



Biomathematical model on Chemotaxis

Dr.M.Rama,

Dept.ofChemistry ,Ch.S.D.St Theresa's (A) College for women Eluru WG

Mail id..manne_rama2001@yahoo.co.in

Mobile...9912107611

ABSTRACT:Mathematics plays a key role in many disciplines of science, primarily as a mathematical modeling tool. New innovations and developments in physics, and chemistry are by the influence of mathematics. Calculus was invented entirely for the use of physics and chemistry. The importance of the role of mathematics in physics and chemistry is understood by the existing disciplines mathematical physics and mathematical chemistry. One can think is mathematics has that same important role or even lesser important role in the field of biology and is mathematics and biology could possibly have anything in common. Even though physics and biology are very different sciences, mathematicians and biologists developed the "mathematical biology" or "biomathematics" as a recent discipline for the mathematical representation in biology to the theoretical and practical applications in biological, biomedical and biotechnology researches. The applications of biomathematics in chemotaxis is explained in this paper.

Keywords: Mathematics, Biology, Mathematical models, Biomathematics, Chemotaxis.

Introduction:Mathematical biology or biomathematics is a fast-growing well-recognized and the most exciting modern application of mathematics. This is an interdisciplinary field has both theoretical and practical applications in research on biological, biomedical and biotechnological fields.

A variety of mathematical techniques are used in mathematical biology where the mathematical representations and modeling of the biological problems using a variety of mathematical theories and techniques. Mathematical areas such as calculus, probability theory, statistics, linear algebra, graph theory, combinatorics, algebraic geometry, topology, dynamical systems, differential equations and coding theory are now being applied in this field.

A large number of insects and animals (including humans) rely on an acute sense of smell for conveying information between members of the species. Chemicals which are involved in this process are called *pheromones*. The acute sense of smell of many deep sea fish is particularly important for communication and predation. Other than for territorial demarcation one of the simplest and important exploitations of pheromone release is the directed movement it can generate in a population. Bacteria such as *Escherichia coli*,



Rhodobactersphaeroides, and *Bacillus subtilis*¹ respond to extracellular changes in their environment by biased random motion towards attractants or away from repellents. Such movement is commonly referred to as chemotaxis and was first reported as early as the late nineteenth century (1,2,3). Convincing evidence suggests that leukocyte cells in the blood move towards a region of bacterial inflammation, to counter it, by moving up a chemical gradient caused by the infection (4,5,6,7). Chemotaxis is essential for survival, if cells fail to reach their proper destinations they die, or the organism dies, so it is expected that the mechanisms for processing chemotactic signals have been optimized during evolution. [5] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. Utilizing the versatile back venturing method, every one of the six DOF are effectively followed utilizing just four control inputs while within the sight of un demonstrated flow and limited unsettling influences. Elements and speed vectors were thought to be inaccessible, along these lines a NN eyewitness was intended to recoup the limitless states. At that point, a novel NN virtual control structure which permitted the craved translational speeds to be controlled utilizing the pitch and the move of the UAV. At long last, a NN was used in the figuring of the real control inputs for the UAV dynamic framework. Utilizing Lyapunov systems, it was demonstrated that the estimation blunders of each NN, the spectator, Virtual controller, and the position, introduction, and speed following mistakes were all SGUUB while unwinding the partition Principle.

Chemotaxis is not always good news, For example during metastasis (the spreading of cancer throughout the body), cancer cells use chemotaxis to direct their motion toward lymphatic vessels and then use the lymphatic system (a component of the circulatory system) in order to spread to different parts of the human body (8). Chemotaxis plays a key role in the survival (and sometimes death) of animal species. Bacterial chemotaxis has been shown to be important in the formation of biofilms - groups of microorganisms that stick to one another on or close to a surface (9). It has been suggested that this could help to make bioremediation a more widely used technique for neutralising difficult to clear sources of contamination such as oil spills in the ocean (10).

The temporal model assumes that the amoeba measures and "remembers" the chemo-attractant concentration at time zero and compares this to a second reading at some time point later as the cell crawls. The spatial model assumes that the cell is able to read the concentration across the span of the cell so that it can compare the number of chemoattractant molecules. A related model is the



pseudospacial model, where the cell tests concentrations by sending out pseudopods at various points to sense where the highest concentrations lie. The temporo-spatial model, detects waves of chemo –attractants coming toward the cell from a particular direction.

A widely studied chemotactic phenomenon is that exhibited by the slime mould *Dictyostelium discoideum* where single-cell amoebae move towards regions of relatively high concentrations of a chemical called cyclic-AMP which is produced by the amoebae themselves. Interesting wavelike movement and spatial patterning are observed experimentally; A discussion of the phenomenon and some of the early mathematical models which have been proposed together with some analysis are given (11). More complex and more biologically realistic models have been proposed by (12,13,14). These new models all exhibit oscillatory behaviour (15) presented a model of excitation and adaptation in bacterial chemotaxis in wider biological contexts. They incorporated detailed biochemical data into their model which they then used to shed light on the actual experimental process.

