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Further Asymptotic Stability of Stochastic Recurrent Neural Networks with Multiple Discrete Delays and Impulses

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***Abstract*— In this paper, the problem on asymptotic stability analysis of stochastic recurrent neural networks with multiple discrete delays and impulses is considered. By employing Lyapunov Krasovskii functional and some well-known inequalities, sufficient conditions are derived in linear matrix inequality form to ensure the asymptotic stability of equilibrium point for the considered neural networks. A numerical example is given to demonstrate the effectiveness of the obtained result.**

Keywords— Stochastic recurrent neural network; Asymptotic stability; Discrete delays; Impulses.

I. INTRODUCTION

Neural networks have attracted much attention due to their applications in many areas of real world problems. In particular, recurrent neural networks with delays have found many applications in some fields such as signal processing, image processing, pattern recognition, associative memory and optimization problems [6], [7], [11]. These applications heavily depend on the stability of the equilibrium point of neural networks. Therefore, the stability analysis is essential for the design and applications of neural networks. In the application of recurrent neural networks with delays, it is often required that the network model has an unique equilibrium point which is globally exponentially stable. In hardware implementation of recurrent neural networks, time delays occur due to finite switching speed of the amplifiers and communication time. In recent years, considerable efforts have been devoted to study the global asymptotic or exponential stability for the neural networks with time delays via Lyapunov function method. In particular, there has been a growing research interest in the study of neural networks with both discrete and distributed delays, see [3], [4], [19].

Further, when performing the computation, there are many stochastic disturbances that affect the stability of neural networks. A neural network could be stabilized or destabilized by certain stochastic inputs [1]. It implies that the stability of stochastic neural networks also has primary significance in the research of neural networks. Hence the stability analysis problem for stochastic neural networks becomes increasingly significant and some results related to this problem have recently been published, see [1], [12], [21]. On the other hand, there is a somewhat new category of neural networks, which is neither purely continuous-time nor purely discrete-time ones; these are called impulsive neural networks. This third category of neural networks displays a combination of characteristics of both the continuous-time and discrete-time systems. However, besides stochastic effects, impulsive effects likewise exist in real systems [25]. Therefore, it is necessary to consider both impulsive and stochastic effect on the dynamical behaviors of recurrent neural networks.

Motivated by the above discussions, the main objective of this paper is to study the global asymptotic stability of stochastic recurrent neural networks with multiple discrete delays and impulses. We establish new stability conditions for the stochastic recurrent neural networks with the help of Lyapunov-Krasovskii functional method and some well-known inequalities. The proposed stability criteria are derived in terms of linear matrix inequalities (LMIs). Furthermore, some examples with simulation results are given to show the effectiveness of the proposed stability result.

II. MODEL DESCRIPTION AND PRELIMINARIES

Consider the following stochastic recurrent neural networks with multiple discrete delay and impulses

$$\begin{aligned} dx_i(t) = {} & [-a_i x_i(t)+\textstyle\sum_{j=1}^{n} b_{ij}\, f_j(x_j(t)) +\sum_{j=1}^{n} b_{ij}^{(\mathcal{M})}\, f_j(x_j(t \\ & -\tau_{\mathcal{M}}(t))) + I_i]\, dt\ +\textstyle\sum_{j=1}^{n} \sigma_{ij}\,(t, f_j\left(x_j(t)\right), f_j(x_j(t \\ & -\tau_{\mathcal{M}}(t))),\ f_j(x_j(t-\tau_r(t)))\, dw_j(t),\ \ t \neq t_k, \end{aligned}$$

$$\Delta x_i(t_k) = I_k x_i(t_k),\ t = t_k, \mathcal{M} = 1, 2, \ldots, r,\ k = 1, 2, \ldots \quad (1)$$

Where $x_i(t)$ is the state of the i^{th} neuron at time t, $a_i > 0$ denotes the passive decay rate, b_{ij} and $b_{ij}^{(\mathcal{M})}$ are the synaptic connection strengths, f_j denotes the neuron activations, I_i is the constant input from outside the system, $\tau_{\mathcal{M}}(t)$ represents the discrete transmission delay and satisfies the following condition.

