



# Multi-Objective Maximization with QAM Modulation for MIMO-OFDM Interference Channels Relying on Beamforming

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**Abstract**—This paper presents three beamforming designs for multiuser multiple-input and multiple-output with orthogonal frequency-division multiplexing, where the transmit and receive beamformers are obtained iteratively with closed-form steps. In the first case, the transmit (Tx) beamformers are set and the receive (Rx) beamformers are calculated. It works in null appropriate channels. This cancels one interference term for each user. By satisfying orthogonality condition, the remaining interferences are eliminated. The second case is jointly optimizing the Tx and Rx beamformers from constrained SNR maximization. It uses the results from the first case. The third case is also for joint optimization of Tx-Rx beamformers but combines constrained SNR and signal-to interference-plus-noise ratio maximization. All cases include Linear Constellation Precoder for extracting multipath diversity. Using the standardized statistical channel model for IEEE 802.11n, the required feedback rates and minimum number of antennas are derived and compared to existing beamforming methods.

**Key Words**—Mobile communications, interference channel, beamforming, optimization, MIMO-OFDM, multipath diversity.

## INTRODUCTION

BEAMFORMING for a multiuser MIMO interference channel is for communications between pairs of terminals where there are several pairs sharing the spectrum simultaneously. Each multi-antenna transmitter strives to direct its data to only one multielement receiver in the presence of interference from all the other users' transmitters [1]. This is a recent subject, and it is different from multiuser uplink and downlink beamforming which has been studied extensively, e.g., [2]–[6]. Of these studies, the use of an SINR constraint, where independent data streams are transmitted from a multi-antenna base station (BS) to several single-antenna mobile units, is discussed in [2] and [3]. Extended work [4] exploited uplink downlink SINR duality. Joint transmit-receive optimization based on null space constraint was introduced in [6]. But none of these can be applied to the more complicated interference channel, which is the subject of this paper. As a further clarification of the context, some recent works refer to interference suppression operations as precoder



matrices and interference alignment (IA), e.g., [7], but in this paper, the signal processing operations are referring to beamforming as a vector operator. Therefore, only single stream data transmission is considered. So multi-stream data transmission for each user, usually discussed in IA design, is not used here. It is clear that there are several different approaches to the beamforming problem. The ultimate metric is a practicable digital communications performance, but it is not yet possible to optimize this directly. Instead, the optimization of some analogue channel performance functions is followed by a calculation of some aspect of the associated communications performance—usually an information-theoretic capacity or throughput rate, along with some error performance. Along these lines, this paper presents three new beamforming design cases. First, a constrained SNR maximization is sought in which the Tx-BFs for all the users are acquired by deploying the null-space of an appropriate channels matrix. This null-space assignment for Tx-BFs eliminates one term of interference at each receiver. The remaining interference terms at each receiver can be eliminated by means of orthogonal vectors. The Rx-BF of each user, in this case, has a closed form solution if its norm is one. The second case is joint Tx–Rx beamformer design for constrained SNR maximization where only the Rx-BF at each receiver terminal nulls out all the

interference. This problem leads to a multi-objective optimization problem which can be solved iteratively because it has guaranteed convergence. The third case is joint constrained SNR and SINR maximization. The new contributions of this paper include: closed-form solutions for multiple cases of the joint beamformers designs in the  $K$ -user MIMO interference channel; a derivation for the minimum number of antennas required for these solutions; a derivation of the required feedback rate (the rate for the first two methods is lower than that of existing solutions if same number of antenna is considered); Finally, all solutions allow for exploiting multipath (i.e., time domain) diversity where the complexity of the decoder is independent of the number of antennas.

## II. SYSTEM MODEL, PROBLEMS FORMULATIONS AND THEIR SOLUTIONS

There are  $K$  pairs of multi-antenna terminals which are striving to share simultaneously the spectrum in time and space. The channel is modeled as 1) a tapped delay line ( $L+1$  taps) according to the IEEE 802.11n propagation model or 2) a single-tap flat fading channel with a perfect spatial correlation matrix. The first channel model justifies both the MIMO-OFDM configuration and deploying the multi-path precoder in this paper.