Most mathematical models for spatial patterning in *Dictyostelium discoideum* are based on continuum models for the chemoattractants and the cells. (14) developed an interesting new model in which the cells are considered as discrete entities with the chemoattractant concentrations continuous. The results agree well with many of the extant experimental results. With their model they were able to investigate the effects of different cell movement rules on aggregation patterns and wave motion, including the origin of the ubiquitous spiral waves.

Let us suppose that the presence of a gradient in an attractant, $a(\mathbf{x}, t)$, gives rise to a movement, of the cells say, up the gradient. The flux of cells will increase with the number of cells, $n(\mathbf{x}, t)$, present. Thus we may reasonably take as the chemotactic flux

$$\mathbf{J} = -D\nabla c \dots\dots\dots 1$$

where D may be a function of \mathbf{x} and c and f a function of c , \mathbf{x} and t .

$$\mathbf{J} = n\chi(a)\nabla a, \dots\dots\dots 2$$

where $\chi(a)$ is a function of the attractant concentration. In the general conservation equation for $n(\mathbf{x}, t)$, namely,

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J} = f(n), \dots\dots\dots 3$$

where $f(n)$ represents the growth term for the cells, the flux

$$\mathbf{J} = \mathbf{J}_{\text{diffusion}} + \mathbf{J}_{\text{chemotaxis}}, \dots\dots\dots 4$$

where the diffusion contribution is from (1) with the chemotaxis flux from (2).

Thus a basic *reaction-diffusion-chemotaxis equation* is

$$\frac{\partial n}{\partial t} = f(n) - \nabla \cdot n\chi(a)\nabla a + \nabla \cdot D\nabla n \dots\dots\dots 5$$



where D is the diffusion coefficient of the cells.

Since the attractant $a(\mathbf{x}, t)$ is a chemical it also diffuses and is produced, by the amoebae, for example, so we need a further equation for $a(\mathbf{x}, t)$. Typically

$$\frac{\partial a}{\partial t} = g(a, n) \nabla \cdot D_a \nabla a \dots \dots \dots 6$$

where D_a is the diffusion coefficient of a and $g(a, n)$ is the kinetics/source term, which may depend on n and a . Normally we would expect $D_a > D$. If several species or cell types all respond to the attractant the governing equation for the species vector is an obvious generalisation of (5) to a vector form with $\chi(a)$ probably different for each species.

In the seminal model of Keller and Segel (1971) for slime mould,

$$g(a, n) = hn - ka,$$

where h, k are positive constants. Here hn represents the spontaneous production of the attractant and is proportional to the number of amoebae n , while $-ka$ represents decay of attractant activity; that is, there is an exponential decay if the attractant is not produced by the cells.

One simple version of the model has $f(n) = 0$; that is, the amoebae production rate is negligible. This is the case during the pattern formation phase in the mould's lifecycle. The chemotactic term $\chi(a)$ is taken to be a positive constant χ_0 . The form of this term has to be determined from experiment. With constant diffusion coefficients, together with the above linear form for $g(a, n)$, the model in one space dimension becomes the nonlinear system

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2} - \chi_0 \frac{\partial}{\partial x} \left(n \frac{\partial a}{\partial x} \right)$$

$$\frac{\partial a}{\partial t} = hn - ka + D_a \frac{\partial^2 a}{\partial x^2} \dots \dots \dots 7$$

which we study in Chapter 1, Volume II. There we consider n to be a bacterial population and a the food which it consumes.

Other forms have been proposed for the chemotactic factor $\chi(a)$. For example,

$$\frac{\chi(a)}{a} = \chi_0, \chi(a) = \frac{\chi_0 K}{(K+a)^2}, \chi_0 > 0, K > 0 \dots \dots 8$$

which are known respectively as the log law and receptor law. In these, as a decreases the chemotactic effect increases.



Conclusion: There are various ways to define a practical measurable *chemotaxis index*, I , which reflects the strength of the chemoattractant. Let us look at one example, and to be specific consider the planar movement of a cell, say, towards a source of chemoattractant at position x_s . Suppose the cell starts at x_A and the source is distance D_1 away. In the absence of chemotaxis the cell's movement is purely random and the mean distance, D_2 say, that the cell moves in a given time T in the direction of x_s is zero. In the presence of chemotaxis the random movement is modified so that there is a general tendency for the cell to move towards the chemoattractant source and over the same time T , $D_2 > 0$. We can define the index $I = D_2/D_1$: the larger I the stronger the chemotaxis. (5) have analysed the detailed chemosensory movement of leukocyte cells with a view to determining its chemotaxis parameters.

The difference in sign in (eqe.5) and (eqe.7) in the diffusion and chemotaxis terms. Each has a Laplacian contribution. Whereas diffusion is generally a stabilising force, chemotaxis is generally *destabilising*, like a kind of negative diffusion. At this stage, therefore, it is reasonable to suppose that the balance between stabilising and destabilising forces in the model system (eqe.7) could result in some steady state spatial patterns in n and a , or in some unsteady wavelike spatially heterogeneous structure. That is, nonuniform spatial patterns in the cell density appear. On the other hand if the chemotactic effect is sufficiently strong there could be a possibility of solution blow-up.

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