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Assumption I. $0 < h_1 \le \tau_{\mathcal{M}}(t) \le h_2 < \infty$, $\dot{\tau}_{\mathcal{M}}(t) \le \mu_{\mathcal{M}}$ where h_1, and h_2 are constants. The stochastic disturbance $w(t) = (w_1(t), w_2(t), \dots, w_m(t))^T$ is a m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\ge 0}$. Let $\sigma(t,x,y)$: $R_+ \times R^n \times R^n \longrightarrow R^{n\times m}$ is locally Lipschitz continuous and satisfies the linear growth condition. $\Delta x_i(t_k) = I_k x_i(t_k) = x_i(t_k^+)$ - $x_i(t_k^-)$ and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$.

The initial conditions of system (1) are described as follows. $x_i(t) = \Phi_i(t), t\,\epsilon\,[-\max_{1\le \mathcal{M}\le r}\{\tau_{\mathcal{M}}\}, 0]$

We assume that the neuron activation functions f_j, $j = 1,2,\dots,n$ are bounded and satisfies the following Assumptions:

Assumption II. $l_j^- \le \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \le l_j^+$, for all $\alpha_1 \ne \alpha_2$

Assume that $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ is an equilibrium point of Eq.(1). It can be easily verified that the transformation $y_i = x_i - x_i^*$ transforms system (1) into the following system:

$$dy(t) = [-Ay(t) + Bg(y(t)) + \textstyle\sum_{\mathcal{M}=1}^{r} B^{(\mathcal{M})} g(y(t -\tau_{\mathcal{M}}(t)))]\, dt + \sigma(t, g(y(t)), g\left(y(t-\tau_1(t))\right), \dots, g\left(y(t-\tau_r(t))\right) dw(t),\ t \ne t_k,$$

$$\Delta y(t_k) = I_k y(t_k),\ t = t_k,\ \mathcal{M} = 1,2,\dots,r,\ k = 1,2,\dots$$

$$y_i(t) = \Phi_i(t), \qquad (2)$$

Where $y = [y_1, y_2, \dots, y_n]^T$, $A = diag[a_1, a_2, \dots, a_n]$, $B = [b_{ij}]$, $B^{(\mathcal{M})} = [b_{ij}{}^{(\mathcal{M})}]$, $g(y) = [g_1(y_1), g_2(y_2), \dots, g_n(y_n)]^T$ with $g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*)$. Obviously, one can check that the functions $g_j(.)$ satisfies $l_j^- \le \frac{g_j(\alpha)}{\alpha} \le l_j^+$, for all $\alpha \ne 0$. Let $C^{2,1}(R^n \times R_+ : R_+)$ denote the family of all non-negative functions $V(y,t)$ on $R^n \times R_+$ which are continuously twice differentiable in y and once differentiable in t. For each $V\,\epsilon\, C^{2,1}$ $([-\tau^*, \infty] \times R^n, R_+)$, define an operator $\mathcal{L}V$ associated with stochastic delayed impulsive neural networks (2) from $R^n \times R_+$ to R by

$$\mathcal{L}V(y(t),t) = V_t(y,t) + V_y(y,t)\,[-Ay(t) + By(t) + \textstyle\sum_{\mathcal{M}=1}^{r} B^{(\mathcal{M})}\, g(y(t-\tau_{\mathcal{M}}(t)))] + \frac{1}{2} trace\,[\sigma^T V_{yy}(y,t)]$$

Here $V_t(y,t) = \frac{\partial V(y,t)}{\partial t}$, $V_y(y,t) = (\frac{\partial V(y,t)}{\partial y_1}, \frac{\partial V(y,t)}{\partial y_2}, \dots, \frac{\partial V(y,t)}{\partial y_n})$ and $V_{yy}(y,t) = (\frac{\partial^2 V(y,t)}{\partial y_i \partial y_j})_{n\times n}$, where $i,j = 1,2,\dots,n$.

Moreover, there exists positive diagonal matrices $F_0 \ge 0$, $F_{\mathcal{M}} \ge 0$ such that

Assumption III. $trace\,[\sigma^T\, t, g(y(t)), g\left(y(t-\tau_1(t))\right), \dots, g(y(t-\tau_r(t)))] \le g^T(y(t))\, F_0\, g(h(t)) + \sum_{\mathcal{M}=1}^{r} g^T(y(t-\tau_{\mathcal{M}}(t)))\, F_{\mathcal{M}}\, g(y(t-\tau_{\mathcal{M}}(t)))$

Lemma 3.1(Schur complement [2]). For a given matrix $= \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} > 0$, Where $\Sigma_{11} = \Sigma_{11}^T$, $\Sigma_{22} = \Sigma_{22}^T$, is equivalent to any one of the following conditions:

