



A REVIEW ON MOMENT REDISTRIBUTION IN CONTINUOUS REINFORCED CONCRETE BEAMS

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ABSTRACT: Structural engineers have long recognised the importance of ductility in the design of reinforced concrete structures and as a consequence of this, the importance of the ability of a reinforced concrete member to redistribute moment to give: prior warning of failure; adjust the structural response to allow for variations in applied load and column drift and to absorb energy during earthquake, blast and other dynamic loadings. Quantifying the ability of a member to redistribute moment and thus to provide the most favourable design to perform moment redistribution by keeping optimum ductility will provide more safe, economical and durable structures. The aim of present study is to find out various parameters affecting moment redistribution in most common structural element such as reinforced concrete continuous beams, comparison of different nation's code provisions for moment redistribution, and finally to establish optimum design criteria for favourable moment redistribution and thus to establish the allowable percentage of redistribution. This paper is based on literature review and initial numerical validation for the proposed study.

INTRODUCTION

Reinforced concrete is the most widely used material around the world. This is due to two major advantages, namely concrete's high compressive strength and ability to be cast in any shape or form. Another feature of concrete is that it can be manipulated to suit most environmental conditions with optimal performance. Concrete also has a major setback, which is that it is a brittle material and has little tensile resistance and thus susceptible to cracking behaviour in the tensile regions. But when combining with reinforcement the reinforced concrete is a ductile material. During the last few decades thousands of experimental tests has been conducted to study the feasibility of the member's structural behaviour to provide effective & economical designs with durability.

Normal we assume linear elastic behaviour when calculating the bending moment and shear force distributions in a reinforced concrete structure. This assumption is reasonable at low levels of loading and it becomes increasingly invalid at higher loads due to cracking and the development of plastic deformations. Due to the nonlinear structural behaviour, linear elastic analysis lead to an wrong assessment of the behaviour and, so, it can become necessary to use more advanced methodologies to achieve sufficiently accurate analysis. The advanced methods can enable a higher degree of performance optimisation of structures than those resulting from the simplified approaches adopted by existing design codes based on linear elastic analysis with redistribution of internal forces. In order to assess the load-carrying capacity at the ultimate limit state, a model combining plastic and nonlinear analysis can be utilised.

MOMENT REDISTRIBUTION

It can be seen that several experimental studies show that indeterminate structures such as continuous concrete

beams does not fail when critical sections reach their ultimate strengths. That is if a structure has adequate ductility, the phenomenon moment redistribution will take place in the flexural members by developing plastic hinges at critical sections which causes the other points of beams to achieve their ultimate strengths and capacities.

Moment redistribution in a statically indeterminate beam is the transfer of moment between high moment regions in the member, while maintaining the overall strength. At the initial stage of loading, the continuous beam will behave linear elastically such that both span moments and support moments will increase proportional to the increase in applied load. Eventually, the ultimate strength will be reached at the maximum moment sections upon further increase in the applied moment. Now as the load is increased further, the moments will redistribute from the maximum moment sections to other parts of the beam, such that the total static moment in the beam remains unchanged.

Moment redistribution concept

Moment redistribution in reinforced concrete members was first observed in 1920 when the results of tests on two beams fully fixed at the end were reported by the German Reinforced Concrete Committee. The first extensive series of tests demonstrating moment redistribution in reinforced concrete beam was carried out by Kazinczy and reported by Yu and Hognestad (1958). Glanville and Thomas (1935) conducted research on moment redistribution. Their tests were conducted on two-span continuous beams loaded with concentrated loads in the middle of each span. Until that time, there had not been any significant results that could relate redistribution to the percentage of reinforcement used in the research.

Consider a beam of length L in Fig. 1.1(c), which is equivalent to an internal span of a continuous beam. For convenience, it is assumed that the same longitudinal reinforcing bars are in the top and bottom of the beam. Hence, the hogging (hog) and sagging (sag) regions have the same moment/curvature (M/Ψ) relationships as shown in Fig. 1.1(a). Let us assume idealised perfectly elastic portion has a flexural rigidity of $(EI)_{el}$ up to a moment capacity of M_u at a curvature Ψ_y , after which there is a perfectly plastic

