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Globally Asymptotic Stability Criteria for Impulsive BAM Neural Networks with Leakage and Time Varying Delays

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Abstract— **This paper investigates the problem of delay dependent asymptotic stability criteria for impulsive BAM neural networks with leakage and time varying delays. By defining a novel Lyapunov functional, an improved delay-dependent asymptotic stability criterion is established in terms of LMI approach. Additionally a numerical example is given to illustrate the effectiveness and benefits of our proposed method.**

***Keywords*— Asymptotic stable; Impulse; BAM Neural Networks; Linear Matrix Inequality; Time varying delays; Leakage delays.**

I. INTRODUCTION

During the past few decades, various kinds of recurrent neural networks have been largely studied including bidirectional associative memory(BAM) neural networks, Hopfield neural networks, Cellular neural networks, Cohen-Grossberg neural networks, neural and social network due to their potential applications in many areas such as classification, signal and image processing, parallel computing, associate memories, optimization, Cryptography and so on. The bidirectional associative memory (BAM) neural network models were first introduced by Kosko. The BAM neural network is composed of neurons arranged in two layers, the X-layer and Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer. Neural networks have to be designed in such a way that, for a given external input, they exhibit only one globally asymptotically stable equilibrium point. Impulsive effect is likely to exist in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time in the fields such as medicine, biology, economics, electronics and telecommunications. Moreover, the existence of time delays may lead to instability (or) bad performance of systems.

However, there has been very little existing work on neural networks with time delay in leakage term. In fact, time delays in the leakage term has also great impact on the dynamic behavior of neural networks. In [1-4], delay-independent asymptotic stability problem is studied. Generally speaking, the delay-dependent stability criteria is less conservative than delay-independent when the time-delay is small. Therefore, authors always consider the delay-dependent type. In [6], a new method has been proposed to obtain delay-dependent stability criteria by introducing an appropriate Lyapunov functional. Although delay-dependent stability criteria for delay NNs were proposed in [6], they have conservatism to some extent, which leaves open room for further improvement.

In this paper, our aim is to study the delay-dependent asymptotic stability problem for a class of Impulsive BAM neural networks with discrete time varying and leakage delays. By utilizing the Lyapunov Stability theory and LMI technique, some novel delay-dependent conditions are obtained. We shall use not only the $u(t-\mu(t))$, $v(t-\tau(t))$ but also the $u(t-h)$, $v(t-m)$ to exploit all possible information for the relationship among $u(t), v(t), u(t-\delta)$, $v(t-\sigma)$, $u(t-\mu(t)$, $v(t-\tau(t)))$, and $\dot{u}(t), \dot{v}(t)$, when constructing Lyapunov functional. Finally, a numerical example is given to indicate significant improvements over the existing results.

II. PROBLEM FORMULATION AND PRELIMINARIES

Nomenclature

R^n	n-dimensional real space
$R^{m\times n}$	set of all real m by n
x^T or A^T	transpose of vector x (or matrix A)
$P>0$	(respectively $P<0$) matrix P is symmetric positive (respectively negative) definite
$*$	the elements below the main diagonal of symmetric block matrix
$\lVert x(t)\rVert$	$(\sum_{i=1}^{m} x_i^2(t))^{\frac{1}{2}}$

In this Paper, the impulsive BAM Neural Networks with leakage and time varying delays is described by the following integro-differential equation system:

$$\dot{x}(t) = -Ax(t-\delta) + W_0 f\big(y(t)\big) + W_1 f\Big(y\big(t-\mu(t)\big)\Big) + I, \ \ t>0, t \neq t_k,$$

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$$\Delta x(t_k) = x(t_k^+) - x(t_k^-), t = t_k, k \epsilon Z_+.$$

$$\dot{y}(t) = -Cy(t-\sigma) + V_0 g(x(t)) + V_1 g\left(x(t-\tau(t))\right) + J, \ t > 0, t \neq t_k,$$

