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Enhanced Result on Stability Analysis of Impulsive Discrete time Stochastic Neural Networks with Additive time Varying delays

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***Abstract-* This paper investigates the problem of exponential stability of impulsive discrete-time stochastic neural networks with additive time varying delays. By defining a novel Lyapunov functional an improved delay-dependent exponential stability criterion is expressed in the form of linear matrix inequality which can be readily solved by using standard numerical software. In addition to that, an illustrative example is provided to show the advantage of our proposed stability condition.**

Key words - Additive time-varying delays; Stochastic neural network; Impulse; Exponential stability; Delay- dependent; LMI.

I. INTRODUCTION

In the past two decades neural network have received considerable attention and successfully applied in many areas such as combinatorial optimization, signal processing and pattern Reorganization [1]. Time delay is one of the major sources of instability which is encountered in many engineering systems such as chemical process long transmission lines in pneumatic systems, networked control systems etc. The study of time delay systems (also called a systems with after effect or dead time, hereditary systems, equations with deviating argument or differential difference equations) has received considerable attention over past years. A great number of research results on time delay system exist in the literature (see [1-4]). The stability of time delay systems is a fundamental problem because of its importance in the analysis and synthesis of such systems.

It is well known that stochastic disturbance is probably the main resource of the performance degradation of the implemented neural networks. Therefore the stability analysis problem for stochastic neural networks with the time delay becomes increasingly significant. Many important and interesting results have been reported for stochastic neural networks with time delays a delay-dependent stability condition was established in [6] where the LMI approach was developed a weak assumption on the activation functions was considered.

On the other hand, an impulsive phenomenon exists universally in a wide variety of evolutionary process where the state is changed abruptly at certain moment of time, involving such field as chemical technology, population dynamics, physics and economics [6]. It has also been shown that the impulsive phenomenon exists likewise in Neural Networks, when a stimulus from body or electronic networks, when a stimulus from body or external environment is received by receptors, the electrical impulses will be conveyed to the neural net and an impulsive phenomenon which is called impulsive perturbation arises naturally[7].

Very recently [8, 9] presented several improved delay dependent stability criteria for discrete stochastic neural networks with time delays by constructing the never lyapunov functional and resorting to the free weight matrices method [10, 11]. However, there still exists room for further improvement because some useful terms are ignored in the Lyapunov functional employed in [8, 9] which may lead to conservatism to some extent. The authors [Xian Ming Tang Jin Shou yu] described additive time varying delays in [12].

Motivated by the discussions the aim of this paper is to study the delay dependent stability analysis of impulsive discrete stochastic neural network with additive time varying delay is investigated. In order to obtain large time delay bounds, a new Lyapunov functional is proposed and a novel delay dependent stability criterion is derived in terms of LMIs. It is shown that the newly established result is less conservative and less computationally complex than the existing ones. Numerical example is given to show the effectiveness of our main results.

Notations

R^n denotes the n-dimensional Euclidean space, $R^{m\times n}$ is the set of all $m\times n$ real matrices. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t>0},\mathrm{P})$ be a complete probability space with a filteration, $\{\mathcal{F}\mathrm{t}\}$; $\mathrm{t}>0$ satisfying the usual conditions $\mathcal{E}\{.\}$ denotes the expectations operator with respect to some probability measure P. The superscript "T" represents the transpose and "*" denotes the

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term that is include by symmetry. The notation $X > P$ (respectively, $X > 0$) means that X is real symmetric, positive definite (resp, semi definite) metric, $\lambda_{\min}(A)$ (resp. $\lambda_{\max}(A)$ represent the smallest (resp, largest) eigenvalues of A.