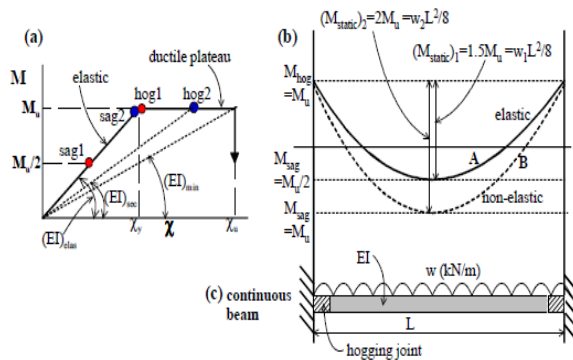


Fig. 1 Moment redistribution concept

ductile plateau in which the secant stiffness $(EI)_{sec}$ reduces up to a curvature of Ψ_u at which failure occurs when the secant stiffness is at its minimum $(EI)_{min}$. The beam in Fig. 1.1(c) is subjected to a uniformly distributed load w , so that whilst the flexural rigidity of the whole beam remains at EI , the moment at the supports M_{hog} is twice that at mid-span M . Hence for this specific beam, there is no moment redistribution whilst the maximum hogging moment M_{hog} is equal to twice the maximum mid-span moment M_{sag} . That is when $M_{hog} = 2M_{sag}$, then there is moment redistribution. Therefore in this context, moment redistribution is defined as occurring when the distribution of moment within a beam is not given by elastic analysis that assumes EI is constant within the beam.

As the uniformly distributed load W is applied to the beam in Fig. 1.1(c) Moment redistribution concept As the uniformly distributed load w is gradually applied to the beam in Fig. 1.1(c), the beam is initially elastic so that $M_{hog} = 2M_{sag}$ and there is no moment redistribution. When the support moment first reaches its moment capacity M_u as shown as the point $hog1$ in Fig. 1.1(a), then the mid-span moment reaches a value of $M_u/2$ which is shown as $sag1$. At this stage, the static moment is $(M_{static})_1 = 1.5 M_u = W L^2/8$ as shown in Fig. 1.1(b) and the distribution of moment is given by line A which is labelled elastic. Up to this point, the beam behaviour remains linear elastic. As the load is increased, the beam deflects further resulting in an increase in M_{sag} above $M_u/2$ in Fig. 1.1(b). However, the moment at the support remains at M_u . The only way that the increase in deflection or deformation, due to the increased load, can be accommodated is for the curvature at the supports to be

increased from $hog1$ to $hog2$ as shown in Fig. 1.1(a) and the hogging curvature will keep increasing until the sagging curvature reaches $sag2$ in Fig. 1.1(a), that is the mid-span moment has reached its capacity M_u whilst the behaviour of the hogging region is no longer elastic. The static moment has now reached $(M_{static})_2 = 2M_u = W L^2/8$ in Fig. 1.1(b), which is the maximum static moment. Hence, the maximum load W_2 that can be applied as all the joints, that is the positions of maximum moments in the hogging and the sagging regions, have reached their moment capacities and a collapse mechanism has formed. The distribution of moment within the beam is now given by line B which has been labelled non-elastic as shown in Fig. 1.1(b). It can be seen in the example shown in Fig. 1.1, that it is the hogging joints that are required to maintain the moment whilst their curvature is increasing. Hence in this example, it is the hogging joints that have to redistribute moment and it is their ductility that governs the amount of moment redistribution. If for example it was necessary for $hog2$ in Fig. 1.1(a) to exceed the curvature capacity of the section k_b to achieve the static moment $(M_{static})_2$ in Fig. 1.1(b), then $sag2$ in Fig. 1.1(a) cannot achieve M_u and the continuous beam would fail before reaching its theoretical plastic capacity. It can be seen in this example that the sagging moment joint has only to reach its moment capacity, M_u in Fig. 1.1(a) at point $sag2$ that is its curvature has only to reach Ψ_y . Hence its ductility, that is its capacity to extend along the plateau in Fig. 1.1(a), is of no consequence. Unless of course the beam is required to absorb energy such as under seismic loads, in which case it may be a requirement that point $sag2$ is also extended into the plastic zone to allow the beam to deflect further and absorb energy without an increase in load.