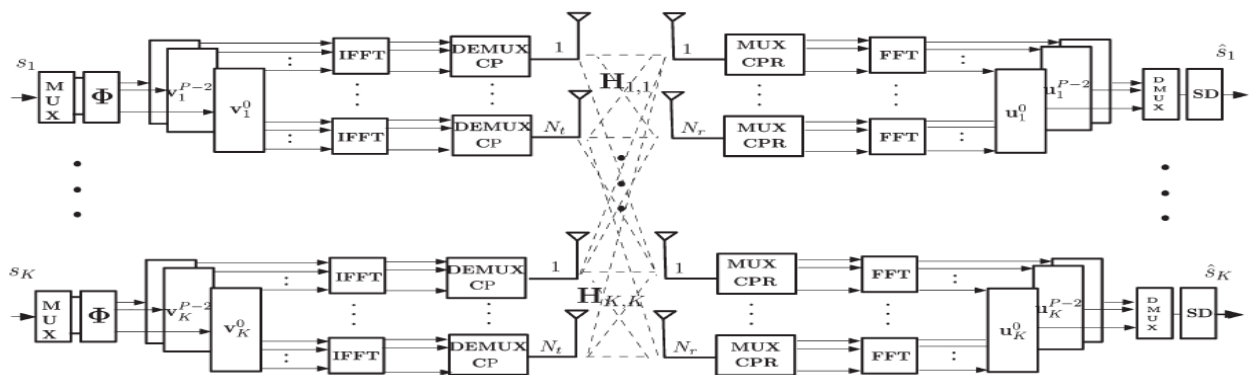




Fig. 1. Simple structure of MIMO-OFDM based on Beamforming

There are  $N$  pairs of multi-projection terminals which remain attempting to share instantaneously the spectrum in time and space. The IEEE 802.11n statistical propagation model assumes: 1) The power azimuth spectrum (PAS) and the power delay profile (or spectrum (PDS)) are separable which means each tap is modeled independently. 2) The PAS and the Doppler spectrum for each tap are separable. In other words, the spatial correlation (correlation matrices) and temporal correlation (Doppler spectrum) for each tap are modeled independently. 3) Each tap is modeled using the Kronecker model. Hence, it is assumed that the transmit and receive correlation matrices are separable for each tap. Perfect channel knowledge is assumed at all the users and perfect timing is also assumed in the usual manner to allow the simplest linear model for the link. The received signal vector is given by,

$$Y_i(P) = \mathbf{H}_{i,i}(P)v_i(p)s_i(p) + \sum_{i' \neq i}^K \mathbf{H}_{i,i'}(p)v_{i'}(p)s_{i'}(p) + n_i(p) \quad (1)$$

In (1), the channel at subcarrier  $p$  is  $\mathbf{H}_{r,i}(P) \in \mathbb{C}^{N_t \times N_r}$ . The entry of  $(v, \mu)$  is defined as  $[H_{r,i}(p)]_{v\mu} \triangleq H_{v\mu}^{r,i}(p)$ , so that

$$H_{v\mu}^{r,i}(p) := \sum_{l=0}^L f_{v\mu}^{r,i}(l) e^{-j2\pi lp/P} \quad (2)$$

$$\text{s.t.} \begin{cases} \mathbf{u}_i^H \mathbf{u}_i = 1 & i = 1, \dots, K \\ \mathbf{v}_i^H \mathbf{v}_i = 1 & i = 1, \dots, K \end{cases} \quad (3)$$

in which it does not have closed-form solution for  $\mathbf{v}_i$  and  $\mathbf{u}_i$  where,  $SINR_i =$

$$S_i / \left( \sum_{j \neq i}^K |I_j|^2 + N_i \right), \quad S_i = \mathbf{u}_i^H \mathbf{H}_{i,i} \mathbf{v}_i \mathbf{v}_i^H \mathbf{H}_{i,i}^H \mathbf{u}_i, \\ I_i = \mathbf{u}_i^H \mathbf{H}_{i,j} \mathbf{v}_j \quad \text{and} \\ N_i = \sigma_{n_i}^2 \mathbf{u}_i^H \mathbf{u}_i \quad (\sigma_{s_i}^2 = 1). \quad \text{Hence, the optimization problem is given as:}$$

$$\begin{aligned} \max_{\mathbf{u}_i \mathbf{v}_i} SINR_i &= S_i / N_i \\ \text{s.t. } I_j &= 0 \quad j \neq i \end{aligned} \quad (4)$$

has closed-form solution for  $\mathbf{v}_i$  and  $\mathbf{u}_i$ . To tackle problem (3), first we assume that one interference term is eliminated before applying the receive beamformer. Therefore, solving constrained SNR maximization is much easier than solving (4).