(i) $\Sigma_{22} > 0,\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T > 0$;

(ii) $\Sigma_{11} > 0,\ \Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12} > 0$;

Lemma 3.2(Gu et al. [9]). For any positive definite matrix $W > 0$, two scalars $b > a$, vector function $\omega : [a,b] \longrightarrow R^n$, such that the integrations concerned are well defined, the following inequality holds:

$$\left(\int_a^b \omega(s)ds\right)^T W \left(\int_a^b \omega(s)ds\right) < (b-a)\int_a^b \omega^T(s) W\, \omega(s)\, ds.$$

For presentation convenience, in the following, we denote $L_1 = diag\,(l_1^- l_1^+,\ l_2^- l_2^+, \dots, l_n^- l_n^+)$, (3)

$$L_2 = diag\,(\tfrac{l_1^- + l_1^+}{2}, \tfrac{l_2^- + l_2^+}{2}, \dots \tfrac{l_n^- + l_n^+}{2}) \qquad (4)$$

III. MAIN RESULTS

In this section, we consider the asymptotic stability for systems with multiple discrete time delays and impulses. Our approach is based on the Lyapunov-Krasovskii stability theory and the LMI technique [2]. It should be noted that the equilibrium point x^* of (2) is asymptotically stable if and only if the equilibrium point of system (2) is asymptotically stable. Thus in the following, we only consider the asymptotic stability of the equilibrium point for system (2).

The following lemma will be essential in establishing the desired LMI based stability criteria.

Theorem 3.1 Assume that (I) – (III) hold. For given Scalars $h_2 > h_1 \ge 0$, $\alpha_{\mathcal{M}}$, $\beta_{\mathcal{M}}, \mu_{\mathcal{M}}$ $(\mathcal{M} = 1,2,3\dots, r)$, system (2) is said to be Globally asymptotically stable in the mean square, if there exist Positive definite matrices $P > 0, Q_1 > 0$, $Q_2 > 0$, Positive diagonal matrices $R_{\mathcal{M}} > 0, S_{\mathcal{M}} > 0$ and a scalar $\delta > 0$ Such that the following LMIs hold:

$$P < \delta I \qquad (5)$$

$$I_k P I_k - P I_k - I_k P < 0 \qquad (6)$$

$$\Sigma_1 = \begin{bmatrix} \Phi_{11} & 0 & 0 & 0 & \Phi_{15} & 0 & 0 \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & \Phi_{46} & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 \\ * & * & * & * & * & \chi^{(\mathcal{M})} & 0 \\ * & * & * & * & * & * & \Phi_{77} \end{bmatrix} < 0, \qquad (7)$$

Where

$\Phi_{11} = -PA - A^T P + Q_1 + Q_2 + (h_2 - h_1)Q_3 - L_1\Gamma$

$\Phi_{15} = PB + L_2\Gamma\, PB^{(\mathcal{M})}$, $\Phi_{44} = -L_1\Omega^{(\mathcal{M})}$, $\Phi_{46} = L_2\Omega^{(\mathcal{M})}$,

$\Phi_{55} = \sum_{\mathcal{M}=1}^{r} \alpha_{\mathcal{M}}\ R_{\mathcal{M}} + \sum_{\mathcal{M}=1}^{r} \frac{1}{1-\beta_{\mathcal{M}}} S_{\mathcal{M}} + \delta F_0 - \Gamma$, $\chi^{(\mathcal{M})} = \sum_{\mathcal{M}=1}^{r} \delta\, F_{\mathcal{M}} - \sum_{\mathcal{M}=1}^{r} \alpha_{\mathcal{M}}\ (1-\mu_{\mathcal{M}}) R_{\mathcal{M}} - \sum_{\mathcal{M}=1}^{r} S_{\mathcal{M}} - \Omega^{(\mathcal{M})}$, $\Phi_{77} = -\frac{1}{h_2 - h_1} Q_3$.