Benefits of utilising moment redistribution are as follows-

- Savings of reinforcing steel, as there is no need to design for the full moments of the moment envelope obtained for different load arrangements.
- Less reinforcement placed in the negative moment zones, thus a reduced magnitude of the internal compression force (in particular beneficial for narrow webs of T-sections).
- Reduction of congestion of bars over supports of continuous beams or slabs and therefore improving the conditions for attaining a good concrete quality in these critical areas.
- More freedom for the designer in arranging the reinforcement.

Ductility

For ensuring the moment redistribution to happen there should be sufficient ductility for members of structures. Ductility is an important property of structural members that allows large deformations and deflections to occur under overload conditions. It provides warning of the imminence of failure for statically determinate beams, and it allows moment



redistribution to occur in statically indeterminate beam at overload. The ductility of a member can be determined from moment curvature relation, where larger deformations indicate better ductility.

LITERATURE REVIEW

As part of study of moment redistribution certain supporting journals were reviewed, details are as follows-

R. H. Scott and R. T. Whittle 2005[1] conducted an investigation which aimed to explore the nature of moment redistribution as load was increased on a structure based on a series of two-span reinforced concrete beams. The effects of the following parameters were investigated depth of section, different values of design moment redistribution, different percentages of tension steel at the centre support and consequently in the spans, different arrangements of reinforcement, different concrete strengths, the effect of brittle reinforcement. Conclusion of study were-total moment redistribution has two components elastic redistribution, and plastic redistribution. A consequence of elastic redistribution is that beams designed for zero redistribution will, in fact, undergo plastic redistribution before the ULS is reached. Reinforcement arrangement large bars have little effect on total redistribution. Concrete strength can influence total redistribution at the ULS since the moment of resistance of the section is increased. Design redistribution from support to span: plastic contribution is less than the designed total redistribution which produces ductile failure mechanism. Design redistribution from span to support: plastic contribution is more than the designed total redistribution which leads to a brittle failure mechanism.

A.K.H. Kwan et al 2002[2] the interrelation between the flexural strength and the flexural ductility that could be simultaneously achieved was evaluated and plotted in the form of charts based on previous studies. Using these charts, a new method of beam design called 'concurrent flexural strength and ductility design' that would allow engineers to consider both the strength and ductility requirements at the same time before deciding on whether to use high-strength concrete or add compression reinforcement has been developed. For application to cases in which the concrete grade is prescribed, a simpler method of first determining the limits of steel ratios that would satisfy the ductility requirement and then designing the reinforcement details according to the strength requirement has also been proposed. The conclusions were- The interrelation between the flexural strength and the flexural ductility that could be simultaneously achieved by a beam section has been evaluated and plotted for different concrete grades and compression steel ratios in the form of charts. The addition of compression reinforcement without increasing the tension reinforcement could produce significant increase in flexural ductility but little increase in flexural strength, whereas the addition of compression reinforcement together with an increase in tension reinforcement could increase both the flexural strength and ductility.

Ricardo N.F. et al. 2005 [3] study the moment redistribution and ductility of continuous high-strength concrete beams. Particular care was given to analysing how the tensile reinforcement ratio and the transverse reinforcement ratio influence the plastic rotation capacity of the beams. A comparative study was carried out on several codes related to the moment redistribution permitted and the experimental findings. It was found that some of the recommendations are unsafe. It was also found that high-strength concrete beams, when properly designed, have enough deformation capacity to be used in plastic analysis. The experimental program consisted of analysing the behaviour of 10 continuous beams. The load was gradually increased by force control until beam failure occurred. The conclusions were- It was found that high-strength concrete can have a good capacity for moment redistribution, depending on the tensile reinforcement ratio in the section at the intermediate support. The evolution of the rotation ductility index with tensile reinforcement ratio and with the transverse reinforcement ratio for beams with high-strength concrete has been presented, and it is concluded that the rotation ductility index was considerably affected by the variation of the tensile reinforcement ratio.

Neha S. Badiger et al 2014[4] Concrete structural components such as beams, columns, walls exist in various buildings and bridges. Understanding the response of these components of structures during loading is crucial for the development of an efficient and safe structure. Recently Finite Element Analysis (FEA) is also used to analyze these structural components. In this paper, four point bending analysis is carried out using reinforced concrete beam. The results of the beam with respect to mesh density, varying depths, use of steel cushions for support and loading points, effect of shear reinforcement on flexure behaviour, impact of tension reinforcement on behaviour of the beam are analyzed and discussed. Finite element software ANSYS 13.0 is used for modelling and analysis by conducting non linear static analysis.