II. PRELIMINARIES

Consider the following n-neuron impulsive discrete-time stochastic neural network with additive time varying delays as:

$$x(k+1) = Ax(k) + Bg\big(x(k)\big) + Cg(x(k - d_1(k) - d_2(k)) + \sigma(k, x(k), x\big(k - d_1(k) - d_2(k)\big)w(k)$$

$$x(k_r) = x(k_r^-) + e_r x(k_r^-), \quad r = 1,2 \ldots, \qquad (1)$$

Where $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$

$g(x(k)) = [g_1\big(x_1(k)\big), g_2\big(x_2(k)\big), \ldots, g_n\big(x_n(k)\big)]^T$ and $y_i(k)$ is the state of i^{th} neuron at time k, $g_i\big(x_i(k)\big)$ denotes the activation function of the i^{th} neuron at time k. $d_1(k)$ $+d_2(k)$ represents the transmission delay that satisfies $\underline{d}$ and $\overline{d}$ are prescribed positive integer representing the lower and upper bound of time delay respectively. $A = diag(c_1, c_2, \ldots, c_n)$ is real constant diagonal matrix with entries $|c_i| < 1$. $B = (b_{ij})_{n\times n}$ is connection weight matrix and $C = (c_{ij})_{n\times n}$ is time delay connection weight matrix. The condition $x(k_r) = x(k_r^-) + e_r x(k_r^-)$, $r = 1,2 \ldots$,are impulse dynamical activities caused by abrupt jumps at certain instants during evolutionary process. Where $x(k_r^-) = \lim_{k\to k_r^-} x(k)$, $e_r: R^n \to R$ are contains over R^n and the impulse instant k_r are assumed to satisfy that $0 = k_0 < k_1 < \cdots k_{r-1} < k_r < \cdots$ and $k_r - k_{r-1} > 1$ for $r = 1,2, \ldots$ $w(k)$ is a scalar weiner process on $(\Omega, \mathcal{F}, P)$ with $\mathcal{E}[w(k)] = 0, \mathcal{E}[w(k)^2] = 1, \mathcal{E}[w(i)w(j)] = 0 \ (i \neq j)$.

Assumption I. [7]

For any $\xi_1, \xi_2 \in R$, $\xi_1 \neq \xi_2$

$$\gamma_i \le \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \le \sigma_i \quad i = 1,2, \ldots, n$$ where γ_i and σ_i are known constant scalars. (2)

Assumption II.

$\sigma(k, x(k), x\big(k - d_1(k) - d_2(k)\big)$: $R^n \times R^n \times R \to R^n$ is the continuous function and is assumed to satisfy

$$\sigma^T\sigma \le \begin{pmatrix} x(k) \\ x(k - d_1(k) - d_2(k)) \end{pmatrix}^T G \begin{pmatrix} x(k) \\ x(k - d_1(k) - d_2(k)) \end{pmatrix}. \qquad (3)$$

Assumption III.

The time delays $d_1(k), d_2(k)$ are assumed to be time varying and satisfy $\underline{d_1}$, $\underline{d_2}$, $\overline{d_1}$, $\overline{d_1}$, are constant positive scalars representing the lower delay and the upper delay respectively. The lower bounds $\underline{d_1}$, $\underline{d_2}$ are merged as $\underline{d}$ to represent minimum time delay. Thus it is assumed that $\underline{d} \le d_1(k) + d_2(k) \le \overline{d}$. (4)

Definition 2.1.

The discrete stochastic time delay neural network (1) is said to be exponentially stable in the mean square if there exist two scalars $\alpha > 0$ and $0 < \beta < 1$ such that

$$\varepsilon\left[||x(k)||^2\right] \le \alpha\beta^k \, sup_{-\overline{d} \le s \le 0}\, \varepsilon\left[||x(s)||^2\right]. \qquad (5)$$

III. MAIN RESULTS

In this section we shall establish our main criterion based on LMI approach. For presentation convenience, in the following we denote $F_1 = diag\{\gamma_1\sigma_1, \gamma_2\sigma_2, \ldots, \gamma_n\sigma_n\}$ and $F_2 = diag\left\{\frac{\gamma_{1+\sigma_1}}{2}, \frac{\gamma_{1+\sigma_1}}{2}, \ldots, \frac{\gamma_{1+\sigma_1}}{2}\right\}$, $\Gamma_1 = diag\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ and $\Gamma_2 = diag\{\sigma_1, \sigma_2, \ldots, n\}$, $\delta = \frac{\underline{d} + \overline{d}}{2} + \frac{\min\{(-1)^{\underline{d}+\overline{d}}, 0\}}{2}$, $\tau = \overline{d} - \delta$, $m = \overline{d} - \underline{d} + 1$.