### III. PROPOSED BEAMFORMING DESIGNS

#### A. Optimal Rx-Bfs For Joint SNR Maximization While The Tx-Bfs Are Known

In this section, the Tx-BFs are found from the null space of an appropriate set of channels, and then the optimal Rx-BFs are sought. For  $K \in \{2(n+1) : n \in \mathbb{N}\}$ , where  $\mathbb{N}$  denotes the positive integers and the beamformer  $\mathbf{v}_i$  is obtained by,

$$\mathbf{v}_i = \mathcal{N}(\mathbf{H}_{K+1-i,i}) \quad (5)$$

Where,

$$\mathcal{N}(\mathbf{A}) \triangleq \{\mathbf{x} | \mathbf{A}\mathbf{x} = \mathbf{0}, \|\mathbf{x}\| = 1\} \quad (6)$$

The next step is to determine  $\mathbf{u}_i$  such that it maximizes the signal-to-noise ratio (SNR) of the  $i$ th user (i.e., after the Rx-BF) while suppressing the  $K-2$  remaining interference terms. This optimization problem is denoted  $\rho$  for the first receiver as an example, and the rest of the receivers' beamformer designs follow by the same methodology.

$$\rho: \max_{\mathbf{u}_1 \in \mathbb{C}^{N_r} \setminus \mathbf{0}} \mathbf{u}_1^H \mathbf{H}_{1,1} \frac{\mathbf{v}_1 \mathbf{v}_1^H \mathbf{H}_{1,1}^H \mathbf{u}_1}{\mathbf{u}_1^H \mathbf{u}_1}$$





### B. Joint Tx-Bf And Rx-Bf For Constrained SNR Maximization

In the previous section, the optimal closed-form Rx-BFs were obtained, while the Tx-BFs are the null space of channels as expressed according to an even or odd number of users, respectively. In this section, joint Tx-BF and Rx-BF are designed for the constrained SNR maximization problem. The value of  $\mathbf{G}_1$  and  $\mathbf{G}_2$  for the channel  $\mathbf{C}_n$  is specified as,

$$\mathbf{G}_1 \triangleq \mathbf{H}_{n,n}^H [\mathcal{N}(\mathbf{C}_n)]_1 [\mathcal{N}(\mathbf{C}_n)]_1^H \mathbf{H}_{n,n}$$

$$\mathbf{G}_2 \triangleq \mathbf{H}_{n,n}^H [\mathcal{N}(\mathbf{C}_n)]_2 [\mathcal{N}(\mathbf{C}_n)]_2^H \mathbf{H}_{n,n}$$

### C. Joint TX-BF and RX-BF design for Constrained SNR and SINR maximization

In the previous section, each Rx-BF nulls its interference and then this solution is inserted to the constrained SNR objective function which yields the Tx beamformer. In this section, multi objective optimization by the fixed point method is applied. Instead of  $G$ , which is optimization w.r.t.  $v_1$  and  $u_1$ , define the problem  $G_1$  as:

$$G_1 : \max_{v_1 \in \mathbb{C}^{N_t}} v_1^H \mathbf{H}_{1,1}^H u_1 u_1^H \mathbf{H}_{1,1}^H v_1 \quad (7)$$

Solving the optimization problem is possible by using evolutionary algorithms for example. But all of the beamformer designs presented here are closed-forms. This is important for implementation.

## VI. FEEDBACK RATE OF PROPOSED BEAMFORMING METHOD IN COMPARISON WITH OTHER BEAMFORMING SCHEMES FOR INTERFERENCE CHANNELS

From the discussion in the Introduction and the start of Section II, an issue in multi-user beamforming is the amount of information

required to be exchanged among receivers and transmitters, which bites into the payload capacity. (The channel sounding is an associated issue, and it is not addressed here.) In this section, the feedback of the presented beamforming

is compared with existing interference channel beamforming schemes. The analysis is for flat channels, and is extended to OFDM via scaling by  $P$ . It is emphasized that the feedback rate, complexity and performance are competing factors in  $K$ -user interference channels. In the previous section, the complexity was demonstrated to be lower than existing systems.

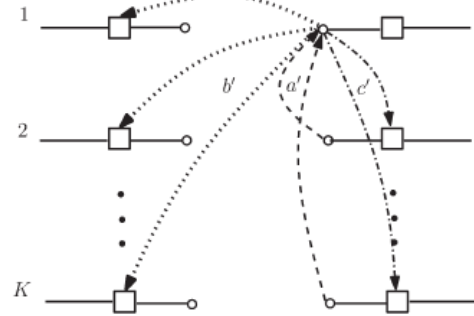


Fig. 3. Feedback graphs for all joint Tx-Rx beamforming designs. The dashed arcs represent the all channels which are feedback from  $N - 1$  receiver nodes to one receiver node and dotted arcs represent Tx-BF which are feedback from one receiver node to transmitter nodes and dashed-dotted arcs show the Rx-BF from one receiver node to  $N-1$  receiver nodes.

Moreover multipath diversity for OFDM transmission is possible by deploying the fixed precoder matrix. The other schemes do not have this capability because their Rx-BFs are also the decoder.

## IV. SIMULATION RESULTS

In this section numerical experiments are described for validating the analysis. For simplicity, all the users use QAM in the evaluation of BER performance. As discussed in the Introduction, digital communications



performance is a tricky aspect of link optimization and using a single modulation cannot create high capacity (efficiency) over a range of average SNRs. (Rayleigh channels, for example, have a very large range of average SNRs.) Similarly, there is no channel coding. Strictly, the digital communications behaviour should be optimized, but this is not yet possible in general as discussed. Nevertheless, optimizing with the analogue objective functions, and then applying a fixed communications configuration allows a fair performance comparison between the differently optimized beamformers.