$\Gamma = diag\{\gamma_1, \gamma_2, \dots, \gamma_n\}$, $\Omega^{(\mathcal{M})} = \{diag(w_1^{(1)}, diag w_2^{(1)}$

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$diagw_n^{(1)}), \dots, diag(diag(w_1^{(r)}, diag(w_2^{(r)}, \dots, diag(w_n^{(r)})\}$

Proof. Define new state variables as follows:

$$y_1(t) = -Ay(t) + Bg(y(t)) + \sum_{\mathcal{M}=1}^{r} B^{(\mathcal{M})} g(y(t-\tau_{\mathcal{M}}(t)))$$

$$y_2(t) = \sigma(t, g(y(t)), g\left(y(t-\tau_1(t))\right), \dots, g\left(y(t-\tau_r(t))\right)$$

Therefore, Eqn (2) can be rewritten as

$$d\,y(t) = y_1(t) + y_2(t)dw(t)$$

Consider the following Lyapunov function defined by

$$\begin{aligned} V(y(t),t) = {} & y_1^T(t)Py_1(t) + \int_{t-h_1}^{t} y^T(s)Q_1y(s)ds \\ & + \int_{t-h_2}^{t} y^T(s)Q_2y(s)ds \\ & + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} y^T(s)Q_3y(s)ds \\ & + \sum_{\mathcal{M}=1}^{r} \alpha_{\mathcal{M}} \int_{t-\tau_{\mathcal{M}}(t)}^{t} g^T(y(s))R_{\mathcal{M}}\, g(y(s))ds \\ & + \sum_{\mathcal{M}=1}^{r} \frac{1}{1-\beta_{\mathcal{M}}} \int_{t-\tau_{\mathcal{M}}(t)}^{t} g^T(y(s))S_{\mathcal{M}}\, g(y(s))ds \end{aligned}$$

By Ito's formula, we can calculate $\mathcal{L}V(y(t),t)$ along the trajectories of the system (2) for $t \neq t_k$, then we have

$$\begin{aligned} \mathcal{L}V(y(t),t) \le {} & y^T(T)[-PA - A^TP + Q_1 + Q_2 + (h_2-h_1) \\ & \times Q_3]\, y(t) + y^T(t)2PB\, g(y(t)) \\ & + \sum_{\mathcal{M}=1}^{r} y^T(t)2PB^{(\mathcal{M})} \times g\left(y(t-\tau_{\mathcal{M}}(t))\right) \\ & - y^T(t-h_1)Q_1y(t-h_1) - y^T(t-h_2)Q_2 \\ & \times y(t-h_2) + g^T(y(t))[\textstyle\sum_{\mathcal{M}=1}^{r} \alpha_{\mathcal{M}}\ R_{\mathcal{M}} \\ & + \sum_{\mathcal{M}=1}^{r} \frac{1}{1-\beta_{\mathcal{M}}} S + \delta\, F_0]g(y(t)) \\ & + g^T\left(y(t-\ \tau_{\mathcal{M}}(t))\right)\, g\left(y(t-\tau_{\mathcal{M}}(t))\right) \\ & \times\ [\textstyle\sum_{\mathcal{M}=1}^{r} \delta\, F_{\mathcal{M}} - \sum_{\mathcal{M}=1}^{r} \alpha_{\mathcal{M}}\ (1-\mu_{\mathcal{M}})\, R_{\mathcal{M}} \\ & \times \textstyle\sum_{\mathcal{M}=1}^{r} S_{\mathcal{M}}] - \begin{pmatrix} y(t) \\ g(y(t)) \end{pmatrix}^T \begin{bmatrix} L_1\Gamma & -L_2\Gamma \\ -L_2\Gamma & \Gamma \end{bmatrix} \\ & \times \begin{pmatrix} y(t) \\ g(y(t)) \end{pmatrix} - \begin{pmatrix} y(t-\tau_{\mathcal{M}}(t)) \\ g(y(t-\tau_{\mathcal{M}}(t))) \end{pmatrix}^T \\ & \times \begin{bmatrix} L_1\Omega^{(\mathcal{M})} & L_1\Omega^{(\mathcal{M})} \\ L_1\Omega^{(\mathcal{M})} & L_1\Omega^{(\mathcal{M})} \end{bmatrix} \begin{pmatrix} y(t-\tau_{\mathcal{M}}(t)) \\ g(y(t-\tau_{\mathcal{M}}(t))) \end{pmatrix} \\ & - \frac{1}{h_2-h_1} \left(\int_{t-h_2}^{t-h_1} y(s)ds\right)^T Q_3 \left(\int_{t-h_2}^{t-h_1} y(s)ds\right) \\ = {} & \xi^T(t)\Xi_1\xi(t), \end{aligned}$$

Where $\Xi_1(t) = [y^T(t),\ y^T(t-h_1),\ y^T(t-h_2),$
$y^T(t-\tau_{\mathcal{M}}(t)),\ g^T(y(t)),\ g^T\left(y(t-\tau_{\mathcal{M}}(t))\right)$
$\int_{t-h_2}^{t-h_1} y(s)ds)^T]^T$ (19)

Thus, for ensuring negativity of $\mathcal{L}V(y(t),t)$ for any Possible state, it suffices to requires Ξ_1 be a negative definite matrix. From (19), $\mathcal{L}V(y(t),t) \le 0$, $\mathcal{L}V(y(t),t) = 0$, if and only if $y(t) = 0$.