Theorem 3.1

For given scalars $\overline{d}$ and $\underline{d}$. Under assumptions I-III the impulsive discrete-time stochastic neural network (1) with additive time varying delay is exponentially stable in the mean square for any additive time varying delay $d_1(k) + d_2(k)$ satisfying $\underline{d} \le d_1(k) + d_2(k) \le \overline{d}$, if there exist symmetric positive definite matrices $P, Q_i (i = 1,2,3,4)$, $Z_j (j = 1,2)$ diagonal matrices $D > 0, H > 0, K > 0, L > 0$ and a scalar $\lambda > 0$ such that following LMI holds.

(i) $P + \delta^2 Z_1 + \tau^2 Z_2 \le \lambda I$ and $\psi_i + \pi < 0, i = 1,2$ (6)

(ii) $\sum_{i=1}^{m} \ln(1 + k_i)^2 + k\ln(1 - \varepsilon) \le \vartheta(k)$ (7)

for every $k \in [k_r, k_{r+1})$ then $\lim_{k\to+\infty} \vartheta(k) = +\infty$.

$$\text{Where} \quad \pi = \begin{bmatrix} \pi_{11} & \lambda G_2 & 0 & Z_1 & \pi_{11} & \pi_{11} \\ * & \pi_{22} & Z_2 & Z_2 & 0 & \pi_{26} \\ * & * & \pi_{33} & 0 & 0 & 0 \\ * & * & * & \pi_{44} & 0 & 0 \\ * & * & * & * & \pi_{55} & \pi_{56} \\ * & * & * & * & * & \pi_{66} \end{bmatrix},$$

$\pi_{11} = -P + Q_1 + mQ_2 + Q_3 - Z_1 - F_1 D - 2m(\Gamma_1 k - \Gamma_2 L) + APA + (A - I)(\delta^2 Z_1 + \tau^2 Z_2)(A - I) + \lambda G_1$,

$\pi_{15} = m(K - L) + F_2 D + APB + (A - I)(\delta^2 Z_1 + \tau^2 Z_2)B$,

$\pi_{16} = APC + (A - I)(\delta^2 Z_1 + \tau^2 Z_2)C$,

$\pi_{22} = -Q_2 - 2Z_2 - F_1 H + 2(\Gamma_1 k - \Gamma_2 L) + \lambda G_3$,

$\pi_{26} = F_2 H - (k - L)$,

$\pi_{33} = -Q_3 - Z_2$,

$\pi_{44} = -Q_1 - Z_1 - Z_2$,

$\pi_{55} = -D + mQ_4 + B^T PB + B^T(\delta^2 Z_1 + \tau^2 Z_2)B$,

$\pi_{66} = -H - Q_4 + C^T PC + C^T(\delta^2 Z_1 + \tau^2 Z_2)C$,

$\Delta_1 = [0 \ \ I \ \ 0 - I \ \ 0 \ \ 0]$ and $\Delta_2 = [0 \ \ I - I \ \ 0 \ \ 0 \ \ 0]$,

$\psi_1 = -\Delta_1^T Z_2 \Delta_1$ and $\psi_2 = -\Delta_2^T Z_2 \Delta_2$.

Defining $\eta(k) = x(k+1) - x(k)$.