$$\Delta y(t_k) = y(t_k^+) - y(t_k^-), t = t_k, k \epsilon Z_+. \quad (1)$$

Where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \epsilon R^n$ & $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T \epsilon R^n$ are the neurons state vectors, $f(y(.))$ and $g(x(.))$ denotes the neuron activation functions, where $f(y(.)) = [f_1(y_1(.)), f_2(y_2(.)), \ldots, f_n(y_n(.))]^T$ and $g(x(.)) = [g_1(x_1(.)), g_2(x_2(.)), \ldots, g_n(x_n(.))]^T$; $A = dia\{a_i\}$, $C = dia\{c_j\}$ are positive diagonal matrices; $a_i, c_j > 0, i, j = 1,2, \ldots, n$ are the neural self inhibitions and $I = [I_1, I_2, \ldots, I_n]^T$ & $J = [J_1, J_2, \ldots, J_n]^T$ are the constant input vectors. $W_0 = (W_{0ij})_{n\times n}$, $V_0 = (V_{0ij})_{n\times n}$ are the connection weight matrices; $W_1 = (W_{1ji})_{n\times n}$, $V_1 = (V_{1ij})_{n\times n}$ are the discretely delayed connection weight matrices; $\mu(t)$ & $\tau(t)$ are time varying continuous functions that satisfies $0 \le \mu(t) \le h, 0 \le \dot{\mu}(t) \le w$ and $0 \le \tau(t) \le m, 0 \le \dot{\tau}(t) \le l$, respectively; where h, w, k & l are constants. The leakage delays $\delta > 0$, $\sigma > 0$ are constants. The impulsive times t_k satisfy $0 = t_0 < t_1 < \cdots < t_k \longrightarrow \infty$, $(i.e., \lim_{k\to\infty} t_k = +\infty)$ and $\inf_{k\epsilon Z_+}\{t_k - t_{k-1}\} > 0$. Suppose that the initial condition of the BAM Neural Network (1) has the form $x(t) = \varphi(t)$ for $t \epsilon [-\overline{\omega}, 0]$ and $y(t) = \psi(t)$ for $t \epsilon [-\overline{\overline{\omega}}, 0]$ Where $\varphi(t)$ and $\psi(t)$ are continuous functions, $\overline{\omega} = \max(h, \delta)$ and $\overline{\overline{\omega}} = \max(m, \sigma)$.

In addition, it is assumed that each neuron activation functions in (1), $f_i(.), g_j(.), i, j = 1, 2, \ldots, n$ satisfies the following conditions:

$$0 \le \frac{f_i(x) - f_i(y)}{x-y} \le k_i, \forall\, x, y \,\epsilon\, \boldsymbol{R}, \ \boldsymbol{x \neq y, i = 1, 2, \ldots, n}$$

$$\text{and } 0 \le \frac{g_j(p) - g_j(q)}{p-q} \le n_j, \forall\, p, q \,\epsilon\, \boldsymbol{R}, \ \boldsymbol{p \neq q, j = 1, 2, \ldots, n} \quad (2)$$

where $k_i, n_j, i, j = 1, 2, \ldots, n$ are positive constants. The equilibrium point, $(x^*, y^*) = [x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_n^*]^T$ of (1) is shifted to the origin by the transformation $u(.) = x(.) - x^*$, $v(.) = y(.) - y^*$, Which converts the system to the following form:

$$\dot{u}(t) = -Au(t-\delta) + W_0 \hat{f}(v(t)) + W_1 \hat{f}\left(v(t-\mu(t))\right), \quad t > 0, t \neq t_k,$$

$$\Delta u(t_k) = u(t_k^+) - u(t_k^-), t = t_k, k \epsilon Z_+.$$

$$\dot{v}(t) = -Cv(t-\sigma) + V_0 \hat{g}(u(t)) + V_1 \hat{g}\left(u(t-\tau(t))\right), \quad t > 0, t \neq t_k,$$

$$\Delta v(t_k) = v(t_k^+) - v(t_k^-), t = t_k, k \epsilon Z_+. \quad (3)$$

Where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$ & $v(t) = [v_1(t), v_2(t), \ldots, v(t)]^T$, $\hat{f}(v(.)) = [\hat{f}_1(v_1(.)), \hat{f}_2(v_2(.)), \ldots, \hat{f}_n(v_n(.))]^T$, $\hat{g}(u(.)) = [\hat{g}_1(u_1(.)), \hat{g}_2(u_2(.)), \ldots, \hat{g}_n(u_n(.))]^T$ and $\hat{f}_i(v_j(.)) = f_i(v_j(.) + v_j^*) - f_i(v_j^*)$, $\hat{g}_j(u_i(.)) = g_j(u_i(.) + u_i^*) - g_j(u_i^*)$, i, j=1, 2,…,n. According to the inequality (2), one can obtain that

$$\hat{f}_i(v_j)[\hat{f}_i(v_j) - k_i v_j] \le 0, \hat{f}_i(0) = 0,$$

$$\hat{g}_j(u_i)[\hat{g}_j(u_i) - n_j u_i] \le 0, \hat{g}_j(0) = 0, \text{ i,j} = 1, 2, \ldots, n. \quad (4)$$