R. Eligehausen P. Langer 1986[5] established a mathematical model for calculating the rotation capacity of plastic hinges in reinforced concrete beams and slabs. The model is based on the integration of the section curvature along the beam, taking into account the contribution of concrete between cracks (tension stiffening) and the shifting of the tensile force by shear cracks. The material behaviour of reinforcing bars, concrete and bond is described as realistically as possible. The analytically predicted rotation capacities of about 70 beams compare favourably with the experimental results. The parameter studies demonstrate that the plastic rotation capacity of hinges given by the CEB-FIP Model Code is unconservative for cold worked deformed reinforcing bars with a low ratio tensile strength to yield stress and a low value of the uniform elongation. With the presented analytical model, the rotation capacity of plastic hinges in reinforced concrete beams or slabs can be predicted with sufficient accuracy for practical purposes. The rotation capacity of plastic hinges is significantly influenced by the



shape of the stress-strain relationships of the reinforcement in the inelastic range. This is especially valid for low percentages of reinforcement.

Adnan Shakir and David M. Rogowsky 2000[6] present an analytical model for computing the plastic rotation capacity ϕ_p and permissible moment redistribution, β , in reinforced concrete beams. Important parameters, affecting ϕ_p and β , are identified and incorporated in the model. The model is validated against experimental results and shows good agreement. A comparison of the moment redistribution limits is made between the model and CSA A23.3-94. Although the code provides a reasonable estimate of β for unfavourable combinations of parameters, the code can be very conservative when conditions are favourable for moment redistribution. Deeper beams with closely spaced stirrups allow significantly more moment redistribution than that predicted by the code. The study shows that the most important factor influencing moment redistribution is plastic rotation capacity and plastic rotation demand. Based on analytical modelling, conclusions were- The CSA A23.3-94 provision for permissible moment redistribution is assessed and found to be deficient in incorporating the key parameters affecting β . Comparison with the code (CSA A23.3-94) limit for moment redistribution shows that the code limit is conservative. Although the code provides a reasonable estimate of β for unfavourable combinations of parameters, it can be very conservative for more favourable combinations of parameters. Deeper beams with closely spaced stirrups allow significantly more moment redistribution than that predicted by the code.

D. Mostofinejad and F. Farahbod 2007[7] conducted a parametric study on moment redistribution in continuous RC beams with equal spans under uniform loading was performed. First, the governing equation for the allowable percent of moment redistribution was extracted using ductility demand and ductility capacity concepts. The effects of different parameters such as the concrete compressive strength, the amount and the strength of reinforcing steel, the magnitude of elastic moment at the support and the ratio of the length to the effective depth of the continuous beam on moment redistribution were then investigated. The results showed that, whereas the permissible moment redistribution in continuous reinforced concrete beams based on the relevant rules in the current codes is not in a safe margin in some cases, it is rather conservative in most cases.

Ali Kheyroddin and Hosein Naderpour 2007[8] conducted a parametric study is performed to assess the influence of the tension reinforcement index, ($\omega = \rho f_y / f_{bc}$), and the bending moment distribution (loading type) on the ultimate deformation characteristics of reinforced concrete (RC) beams. The analytical results for 15 simply supported beams with different amounts of tension reinforcement ratio under three different loading conditions are presented and compared with the predictions of the various formulations and the experimental data, where available. The plastic hinge rotation capacity increases as the loading is changed from the concentrated load at the middle to the third-point loading, and

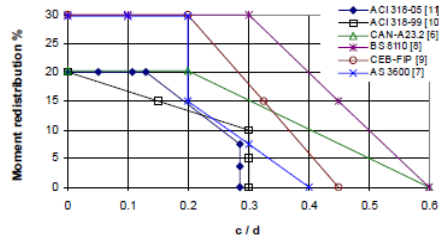
it is a maximum for the case of the uniformly distributed load. The analytical results indicate that the proposed equations can be used for analysis of ultimate capacity and the associated deformations of RC beams with sufficient accuracy.

M.Z.Cohn and Z.Lounis 1990[9] compared various nations design codes in the causes of moment redistribution and found that there is difference in percentage allowable redistribution. The parameter of crack width's importance is mentioned in this paper. Regarding the parameters X_u/d , ratio of LL to total Load is discussed.