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Proof. Consider the following Lyapunov-Krasovskii functional for system (1) to prove the exponential stability result

$$V(k) = V_1(k) + V_2(k), \tag{8}$$

Where,

$$\begin{aligned} V_1(k) = {} & x^T(k)Px(k) + \textstyle\sum_{i=k-\delta}^{k-1} x^T(i)Q_1x(i) \\ & + \textstyle\sum_{j=-\overline{d}+1}^{-\underline{d}+1} \sum_{i=k-1+j}^{k-1} x^T(i)Q_2x(i) \\ & + \textstyle\sum_{j=-\overline{d}+1}^{-\underline{d}+1} \sum_{i=k-1+j}^{k-1} g^T(x(i))Q_4\, g(x(i)) \\ & + \delta \textstyle\sum_{j=-\delta}^{-1} \sum_{i=k+j}^{k-1} \eta^T(i)Z_1\eta(i) \\ & + 2\textstyle\sum_{j=-\overline{d}+1}^{-\underline{d}+1} \sum_{i=k-1+j}^{k-1} \{[g(x(i)) - \Gamma_1 x(i)]^T k. \\ & + [\Gamma_2 x(i) - g(x(i))]^T L\} x(i), \end{aligned}$$

$$V_2(k) = \begin{cases} \sum_{i=k-\underline{d}}^{k-1} x^T(i)Q_3x(i) + (\overline{d} - \delta) \\ \quad \times \sum_{j=\overline{d}}^{-\delta-1} \sum_{i=k+j}^{k-1} \eta^T(i)Z_2\eta(i)\,, \\ \qquad \delta \le d_1(k) + d_2(k) \le \overline{d} \\ \sum_{i=k-\underline{d}}^{k-1} x^T(i)Q_3x(i) + (\delta - \underline{d}) \\ \quad \times \sum_{j=-\delta}^{-\underline{d}-1} \sum_{i=k+j}^{k-1} \eta^T(i)Z_2\eta(i), \\ \qquad \underline{d} \le d_1(k) + d_2(k) \le \delta. \end{cases}$$

Then, along the solution of system (1), we have

$$\begin{aligned} \varepsilon[\Delta V_1(k)] \le {} & \varepsilon[x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ & + x^T(k)Q_1x(k) \; - x^T(k-\delta)Q_1x(k-\delta) \\ & + (\overline{d} - \underline{d} + 1)x^T(k)Q_2x(k) \; - x^T(k - d_1(k) - d_2(k))Q_2 \\ & \times x(k - d_1(k) - d_2(k)) + (\overline{d} - \underline{d} + 1)g^T(x(k))Q_4 g(x(k)) \\ & - g^T(x(k - d_1(k) - d_2(k)))Q_4\, g(x(k - d_1(k) - d_2(k))) \\ & + \delta^2\eta^T(k)Z_1\eta(k) - \delta\textstyle\sum_{i=k-\delta}^{k-1}\eta^T(i)Z_1\eta(i) + 2(\overline{d} - \underline{d} + 1) \\ & \times [g(x(k)) - \Gamma_1 x(k)]^T k\, x(k) \; - 2g^T(x(k - d_1(k) - d_2(k)) \\ & \times k\, x(k - d_1(k) - d_2(k)) + 2x^T(k - d_1(k) - d_2(k))\Gamma_1 k \\ & \times x(k - d_1(k) - d_2(k)) + 2(\overline{d} - \underline{d} + 1) \\ & \times [\Gamma_2 x(k) - g(x(k))^T\, L\, x(k) + 2g^T(x(k - d_1(k) - d_2(k))) \\ & \times L\, x(k - d_1(k) - d_2(k)) - 2x^T(k - d_1(k) - d_2(k)) \\ & \times \Gamma_2 L\, x(k - d_1(k) - d_2(k)). \end{aligned}$$

It is easy to get that

$$-\delta\textstyle\sum_{i=k-\delta}^{k-1}\eta^T(i)Z_1\eta(i) = \begin{pmatrix} x(k) \\ x(k-\delta) \end{pmatrix}^T \begin{bmatrix} -Z_1 & Z_1 \\ * & -Z_1 \end{bmatrix} \begin{pmatrix} x(k) \\ x(k-\delta) \end{pmatrix}.$$

When $\delta \le d_1(k) + d_2(k) \le \overline{d}$

$$\begin{aligned} \varepsilon[\Delta V_2(k)] = {} & \varepsilon[x^T(k)Q_3x(k) - x^T(k-\overline{d})Q_3x(k-\overline{d}) \\ & + (\overline{d} - \delta)^2\, \eta^T(k)Z_2\eta(k) - (\overline{d} - \delta)\textstyle\sum_{i=k-\overline{d}}^{k-\delta-1} \eta^T(i)Z_2\eta(i)]. \end{aligned}$$