To determine the benefit of LCP in the multiuser MIMO-OFDM interference channel, the IEEE 802.11n channel model B is used with the following settings: 3 Hz maximum Doppler shift for all paths with Bell Doppler spectrum; 15 ns rms delay spread;  $\lambda/2$  element spacing at the transmit and receive antennas; and for both clusters 1 and 2: average path gains; angular spread (AS) at the receiver and at the transmitter; mean angles of departure (AoD); and mean angles of arrival (AoA) are all set according to the standard.

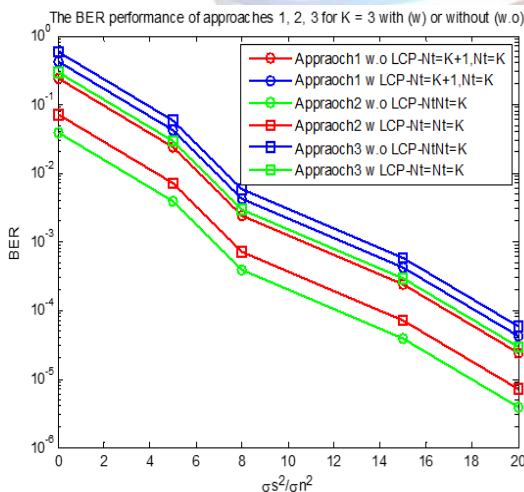


Fig. 3. The BER performance of approaches 1, 2, 3 for  $K=3$  with (w) or without (w.o) deploying LCP precoder and sphere decoder with IEEE 802.11n channel model B.

In summary, Fig. 3 shows the BER for  $K=3$  users over IEEE 802.11n channel model B. The joint multi-objective Tx-BF and Rx-BF design (approach 3) has the best performance,

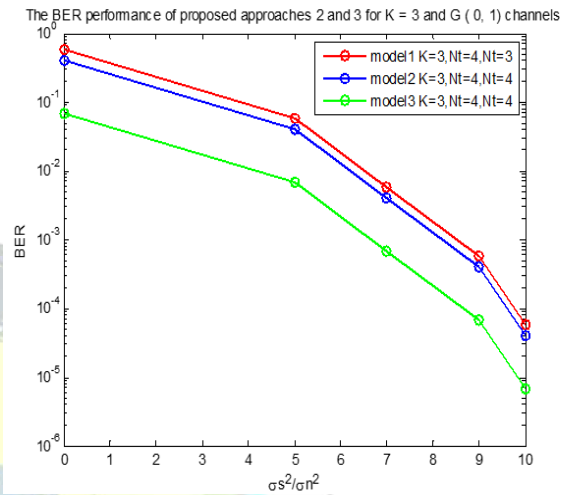


Fig. 4. The BER performance of proposed approaches 2 and 3 for  $K=3$  and  $G(0, 1)$  channels.

then the joint single-objective Tx-BF and Rx-BF design (approach 2), and finally the individual design. This simulation also demonstrates that for this MIMO channel for WLAN with 9 paths, using the LCP matrix still improves the system performance. Finally it is recalled that the complexity of SD decoding at the receiver increases with the constellation size but not with the number of antennas.

The BER performance and computational complexity of proposed methods have been compared with existing known methods, such as ZF, LI and sum-rate maximization. From Fig. 4, approaches 2 and 3 have better performance than ZF with selection and also much better than LI. The channel for this simulation is flat fading in time domain denoted by  $G(0, 1)$ .

## V. CONCLUSION

Three new beamforming algorithms are presented for a multiuser MIMO-OFDM interference channel which can also develop



multipath diversity using the known technique of applying an LCP matrix. With a unit norm for the transmit and receive beamformers, the algorithms comprise iterative procedures with closed-form steps, allowing a fast solution. Because no derivative or Lagrangian multiplier is needed, the computational complexity is less than existing beamforming methods. It is shown that the third algorithm—joint constrained SNR and SINR maximization—outperforms the least-square beamforming design, with a much lower computational time. For quasi-realistic channels (exponential power delay profile, Kronecker antenna correlations, as in the IEEE 802.11n channel model), the second algorithm may be better than the third algorithm and also it requires less feedback. The simplicity of the presented algorithms comes at the price of one more antenna element at each terminal, compared to existing methods. The results of this paper can also be viewed as some quantification of the trade-offs of between algorithmic simplicity, a minimum number of antennas, feedback rate, and the capability of extracting multipath diversity, in beamforming for the MIMO-OFDM interference channel.

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