When $t = t_k$, We have

$$\begin{aligned} V(y(t_k^+),t_k^+) = {} & y_1^T(t_k)P\,y_1(t_k) - y_1^T(t_k)PI_k\,y_1(t_k) \\ & - y_1^T(t_k)I_k^TP\,y_1(t_k) + y_1^T(t_k)I_k^TP\,I_ky_1(t_k) \\ & + \int_{t_k-h_1}^{t_k} y^T(s)Q_1y(s)ds \\ & + \int_{t_k-h_2}^{t_k} y^T(s)Q_2y(s)ds \\ & + \int_{-h_2}^{-h_1}\int_{t_k+\theta}^{t_k} y^T(s)Q_3y(s)ds \\ & + \sum_{\mathcal{M}=1}^{r} \alpha_{\mathcal{M}} \int_{t_k-\tau_{\mathcal{M}}(t_k)}^{t_k} g^T(y(s))R_{\mathcal{M}}\, g(y(s))ds \\ & + \sum_{\mathcal{M}=1}^{r} \frac{1}{1-\beta_{\mathcal{M}}} \int_{t_k-\tau_{\mathcal{M}}(t_k)}^{t_k} g^T(y(s))S_{\mathcal{M}}\, g(y(s))ds \\ = {} & V(t(t_k),t_k) + y_1^T(t_k)[I_kPI_k - PI_k - \\ & I_kP]y_1(t_k) \\ \le {} & V(y(t_k),t_k) \end{aligned}$$

Based on the Lyapunov-Krasovskii stability theorem, the system (2) is globally asymptotically stable. The Proof is Completed.

IV NUMERICAL EXAMPLE

In this section, we provide a numerical example to demonstrate the effectiveness of the proposed asymptotic stability result.

Example 4.1 Consider the following two-neuron stochastic recurrent neural networks with impulses:

$$\begin{aligned} dy(t) = {} & [-Ay(t) + Bg(y(t)) + \textstyle\sum_{\mathcal{M}=1}^{r} B^{(\mathcal{M})}\, g(y(t \\ & -\tau_{\mathcal{M}}(t)))]\, dt + \sigma(t, g(y(t)), g\left(y(t-\tau_1(t))\right), \\ & \dots, g\left(y(t-\tau_r(t))\right) dw(t),\ t \neq t_k, \end{aligned}$$

$$\Delta y(t_k) = I_ky(t_k),\ \ t = t_k,\ \mathcal{M} = 1,2,\dots,r,\ k = 1,2,\dots$$

where the activation function is described by $g_1(x) = \tanh(0.7x) - 0.1\sin x$, $g_2(x) = \tanh(0.4x) - 0.2\cos x$, $\tau_1(t) = 0.5 + 0.5\sin t$, $\tau_2(t) = 0.5 - 0.5\cos t$, $\alpha_1 = \alpha_2 = 0.5$. The delayed feedback matrices $A, B, B^{(1)}, B^{(2)}$ are

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix},\ B = \begin{bmatrix} 0.4 & 0.3 \\ 0.5 & 0.1 \end{bmatrix},$$

$$B^{(1)} = \begin{bmatrix} 0.2 & -0.4 \\ -0.1 & 0.5 \end{bmatrix}, B^2 = \begin{bmatrix} 0.3 & -0.5 \\ 0.3 & -0.6 \end{bmatrix}$$

Clearly the activation function satisfies the Assumptions (I)-(III) with

$$F_0 = F_1 = \begin{bmatrix} -0.08 & 0 \\ 0 & -0.12 \end{bmatrix}, F_2 = \begin{bmatrix} -0.08 & 0 \\ 0 & -0.12 \end{bmatrix},$$

$$I_1 = I_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Solving the LMIs (5)-(7) in Theorem 3.1, a following feasible solution is obtained by using LMI toolbox

$$P = \begin{bmatrix} 1.9792 & 0.0941 \\ 0.0941 & 1.8401 \end{bmatrix},\ Q = \begin{bmatrix} 1.3455 & -0.1864 \\ -0.1864 & 1.4890 \end{bmatrix},$$

Thus the system (2) satisfies all the conditions stated in Theorem 3.1. Hence the stochastic recurrent neural network (2) with impulsive effect is globally asymptotically stable.