Let $g = \frac{\overline{d} - d_1(k) - d_2(k)}{\overline{d} - \delta}$ then it is easy to get that $0 \le g \le 1$, $d_1(k) + d_2(k) - \delta = (1-g)(\overline{d} - \delta)$ and $(-\overline{d} - \delta) \times \sum_{i=k-\overline{d}}^{k-\delta-1}\eta^T(i)Z_2\,\eta(i)$

$$\begin{aligned} & \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \\ x(k-\overline{d}) \end{pmatrix}^T \begin{pmatrix} -2Z_2 & Z_2 & Z_2 \\ * & -Z_2 & 0 \\ * & * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \\ x(k-\overline{d}) \end{pmatrix} \\ & + g \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \end{pmatrix}^T \begin{pmatrix} -Z_2 & Z_2 \\ * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \end{pmatrix} \\ & + (1-g) \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\overline{d}) \end{pmatrix}^T \\ & \begin{pmatrix} -Z_2 & Z_2 \\ * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\overline{d}) \end{pmatrix}. \end{aligned}$$

Similarly, $\underline{d} \le d_1(k) + d_2(k) \le \delta$

$$\begin{aligned} & \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \\ x(k-\underline{d}) \end{pmatrix}^T \begin{pmatrix} -2Z_2 & Z_2 & Z_2 \\ * & -Z_2 & 0 \\ * & * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \\ x(k-\underline{d}) \end{pmatrix} \\ & + g \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \end{pmatrix}^T \begin{pmatrix} -Z_2 & Z_2 \\ * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\delta) \end{pmatrix} \\ & + (1-g) \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\underline{d}) \end{pmatrix}^T \\ & \begin{pmatrix} -Z_2 & Z_2 \\ * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ x(k-\underline{d}) \end{pmatrix}. \end{aligned}$$

We have from (2) and [20]

$$\begin{pmatrix} x(k) \\ g(x(k)) \end{pmatrix}^T \begin{bmatrix} F_1D & -F_2D \\ * & D \end{bmatrix} \begin{pmatrix} x(k) \\ g(x(k)) \end{pmatrix} \le 0 \text{ and}$$

$$\begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ g(x(k-d_1(k)-d_2(k))) \end{pmatrix}^T \begin{bmatrix} F_1H & -F_2H \\ * & H \end{bmatrix} \begin{pmatrix} x(k-d_1(k)-d_2(k)) \\ g(x(k-d_1(k)-d_2(k))) \end{pmatrix} \le 0.$$

We have get from (3) and (6) that

$$\begin{aligned} & \sigma(k, x(k), x(k - d_1(k) - d_2(k))^T (P + \delta^2 Z_1 + \tau^2 Z_2) \\ & \qquad \times \sigma(k, x(k), x(k - d_1(k) - d_2(k)) \\ & \qquad \le \lambda \begin{pmatrix} x(k) \\ x(k-d_1(k)-d_2(k)) \end{pmatrix}^T \begin{pmatrix} G_1 & G_2 \\ * & G_3 \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-d_1(k)-d_2(k)) \end{pmatrix}. \end{aligned}$$

Thus

$$\varepsilon[\Delta V(k)] \le \varepsilon[\zeta^T(k)(g(\pi + \psi_1) + (1-g)(\pi + \psi_2))\zeta(k)],$$