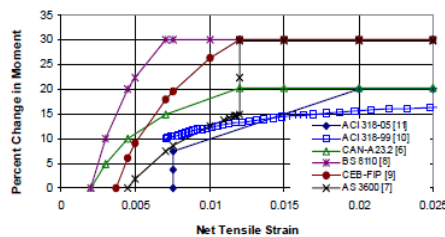
Ricardo N.F. do Carmo and Sérgio M.R. Lopes 2014[10] suggests that evaluation of the ductility of reinforced concrete beams is very important, since it is essential to avoid a fragile collapse of the structure by ensuring adequate deformation at the ultimate limit state. One of the procedures used to quantify ductility is based on deformations, namely, the plastic rotation capacity. Knowledge of the plastic rotation capacity of certain regions of the structure is important for a plastic analysis or a linear analysis with moment redistribution. An experimental program is described in this article. It is composed of 10 tests designed to study the moment redistribution and ductility of continuous high-strength concrete beams. Particular care was given to analysing how the tensile reinforcement ratio and the transverse reinforcement ratio influence the plastic rotation capacity of the beams. A comparative study was carried out on several codes related to the moment redistribution permitted and the experimental findings. It was found that some of the recommendations are unsafe. It was also found that high-strength concrete beams, when properly designed, have enough deformation capacity to be used in plastic analysis.

CODAL COMPARISON

Provision of moment redistribution in various codes such as ACI 318-14, IS 456-2000, BS 8110, Eurocode 2 were studied and comparison is tabulated in Table 1. It can be seen that as per the ACI code the condition for moment redistribution is based on minimum strain of reinforcement and based on Eurocode and Indian standard the base is ratio of neutral axis depth to effective depth.



Permissible moment redistribution versus the ratio of c/d based on different codes



Permissible moment redistribution versus the net tensile strain based on different codes

Fig. 2 Comparison of codes

The Table 1 shows clear comparison results of moment redistribution in different nation codes briefly.

Table 1 Comparison of moment redistribution based on various codes

Description	ACI 318-14	IS 456-2000	BS 8110	EC 2
Clause No	6.6.5	37	3.2.2.1.	
Maintain equilibrium between external and internal loads	Yes	Yes	Yes	$B \geq k_1 + k_2 X_u/d$ 50 Mpa $\geq f_{ck} B \geq k_3 + k_4 X_u/d$ $f_{ck} \geq 50$ Mpa $B \geq k_5$ for Class B and C steel
	$\epsilon_t \geq 0.0075$ at the section	$X_u/d \leq (0.6 - dM/M)$	$d > (b - 0.4)$	For rotation capacity X_u/d should be less than 0.23-0.30 depending on grade of concrete
	flexural members are continuous			For plastic analysis X_u/d should be less than 0.10-0.15 depending on grade of concrete
Max percentage redistribution up to 4 storey	least of $100\epsilon_t$ % and 20 %	< 30%	< 30%	
	Moment should not be from co-efficients in cide	Moment should not be from co-efficients in cide		are predominantly subject to flexure
Max percentage redistribution after 4 storey		< 10%	< 10%	have the ratio of the lengths of adjacent spans in the range of 0.5 to 2
				No allowed for Cass A steel

NUMERICAL MODELLING RC BEAMS

Numerical modelling of continuous two span reinforced concrete beam is done based on the article by Tiejiong Lou et al. 2014, "Evaluation of Moment Redistribution in Normal-Strength and High-Strength Reinforced Concrete Beams" ASCE -J.Struct. Eng. 2014[10]. This article presents an

investigation of redistribution of moments in normal-strength concrete (NSC) and high-strength concrete (HSC) continuous beams.