Lemma 2.1 For any constant symmetric matrix Q_{11}, $Q_{12}, Q_{22} \,\epsilon\, R^{n\times n}$, $Q_{11} = Q_{11}^T > 0$, $Q_{22} = Q_{22}^T > 0$, $\begin{pmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{pmatrix} > 0$, scalar $\bar{\tau} > 0$, and vector function $\dot{x}(t): [-\bar{\tau}, 0] \longrightarrow R^n$, such that the integrations are well defined, then

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t} \begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix}^T \begin{pmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{pmatrix} \begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix} dt \le \begin{pmatrix} \int_{t-\bar{\tau}}^{t} x(s)ds \\ \int_{t-\bar{\tau}}^{t} \dot{x}(s)ds \end{pmatrix}^T \begin{pmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{pmatrix} \begin{pmatrix} \int_{t-\bar{\tau}}^{t} x(s)ds \\ \int_{t-\bar{\tau}}^{t} \dot{x}(s)ds \end{pmatrix}. \quad (5)$$

Lemma 2.2 For any constant matrix $\varphi \,\epsilon\, R^{n\times n}, \varphi = \varphi^T > 0$, scalar $\gamma > 0$, vector function $\dot{\omega}: [0, \gamma] \longrightarrow R^n$ such that the integrations concerned are well defined, then

$$-\gamma \int_{-\gamma}^{0} \dot{\omega}^T(t+s)\varphi\dot{\omega}(t+s)ds \le \left(\int_{-\gamma}^{0} \dot{\omega}^T(t+s)ds\right)^T \varphi \left(\int_{-\gamma}^{0} \dot{\omega}^T(t+s)\, ds\right). \quad (6)$$

Rearranging the term $\left(\int_{-\gamma}^{0} \dot{\omega}^T(t+s)\right) ds$ with $\omega(t) - \omega(t-\gamma)$, we can yield the following inequality:

$$-\gamma \int_{-\gamma}^{0} \dot{\omega}^T(t+s)\varphi\dot{\omega}(t+s)ds \le (\omega(t) - \omega(t-\gamma)) \begin{pmatrix} -\varphi & \varphi \\ \varphi & -\varphi \end{pmatrix} \begin{pmatrix} \omega(t) \\ \omega(t-\gamma) \end{pmatrix}. \quad (7)$$

III. MAIN RESULTS

In this section we investigates the globally asymptotically stability of the system (3).

Theorem 3.1 For given scalars $K = dia(k_1, k_2, \ldots, k_n)$, $\widehat{N} = dia(n_1, n_2, \ldots, n_r)$, $h > 0, w \ge 0$ & $m > 0, l \ge 0$, the origin of system (3) with (4) and $0 \le \mu(t) \le h, 0 \le \dot{\mu}(t) \le w$, $0 \le \tau(t) \le m, 0 \le \dot{\tau}(t) \le l$ is globally asymptotically stable if there exist symmetric positive matrices $P_1, N_1, T_{22}, R_{22}, D_k, E_k, Q_{11}, Q_{22}, U_{11}, U_{22}, R_1, R_2, Q_i, U_j$ $(i, j = 1, 2\,3, 4)$ symmetric and positive definite matrices $D_1, D_2, G_1, G_2, \Lambda = dia(\lambda_1, \lambda_1, \ldots, \lambda_n)$, $\Pi = dia(\varphi_1, \varphi_2, \ldots, \varphi_n)$ and any matrices $T_{12}, R_{12}, Q_{12}, U_{12}$, P_i & $N_j (i, j = 2, 3, \ldots, 21)$ with appropriate dimensions, such that the following LMIs hold:

$$\begin{bmatrix} P_1 & T_{12} \\ * & T_{22} \end{bmatrix} \ge 0, \ \begin{bmatrix} N_1 & R_{12} \\ * & R_{22} \end{bmatrix} \ge 0 \text{ with } P_1 > 0 \,\&\, N_1 > 0, \quad (8)$$

$$\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \ge 0 \,\&\, \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix} \ge 0, \quad (9)$$

$$[D_k^T Q_4 D_K - Q_4] < 0 \ , \quad [E_k^T\, U_4\, E_k - U_4] < 0 \quad (a)$$

$$\Omega = (\Omega_{ij})_{n\times n} < 0 \text{ and } \Xi = (\Xi_{ji})_{n\times n} < 0, \quad (10)$$