Where

$$\begin{aligned} \zeta(k) = [& x^T(k),\;\; x^T(k - d_1(k) - d_2(k)),\;\; \theta^T(k), \\ & x^T(k-\delta),\;\; g^T(x(k)),\;\; g^T(x(k - d_1(k) - d_2(k)))]^T \end{aligned}$$

$$\theta(k) = \begin{cases} x(k - \overline{d}\,), \delta \le d_1(k) + d_2(k) \le \overline{d} \\ x(k - \overline{d}\,), \underline{d} \le d_1(k) + d_2(k) \le \delta \end{cases}$$

$$g = \begin{cases} \frac{\overline{d} - d_1(k) - d_2(k)}{\overline{d} - \delta}, \delta \le d_1(k) + d_2(k) \le \overline{d} \\ \frac{d_1(k) + d_2(k) - \underline{d}}{\delta - \underline{d}}, \underline{d} \le d_1(k) + d_2(k) \le \delta. \end{cases}$$

From the definition of Operator Δ, one can observe that $\Delta V(k, x(k)) = V(k+1, x(k+1)) - V(k, x(k)) < 0$.

Then there exist a +ve Constant $\epsilon \le 1$ such that

$$V(k+1,\; x(k+1)) \le (1-\epsilon)V(k, x(k)),\;\; k \ne k_r.$$

Then

$$\begin{aligned} & V(k_0 + 1, x(k_0 + 1)) \le (1-\epsilon)V(k_0, x(k_0)) \\ & \quad \vdots \\ & V(k_1 + 1,\; x(k_1)) \le (1-\epsilon)^{k_1 - k_0} V(k_0, x(k_0)) \end{aligned}$$

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and hence

$V(k, x(k)) \leq (1-\epsilon)^{k-k_r} V(k_r, x(k_r)), k \in [k_r, k_{r+1}).$ (9)

Here we note that $k = k_r^-$

$V(k_r, x(k_r)) \leq (1+k_r)^2\, V(k_r^-, x(k_r^-)).$ (10)

By applying (9) and (10) successively in each Interval of $[k_r, k_{r+1}]$ yields the following equations

For$k \in [k_1, k_2)$,

$V(k_1, x(k_1)) \leq (1+k_1)^2 (1-\epsilon)^{k-k_0} V(k_0, x(k_0))$ and

$V(k_2, x(k_2)) \leq$

$(1+k_2)^2 (1+k_2)^2 (1-\epsilon)^{k_2-k_0} V(k_0, x(k_0))$

For$k \in [k_0, k_1)$,

$V(k, x(k)) \leq (1+k_0)^2 (1-\epsilon)^{k_1-k_0} V(k_0, x(k_0)).$

Therefore by Induction , for any $[k_r, k_{r+1}), r = 0,1,2$ ...we obtain $V(k, x(k)) \leq V(k_0, x(k_0)) \exp\{\vartheta(k)\}$. (11)

On the other hand , it is easily to get that

$V(k) \leq \alpha_1 \varepsilon\left\{||x(k)||^2\right\} + \alpha_2 \sum_{i=k-\bar{d}}^{k-1} \varepsilon\{||x(i)||^2\}.$ (12)

For any $\varpi > 1$, it follows from(12) that

$\varpi^{s+1}\varepsilon\{V(s+1)\} - \varpi^s \varepsilon\{V(s)\} \leq \varpi^s(-\epsilon\, \varpi\, \varepsilon\left\{||x(s)||^2\right\}$

$+(\varpi-1)\alpha_1\, \varepsilon\{\,||x(s)||^2\} \;+(\varpi-1)\alpha_2 \sum_{i=s-\bar{d}}^{s-1} \varepsilon\{||x(s)||^2\}.$ (13)

Summing upon both sides of (13) from 0 to $k-1$ we obtain that $\varpi^k\, \varepsilon\{V(k)\} - \varepsilon\{V(0)\} \leq \mu_1(\varpi) sup_{-\bar{d}\leq s\leq 0} \varepsilon\{||x(s||^2\}$

$+\mu_2(\varpi) \sum_{s=0}^{k} \varpi^k \varepsilon\left\{||x(s)||^2\right\}.$ (14)

Since $\mu_2(1) = -\epsilon\varpi < 0$ there must exist a +ve $\varpi_0 > 1$ such that $\mu_2(\varpi_0) < 0$. Then we have

