^0 \left(t-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(t-(s-\theta)\right)^\alpha\right) \times B(s-\theta,\theta)u_0(s)ds]$$
$$+ \int_0^t [\int_{-h}^{0} \left(t-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(t-(s-\theta)\right)^\alpha\right) \times d_\theta B_t(s-\theta,\theta)]u(s)ds$$

Next, we introduce the following notations:

$a_1 = sup \parallel E_\alpha(AT^\alpha)y_0 \parallel$; $a_2 = sup \parallel E_\alpha(A(T-s)^\alpha)y_0 \parallel$;

$a_3 = sup \parallel E_\alpha\left(A\left(T-(s-\theta)\right)^\alpha\right) \parallel$;

$$a_4 = \parallel \int_\theta^0 \left(T-(s-\theta)\right)^{\alpha-1} E_{\alpha,\alpha}\left(A\left(T-(s-\theta)\right)^\alpha\right) B(s-\theta,\theta)u_0(s)ds \parallel;$$

$a_5 = sup \parallel M^*(T,t) \parallel$; $a_6 = sup \parallel M^*(T,t) \parallel$;

$a = max\{a_3 T \parallel M(T,s) \parallel ,1\}$; $d_1 = 4a_5 |W^{-1}| \alpha^{-1}[|y_1| + a_1 + a_4]$; $d_2 = 4[a_1 + a_4]$;

$c_1 = 4a_2 a_5\, T^\alpha \left|W^{-1}\right| \alpha^{-1}$; $c_1 = 4a_2\, T^\alpha\, \alpha^{-1}$;

$c = \max\{c_1, c_2\}$; $d = \max\{d_1, d_2\}$;

$sup \mid f \mid = \sup \{ \mid f(s,z(s),v(s) \mid ; s \,\epsilon\, J \}$.

Then $\mid u(t) \mid \leq \frac{1}{4a}[\ d + c \sup \mid f \mid\ ]$, and

$\mid y(t) \mid \leq (a_1 + a_4) + a_3 \int_0^t \|G(t,s)\|\, \|u(s)\| ds + a_2 \int_0^t (t-s)^{\alpha-1} sup \mid f \mid ds \leq \frac{d}{2} + \frac{c}{2} sup \mid f \mid$.

By hypothesis the function f satisfies the following condition [20]. For each pair of positive constants c and d, there exists a positive constant r such that, if $\mid p \mid \leq r$, then $c \mid f(t,p) \mid + d \leq r$, for all $t \,\epsilon\, J$. Also for given c and d, if r is a constant such that the inequality (12) is satisfied, then r_1 such that $r < r_1$ will also satisfy (12). Now, take c and d as given above, and let r be chosen so that (12) is satisfied.

Therefore, if $\|z\| \leq \frac{r}{2}$ and $\|v\| \leq \frac{r}{2}$, then $\mid z(s) \mid + \mid v(s) \mid \leq r$, for all $s \,\epsilon\, J$. It follows that $d + c \sup \mid f \mid \leq r$. Therefore, $\mid u(s) \mid \leq \frac{r}{4a}$, for all $s \,\epsilon\, J$, and hence $\|u\| \leq \frac{r}{4a}$, which gives $\|y\| \leq \frac{r}{2}$. Thus, we have proved that, if $(r) = \{(z,v) \,\epsilon\, Q:\ \|z\| \leq \frac{r}{2}$ and $\|v\| \leq \frac{r}{2}\}$, then η maps $Q(r)$ into itself. Since f is continuous, it implies that the operator is continuous, and hence is completely continuous by the application of Arzela-Ascoli's theorem. Since $Q(r)$ is closed, bounded and convex, the schaunder fixed point theorem guarantees that η has a fixed point $(z,v) \in Q(r)$ such that $\eta(z,v) = (z,v) \equiv (y,u)$. Hence $y(t)$ is the solution of the system (7), and it is easy to verify that $y(T) = y_1$. Further the control function $u(t)$ Steers the system (7) from initial complete state $y(0)$ to y_1 on J. Hence the system (7) is globally relatively controllable on J.

IV. NUMERICAL EXAMPLE

In this section we apply the results obtained in the previous section for the following fractional dynamical systems with distributed delays in control with impulses.

Consider the Nonlinear fractional dynamical system

$$^{c}D^{\alpha}y_1(t) = y_2(t) + \int_{-1}^{0} e^\theta[\,cost u_1(t+\theta) + sint u_2(t+\theta)]d\theta + \frac{y_1(t)}{1+y_2^2(t)}$$

$$^{c}D^{\alpha}y_2(t) = -y_1(t) + \int_{-1}^{0} e^\theta[\,-sint u_1(t+\theta) + cost u_2(t+\theta)]d\theta + \frac{y_2(t)}{1+y_1^2(t)}$$

$$\Delta y|_{t=\frac{1}{2}} = \frac{|y(\frac{1}{2}^-)|}{3+|y(\frac{1}{2}^-)|}$$

For $t \in J$ and $0 < \alpha < 1$. In matrix form

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\quad B(t,\theta) = \begin{pmatrix} e^\theta cost & e^\theta sint \\ -e^\theta sint & e^\theta cost \end{pmatrix}$$

and $f(t, y(t)) = \begin{pmatrix} \frac{y_1}{1+y_2^2(t)} \\ \frac{y_2}{1+y_1^2(t)} \end{pmatrix}$

Here $y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$ with $y(t) = y(t); D^{\frac{Q}{2}}y_1(t) = y_2(t)$.

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