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Where $\Omega_{11} = T_{12} + Q_1 + Q_3 + h^2 Q_4 - R_1$, $\Omega_{12} = -P_{12} - T_{12}$, $\Omega_{13} = -P_2 A$, $\Omega_{14} = P_1 - P_2$, $\Omega_{15} = W_0 P_2 + K D_1$, $\Omega_{16} = P_{12} + R_1$, $\Omega_{17} = W_1 P_2$, , $\Omega_{110} = -P_{12}$, $\Omega_{22} = -P_{13} - P_{20} - Q_3$, $\Omega_{23} = P_{10} A - P_3 A$, $\Omega_{24} = -P_{10} - P_3$, $\Omega_{25} = P_{10} W_0 + P_3 W_0$, $\Omega_{26} = P_{20} + P_{13}$, $\Omega_{27} = W_1 P_{10} + P_3 W_1$, $\Omega_{28} = -P_{19}$, $\Omega_{210} = -P_{20}$, $\Omega_{33} = P_4 A$, $\Omega_{34} = -P_4$, $\Omega_{35} = P_4 W_0$, $\Omega_{36} = P_{14}$, $\Omega_{37} = P_4 W_1$, $\Omega_{38} = P_9 A^T$, $\Omega_{310} = -P_{14}$, $\Omega_{44} = h^2 Q_{11} - P_5$, $\Omega_{45} = h^2 Q_{12} + P_5 W_0$, $\Omega_{46} = P_{15}$, $\Omega_{47} = P_5 W_1$, $\Omega_{48} = -P_9$, $\Omega_{410} = -P_{15}$, $\Omega_{55} = -D_1 + Q_2 + \Lambda W_0 + h^2 Q_{22} + P_6 W_0$, $\Omega_{56} = P_{15}$, $\Omega_{57} = \Lambda W_1 + P_6 W_1$, $\Omega_{58} = P_9 {W_0}^T$, $\Omega_{510} = -P_{16}$, $\Omega_{66} = -(1-w) Q_1 - R_1 + P_{17}$, $\Omega_{67} = K D_2 + P_7 W_1$, $\Omega_{69} = P_{19}$, $\Omega_{610} = -P_{17}$, $\Omega_{77} = -D_2 - (1-w) Q_2 + P_8 W_1$, $\Omega_{78} = P_9 {W_1}^T$, $\Omega_{710} = P_{18}$, $\Omega_{88} = Q_4$, $\Omega_{810} = -P_{19}$, $\Omega_{99} = Q_{22}$, $\Omega_{911} = \frac{Q_{12}}{2}$, $\Omega_{912} = -\frac{Q_{12}}{2}$, $\Omega_{1010} = -R_1 - P_{21}$, $\Omega_{1111} = -\frac{Q_{11}}{2}$, $\Omega_{1112} = -Q_{11}$, $\Omega_{1212} = \frac{Q_{11}}{2}$, $\Omega_{18} = \Omega_{19} = \Omega_{111} = \Omega_{112} = \Omega_{29} = \Omega_{211} = \Omega_{212} = \Omega_{39} = \Omega_{311} = \Omega_{312} = \Omega_{49} = \Omega_{411} = \Omega_{412} = \Omega_{59} = \Omega_{511} = \Omega_{512} = \Omega_{68} = \Omega_{611} = \Omega_{612} = \Omega_{79} = \Omega_{711} = \Omega_{712} = \Omega_{89} = \Omega_{811} = \Omega_{812} = \Omega_{910} = \Omega_{1011} = \Omega_{1012} = 0$ and $\Xi_{11} = R_{12} + U_1 + U_3 + m^2 U_4 - R_2$, $\Xi_{12} = -N_{12} - R_{12}$, $\Xi_{13} = -N_2 C$, $\Xi_{14} = N_1 - N_2$, $\Xi_{15} = V_0 N_2 + N G_1$, $\Xi_{16} = N_{12} + R_2$, $\Xi_{17} = V_1 N_2$, $\Xi_{110} = -N_{12}$, $\Xi_{22} = -N_{13} - N_{20} - U_3$, $\Xi_{23} = N_{10} C - N_3 C$, $\Xi_{24} = -N_{10} - N_3$, $\Xi_{25} = N_{10} V_0 + N_3 V_0$, $\Xi_{26} = N_{20} + N_{13}$, $\Xi_{27} = V_1 N_{10} + N_3 V_1$, $\Xi_{28} = -N_{19}$, $\Xi_{210} = -N_{20}$, $\Xi_{33} = N_4 C$, $\Xi_{34} = -N_4$, $\Xi_{35} = N_4 V_0$, $\Xi_{36} = N_{14}$, $\Xi_{37} = N_4 V_1$, $\Xi_{38} = N_9 C^T$, $\Xi_{310} = -N_{14}$, $\Xi_{44} = m^2 U_{11} - N_5$, $\Xi_{45} = m^2 U_{12} + N_5 V_0$, $\Xi_{46} = N_{15}$, $\Xi_{47} = N_5 V_1$, $\Xi_{48} = -N_9$, $\Xi_{410} = -N_{15}$, $\Xi_{55} = -G_1 + U_2 + \Pi V_0 + m^2 U_{22} + N_6 V_0$, $\Xi_{56} = N_{15}$, $\Xi_{57} = \Pi V_1 + N_6 V_1$, $\Xi_{58} = N_9 {V_0}^T$, $\Xi_{510} = -N_{16}$, $\Xi_{66} = -(1-l) U_1 - R_2 + N_{17}$, $\Xi_{67} = N G_2 + N_7 V_1$, $\Xi_{69} = N_{19}$, $\Xi_{610} = -N_{17}$, $\Xi_{77} = -G_2 - (1-l) U_2 + N_8 V_1$, $\Xi_{78} = N_9 {V_1}^T$, $\Xi_{710} = N_{18}$, $\Xi_{88} = U_4$, $\Xi_{810} = -N_{19}$, $\Xi_{99} = U_{22}$, $\Xi_{911} = \frac{U_{12}}{2}$, $\Xi_{912} = -\frac{U_{12}}{2}$, $\Xi_{1010} = -R_2 - N_{21}$, $\Xi_{1111} = -\frac{U_{11}}{2}$, $\Xi_{1112} = -U_{11}$, $\Xi_{1212} = \frac{U_{11}}{2}$, $\Xi_{18} = \Xi_{19} = \Xi_{111} = \Xi_{112} = \Xi_{29} = \Xi_{211} = \Xi_{212} = \Xi_{39} = \Xi_{311} = \Xi_{312} = \Xi_{49} = \Xi_{411} = \Xi_{412} = \Xi_{59} = \Xi_{511} = \Xi_{512} = \Xi_{68} = \Xi_{611} = \Xi_{612} = \Xi_{79} = \Xi_{711} = \Xi_{712} = \Xi_{89} = \Xi_{811} = \Xi_{812} = \Xi_{910} = \Xi_{1011} = \Xi_{1012} = 0$.