$\varepsilon\{V(k)\} \leq \mu_1(\varpi_0)\, (\frac{1}{\varpi_0})^k \;\; sup_{-\bar{d}\leq s\leq 0}\, \varepsilon\left\{||x(s)||^2\right\}.$

$+\left(\frac{1}{\varpi_0}\right)^k V(0)$ (15)

It follows that $\left\{||x(s)||^2\right\} \leq \;\alpha\beta^k \;\; sup_{-\bar{d}\leq s\leq 0}\; \varepsilon\left\{||x(s)||^2\right\}$

Where $\beta = (\varpi_0)^{-\frac{1}{2}},\; \alpha = \sqrt{\frac{\mu_1(\varpi_0)+\phi}{\lambda_{min}(Q)}}$. By definition of (2.1) the system is globally exponentially stable in the mean square.

IV NUMERICAL EXAMPLE

In this section, a numerical example is introduced to demonstrate the less conservativeness of the proposed method.

Example 4.1

Consider the impulsive discrete stochastic additive time delay neural networks of the system (1)

$$A = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0.2 & -1 \\ -0.8 & 0.1 \end{bmatrix}, C = \begin{bmatrix} 0.2 & -0.9 \\ 1 & 0.8 \end{bmatrix}$$

$d_1 = [4(\cos(\pi \times 0.5)) \;\; 0],\; d_2 = [9(\sin(\pi \times 0.5) - 1)), \;\; 0]$

$e_r = -0.6.$

The activation function is defined by $g(x) = e^{-8x}$.

By solving the LMI in (6) we can obtain the feasible solution. Here in our paper, we have not provided such kind of solutions due to the restriction of page limitation. This ensures that all the conditions in Theorem 3.1 are satisfied and hence system (1) exponentially stable in the mean square.

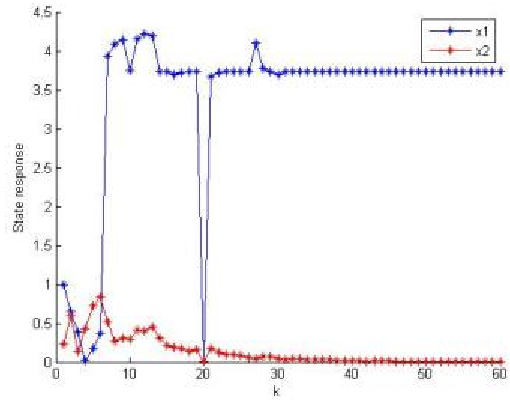

Figure 1 $e_r = -0.6$ and the state response $x(k)$ with impulse

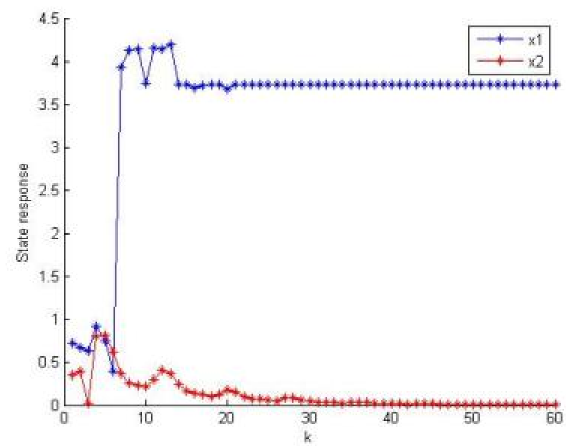

Figure 2 The state response $x(k)$ without impulse

Figures 1 and 2 depict the time response of state variables x1 and x2 with and without impulsive effects. The simulation result reveals that by taking external disturbances like stochastic and impulsive effects into account our results quickly lead to the stable state for the above given parameters.

V. CONCLUSIONS

A new augmented Lyapunov–Krasovskii functional is constructed to achieve the delay-dependent stability result of the considered discrete–time stochastic impulsive neural network with two additive time-varying delays in state and some new improved sufficient conditions ensuring globally exponentially stable are obtained. The merit of the proposed conditions lies in its less conservatives and making full use of

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the delay information. Finally, an illustrative example has been provided to show the advantage of the obtained results.

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