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TABLE I

COMPARISONS OF UPPER BOUNDS OF TIME DELAYS

Methods	$\mu = 0$ $h_1 = 0$	$\mu = 0.5$ $h_1 = 0.5$	$\mu = 0.8$ $h_1 = 0.8$
In Ref [8]	$h_2 = 3.3294$	$h_2 = 2.6363$	$h_2 = 1.7200$
In Ref [10]	$h_2 = --$	$h_2 = 2.2245$	$h_2 = 1.5847$
In Ref [13]	$h_2 = --$	$h_2 = 2.1502$	$h_2 = 1.3164$
In Ref[14]	$h_2 = 5.8415$	$h_2 = 4.0130$	$h_2 = 3.6574$
In Ref [15]	$h_2 = 2.2931$	$h_2 = 2.1019$	$h_2 = 1.7939$
In Ref [16]	$h_2 = --$	$h_2 = 2.2245$	$h_2 = 1.5847$
In Ref [17]	$h_2 = --$	$h_2 = 2.4895$	$h_2 = 1.7778$
In Ref[18]	$h_2 = 7.2820$	$h_2 = 5.8958$	$h_2 = 4.0847$
In Ref[32]	$h_2 = 9.6503$	$h_2 = 8.0394$	$h_2 = 5.8974$
In Ref[32]	$h_2 = --$	$h_2 = 2.5196$	$h_2 = 1.7808$
In this paper	large finite h_2	large finite h_2	large finite h_2

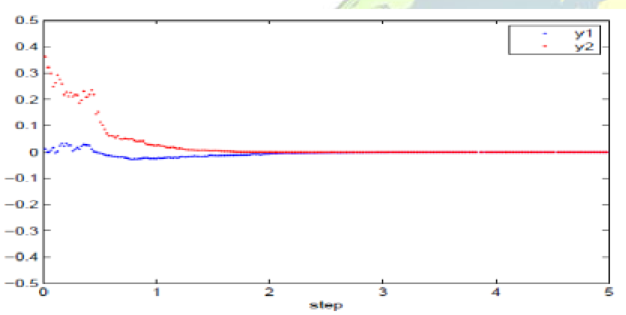

Fig. 1 State responses of $y_1(t), y_2(t)$ of the network (2) without impulsive effects.

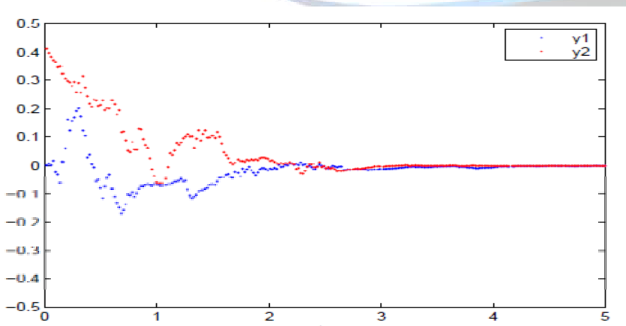

Fig. 2 State responses of $y_1(t), y_2(t)$ of the network (2) with impulsive effects.

In order to show the significant contribution of this paper, we summarize the comparisons between the previous works and the obtained result. Table.1 gives the comparison results on the maximum allowable upper bound h2. Therefore, we conclude that the equation (2) is asymptotically stable for any constant allowable upper bound h2. Hence the proposed method is finer than the previous works based on the upper bound techniques.

V CONCLUSIONS

In this paper, the problem of stability criterion for stochastic recurrent neural networks with multiple discrete delays and impulses has been investigated by the use of Lyapunov method and LMI framework. By constructing an appropriate Lyapunov function and combined with stochastic analysis approach, a new set of sufficient conditions have been obtained to ensure the global asymptotic stability of the addressed neural networks. The methods of this paper can also be used to study the global exponential stability of the equilibrium point. Finally, a numerical example is given to show the effectiveness of our stability result.

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