Proof.

Consider a class of Lyapunov functional candidate as follows:

$$
\begin{aligned}
V\big(u(t), v(t)\big) = {} & X^T(t) SPX(t) + \\
& + \begin{bmatrix} v(t) \\ \int_{t-h}^{t} v(s) ds \end{bmatrix}^T \begin{bmatrix} 0 & T_{12} \\ * & T_{22} \end{bmatrix} \begin{bmatrix} v(t) \\ \int_{t-h}^{t} v(s) ds \end{bmatrix} + Y^T(t) LNY(t) \\
& + \begin{bmatrix} u(t) \\ \int_{t-m}^{t} u(s) ds \end{bmatrix}^T \begin{bmatrix} 0 & R_{12} \\ * & R_{22} \end{bmatrix} \begin{bmatrix} u(t) \\ \int_{t-m}^{t} u(s) ds \end{bmatrix} \\
& - h \int_{-h}^{0} \int_{t+\theta}^{t} \begin{bmatrix} \dot{u}(s) \\ \hat{f}\big(v(s)\big) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} \dot{u}(s) \\ \hat{f}\big(v(s)\big) \end{bmatrix} ds\, d\theta \\
& + m \int_{-m}^{0} \int_{t+\theta}^{t} \begin{bmatrix} \dot{v}(s) \\ \hat{g}\big(u(s)\big) \end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix} \begin{bmatrix} \dot{v}(s) \\ \hat{g}\big(u(s)\big) \end{bmatrix} ds\, d\theta \\
& + h \int_{-h}^{0} \int_{t+\theta}^{t} \dot{v}^T(s) R_1 \dot{v}(s) ds\, d\theta + m \int_{-m}^{0} \int_{t+\theta}^{t} \dot{u}^T(s) R_2 \dot{u}(s) ds\, d\theta + 2 \sum_{i=1}^{n} \lambda_i \int_0^{u_i} \hat{f}_i(s) ds + 2 \sum_{j=1}^{n} \varphi_j \int_0^{v_j} \hat{g}_j(s) ds \\
& + \int_{t-\mu(t)}^{t} \big[v^T(s) Q_1 v(s) + \hat{f}^T\big(v(s)\big) Q_2 \hat{f}\big(v(s)\big)\big] ds \\
& + \int_{t-h}^{t} v^T(s) Q_3 v(s) ds + \int_{t-\tau(t)}^{t} \big[u^T(s) U_1 u(s) \\
& + \hat{g}^T\big(u(s)\big) U_2 \hat{g}\big(u(s)\big)\big] ds + \int_{t-m}^{t} u^T(s) U_3 u(s) ds
\end{aligned}
$$