Based on the experimental program has been carried out to study the ductile behavior and moment redistribution in continuous HSC beams (Carmo and Lopes 2005, 2008). The experiment consisted of a series of six specimens, designated

Compressive strength	71	Mpa
Ex	40899	Mpa
Prxy	0.3	
Stress Strain Curve	Elastic strain	Stress
	Strain	Stress
	0	0.00
	0.0009	36.18
	0.0015	53.873
	0.002	66.2314
	0.0025	75.9513
	0.003	83.1226
	0.0035	87.8311
	0.00426	90.455
	0.00454	90.455
Shear transfer coefficient open crack	0.20	
Shear transfer coefficient closed crack	0.90	
Uniaxial cracking stress $0.7 \cdot \sqrt{f_{ck}}$	1.85	Mpa
Uniaxial crushing stress	71.00	Mpa
Reinforcing Steel		
Yield strength	569	Mpa
Ultimate strength	669	Mpa
Ex	200000	MPa
Prxy	0.2	
Tangent Modulus	1370	MPa

as V1-0.7, V1-1.4, V1-2.1, V1-2.9, V1-3.8, and V1-5.0, which were fabricated and tested in Coimbra. Parameters of specimen V1-1.4 taken whose details is shown in Fig. 3.

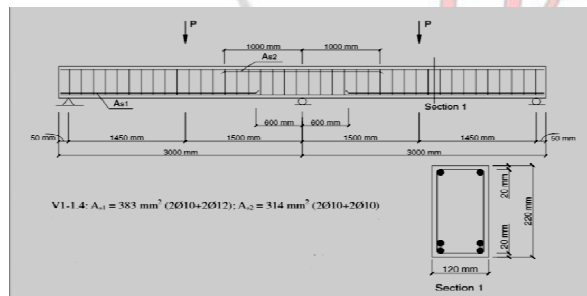


Fig. 3. RC beam details

Modelling is done using Ansys workbench 16.2 and the parameters are tabulated in Table. Two top and two bottom steel bars of 10, 12, or 16mm diameter were provided over the entire length of the beams

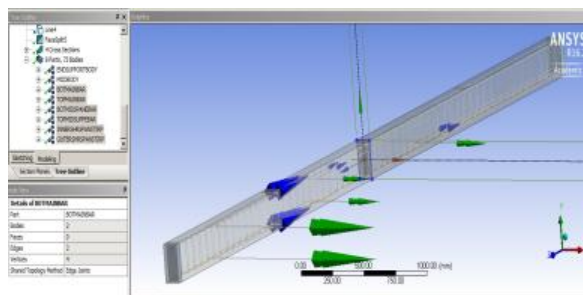


Fig. 4. Model in Ansys

Material Properties

Based on the information from the article and from the Eurocode -2, material properties are assigned. Since we are modelling ultimate load behaviour multilinear properties are assigned based on MC 90 equations in the Eurocode for confined concrete and for reinforcement bilinear parameters are assigned. For failure modelling cracking and crushing parameters also provided as per Table 2. Left support was provided as hinged and other supports are provided as simply supported and gravity loads provided. Then loads are provided as different steps and after meshing analysis done. Load deflection graph plotted are shown in Fig.5

Table 2 Material properties

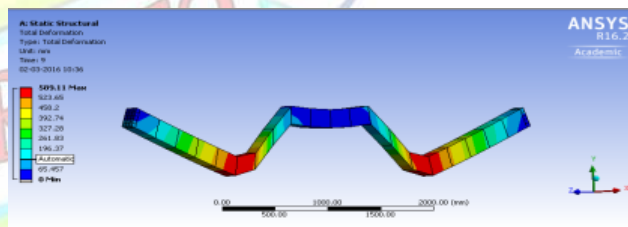


Fig. 5. Deflected Profile

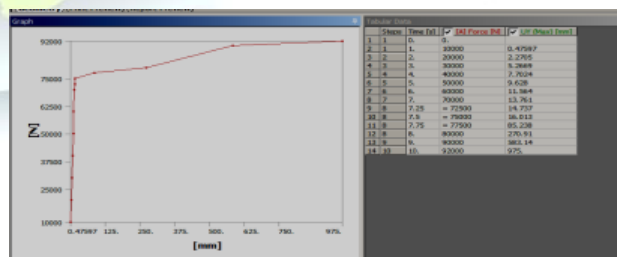


Fig. 6. Load deflection graph

CONCLUSIONS

Based on the papers reviewed it can be see that moment redistribution is a phenomenon which can be utilized by providing optimum design concept of proportioning of beam dimension, reinforcement ratios of both compression and tension, by changing strength of concrete .Also by the study of different codes it can be see that our IS code provisions are



to be give more importance for providing provisions for moment redistribution.

As part of study of moment redistribution numerical modeling of ultimate load behavior of a two span continuous beam done in Ansys Workbench 16.2 and load deflection curve plotted and validation for thesis also done.

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