$X^T(t) = [v^T(t)\ v^T(t-h)\ u^T(t-\delta) \dot{u}^T(t)\ \hat{f}^T\big(v(t)\big)\ v^T\big(t-\mu(t)\big)\ \hat{f}^T\Big(v\big(t-\mu(t)\big)\Big)\ \Big(\int_{t-h}^{t} v(s)\, ds\Big)^T \Big(\int_{t-\mu(t)}^{t} f\big(v(s)\big)\, ds\Big)^T \Big(\int_{t-h}^{t-\mu(t)} \dot{v}(s) ds\Big)^T u^T(t)\ u^T\big(t-\mu(t)\big)]$, $Y^T(t) = [u^T(t)\ u^T(t-m)\ v^T(t-\sigma)\ \dot{v}^T(t)\ \hat{g}^T\big(v(t)\big)\ u^T\big(t-\tau(t)\big)\ \hat{g}^T\Big(u\big(t-\tau(t)\big)\Big) \Big(\int_{t-m}^{t} u(s)\, ds\Big)^T \Big(\int_{t-\tau(t)}^{t} g\big(v(s)\big)\, ds\Big)^T \Big(\int_{t-m}^{t-(t)} \dot{u}(s) ds\Big)^T v^T(t)\ v^T\big(t-\tau(t)\big)]$,

and $S = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,

$P = \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} & P_{11} \\ P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & P_{17} & P_{18} & P_{19} & P_{20} & P_{21} \end{bmatrix}$

and $L = \begin{bmatrix} \hat{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,

$N = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 & N_9 & N_{10} & N_{11} \\ N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} & N_{19} & N_{20} & N_{21} \end{bmatrix}$.

Using the facts: $v\big(t-\mu(t)\big) - v(t-h) - \int_{t-h}^{t-\mu(t)} \dot{v}(s) ds = 0$, $u\big(t-\tau(t)\big) - u(t-m) - \int_{t-m}^{t-\tau(t)} \dot{u}(s) ds = 0$ and the Lemmas (1) & (2), the time derivative of $V(u(t), v(t))$ along the trajectory of syste m(3) is given by:

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$$\dot{V}\big(u(t),v(t)\big) = 2X^T(t)P^T \begin{bmatrix} \dot{u}(t) \\ -Au(t-\delta) - \dot{u}(t) + W_0\hat{f}\big(v(t)\big) + W_1\hat{f}\Big(v\big(t-\mu(t)\big)\Big) \\ v\big(t-\mu(t)\big) - v(t-h) - \int_{t-h}^{t-\mu(t)} \dot{v}(s)\,ds \end{bmatrix}$$

$$+2\left[v^T(t) \quad \left(\int_{t-h}^{t} v(s)\,ds\right)^T\right] \begin{bmatrix} T_{12}v(t) - T_{12}v(t-h) \\ T_{12}^T\dot{v}(t) + T_{22}v(t) - T_{22}v(t-h) \end{bmatrix}$$

$$+2\left[u^T(t) \quad \left(\int_{t-m}^{t} u(s)\,ds\right)^T\right] \begin{bmatrix} R_{12}u(t) \\ R_{12}^T\dot{u}(t) + R_{22}u(t) \end{bmatrix} \begin{matrix} -R_{12}u(t-m) \\ -R_{22}u(t-m) \end{matrix} + h^2 \begin{bmatrix} \dot{u}(t) \\ \hat{f}(v(t)) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} \dot{u}(t) \\ \hat{f}(v(t)) \end{bmatrix}$$

$$- \begin{bmatrix} u(t) - u(t-\mu(t)) \\ \int_{t-\mu(t)}^{t} \hat{f}(v(s))\,ds \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} u(t) - u(t-\mu(t)) \\ \int_{t-\mu(t)}^{t} \hat{f}(v(s))\,ds \end{bmatrix}$$

$$+m^2 \begin{bmatrix} \dot{v}(t) \\ \hat{g}(u(t)) \end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix} \begin{bmatrix} \dot{v}(t) \\ \hat{g}(u(t)) \end{bmatrix}$$

$$- \begin{bmatrix} v(t) - v(t-\tau(t)) \\ \int_{t-\tau(t)}^{t} \hat{g}(u(s))\,ds \end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix} \begin{bmatrix} v(t) - v(t-\tau(t)) \\ \int_{t-\tau(t)}^{t} \hat{g}(u(s))\,ds \end{bmatrix}$$

$$+h^2\dot{v}^T(t)R_1\dot{v}(t) - \left(\int_{t-h}^{t-\mu(t)} \dot{v}(s)\,ds\right)^T R_1 \left(\int_{t-h}^{t-\mu(t)} \dot{v}(s)\,ds\right)$$

$$-\begin{pmatrix} v(t) \\ v(t-\mu(t)) \end{pmatrix}^T \begin{pmatrix} R_1 & -R_1 \\ -R_1 & R_1 \end{pmatrix} \begin{pmatrix} v(t) \\ v(t-\mu(t)) \end{pmatrix}$$

$$+m^2\dot{u}^T(t)R_2\dot{u}(t) - \left(\int_{t-m}^{t-\tau(t)} \dot{u}(s)\,ds\right)^T R_2 \left(\int_{t-m}^{t-\tau(t)} \dot{u}(s)\,ds\right)$$

$$-\begin{pmatrix} u(t) \\ u(t-\tau(t)) \end{pmatrix}^T \begin{pmatrix} R_2 & -R_2 \\ -R_2 & R_2 \end{pmatrix} \begin{pmatrix} u(t) \\ u(t-\tau(t)) \end{pmatrix}$$

$$+2\hat{f}^T\big(v(t)\big)\Lambda\,[-Au(t-\delta) + W_0\hat{f}\big(v(t)\big) + W_1 \hat{f}\Big(v\big(t-\mu(t)\big)\Big)] + 2\hat{g}^T\big(u(t)\big)\Pi\,[-Cv(t-\sigma) + V_0\hat{g}\big(u(t)\big) + V_1\hat{g}\Big(u\big(t-\tau(t)\big)\Big)] + v^T(t)Q_1v(t) - (1-w)v^T\big(t-\mu(t)\big) Q_1v\big(t-\mu(t)\big) + \hat{f}^T\big(v(t)\big)Q_2\hat{f}\big(v(t)\big) - (1-w) \hat{f}^T\Big(v\big(t-\mu(t)\big)\Big)Q_2\hat{f}\Big(v\big(t-\mu(t)\big)\Big) + v^T(t)Q_3v(t) - v^T(t-h)Q_3v(t-h) + u^T(t)U_1u(t) - (1-l) u^T\big(t-\tau(t)\big)U_1\,u\big(t-\tau(t)\big) + \hat{g}^T\big(u(t)\big)\,U_2\hat{g}\big(u(t)\big) - (1-l) \hat{g}^T\Big(u\big(t-\tau(t)\big)\Big)U_2\hat{g}\Big(u\big(t-\tau(t)\big)\Big) + u^T(t)U_3u(t) - u^T(t-m)Q_3\,v(t-m) + h^2v^T(t)Q_4v(t) + \left(\int_{t-h}^{t} v(s)\,ds\right)^T Q_4\left(\int_{t-h}^{t} v(s)\,ds\right) + m^2u^T(t)U_4u(t) + \left(\int_{t-m}^{t} u(s)\,ds\right)^T U_4\left(\int_{t-m}^{t} u(s)\,ds\right).$$

Furthermore, there exist positive diagonal matrices D_1, D_2 & G_1, G_2 such that the following inequalities hold based on (4)

$$2\left(v^T(t)KD_1\hat{f}\big(v(t)\big)\right) - \hat{f}^T\big(v(t)\big)D_1\hat{f}\big(v(t)\big) \geq 0$$

$$2v^T\big(t-\mu(t)\big)KD_2\hat{f}\Big(v\big(t-\mu(t)\big)\Big) - \hat{f}^T\big(v(t - \mu(t))\big)D_2\hat{f}\Big(v\big(t-\mu(t)\big)\Big) \geq 0$$

$$2\left(u^T(t)NG_1\hat{g}\big(u(t)\big)\right) - \hat{g}^T\big(u(t)\big)G_1\hat{g}\big(u(t)\big) \geq 0$$

$$2u^T\big(t-\tau(t)\big)NG_2\hat{g}\Big(u\big(t-\tau(t)\big)\Big) - \hat{g}^T\big(u(t - \tau(t))\big)G_2\hat{g}\Big(u\big(t-\tau(t)\big)\Big) \geq 0.$$

Finally we obtain, $\dot{V}\big(u(t),v(t)\big) \leq X^T(t)\Omega X(t) + Y^T(t)\Xi Y(t)$. If $\Omega<0$ & $\Xi<0$, there exist a scalar $\alpha>0$, such that $\dot{V}\big(u(t),v(t)\big) \leq -\alpha\|(u(t),v(t))\|^2$, thus according to Ref. [31], system (3) is globally asymptotically stable.

For $t = t_k$, from the Lyapunov functions one can obtain the following:

$$V_6\big(t_k,u(t_k),v(t_k)\big) - V_6\big(t_k^-,u(t_k^-),v(t_k^-)\big)$$
$$= h\int_{-h}^{0}\int_{t_k+\theta}^{t_k} v^T(s)\,Q_4v(s)\,ds\,d\theta$$
$$- h\int_{-h}^{0}\int_{t_k^-+\theta}^{t_k^-} v^T(s)Q_4\;v(s)\,ds\,d\theta$$
$$+m\int_{-m}^{t_k^-+\theta}\int_{t_k+\theta}^{t_k} u^T(s)\;U_4u(s)\,ds\,d\theta$$
$$-m\int_{-m}^{0}\int_{t_k^-+\theta}^{t_k^-} u^T(s)\;U_4u(s)\,ds\,d\theta$$
$$= h\int_{-h}^{0}\int_{t_k^-+\theta}^{t_k^-} v^T(s)[D_k^TQ_4D_K - Q_4]v(s)ds\,d\theta$$
$$+m\int_{-m}^{0}\int_{t_k+\theta}^{t_k^-} u^T(s)\,[E_k^T\,U_4\,E_k - U_4\,]u(s)ds\,d\theta \;\leq 0.$$
$$\Longrightarrow V_6(t_k,u(t_k),v(t_k) \;\leq\; V_6(t_k^-,u(t_k^-),v(t_k^-),\; k \in \mathbb{Z}_+.$$

Which implies that,

$$V(t_k,u(t_k),v(t_k) \;\leq\; V(t_k^-,u(t_k^-),v(t_k^-),\; k \in \mathbb{Z}_+.$$

Therefore, In both cases the system (3) is globally asymptotically stable.

Hence, this completes the proof.

V. NUMERICAL EXAMPLE

In this section, a numerical example is provided to illustrate the effectiveness of the proposed method.

Example 1. Consider the delayed BAM NNs (3) with

$$A = diag(1.2679, 0.4563, 0.9238, 0.4480),$$
$$C = diag(0.6329, 0.4328, 1.6542, 0.4890),$$

$$W_0 = \begin{bmatrix} -0.0326 & 0.4536 & -0.2335 & 0.2331 \\ -1.9076 & 0.5439 & -0.2610 & 1.2236 \\ 0.3394 & -0.0860 & -0.9321 & -0.5785 \\ -0.1311 & 0.3235 & -0.9432 & -0.5015 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.8647 & -1.2405 & -0.5325 & 0.0220 \\ 0.0472 & -0.9126 & 0.0342 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2774 \end{bmatrix},$$

$$V_0 = \begin{bmatrix} 1.0271 & 1.9987 & 1.4423 & -0.1981 \\ -2.2896 & -0.3320 & -0.7632 & -0.8456 \\ 1.1430 & 0.0360 & 1.2541 & -1.2079 \\ 0.0765 & 1.4065 & -2.5420 & -2.3652 \end{bmatrix},$$

$$V_1 = \begin{bmatrix} -1.2540 & -0.3642 & -2.2551 & 0.1667 \\ 0.9100 & 0.9126 & 1.0754 & 0.1268 \\ -1.1902 & -0.2445 & 1.1556 & -0.6724 \\ 0.5652 & -0.1982 & -0.0987 & -2.7254 \end{bmatrix},$$

$$k_1 = 0.1156,\; k_2 = 0.1652,\, k_3 = 0.7231,\;\; k_4 = 0.3162,$$
$$n_1 = 0.3233,\, n_2 = 0.0219,\, n_3 = 0.5463,\;\; n_4 = 0.5455.$$

When $w = 0$, $l = 0$, Applying the criteria in Refs. [1-6], the maximum value of h & m for globally asymptotically stable of system (3) are 1.4244, 1.9321, 3.5824, 4.0120 & 2.6541,

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1.5443, 1.6598, 3.321, respectively, while by using the Theorem 1 in this paper, we have h = 67:7328, & m = 54.8796, which shows that our result is less conservative than those in [4-6]. By solving the LMI in (10) we can obtain the feasible solution. Here in our paper, we have not provided such kind of solutions due to the restriction of page limitation. This ensures that all the conditions in Theorem 3.1 are satisfied and hence system (3) is globally asymptotic stable.

V. CONCLUSIONS

The problem of delay-dependent globally asymptotic stability criteria for Impulsive BAM neural networks with time-varying & leakage delays is investigated. A new class of Lyapunov functional is constructed to derive some novel delay-dependent stability criteria. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

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