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Plane Symmetric String Cosmological Models in Bimetric Theory of Gravitation

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Abstract: In this paper we obtain plane symmetric string cosmological models in Bimetric theory of gravitation. The field equations in the presence of cosmic strings have been obtained and the corresponding string cosmological models are presented.

1. Introduction

Rosen (1973) proposed a biometric theory of gravitation incorporating the covariance and equivalence principles. In this theory a Riemannian metric tensor g_{ij} and an auxiliary flat space, having the metric tensor γ_{ij}, are used. In this theory γ_{ij} describes the properties flat space-time whereas g_{ij} represents gravitational potential tensor, which describes the interaction between matter and gravitation. Rosen (1973, 1975), Yilmaz (1975), Karade and Dhoble (1980), Karade (1980) and Reddy and Venkateswarlu (1989) have studied several aspects of biometric theory of gravitation. Mohanty et al. (2002) have studied on Bianchi type-I mesonic cosmological model in biometric theory. Recently, Rao et al. (2008) have established the existence of Bianchi type-I string models in Bimetric theory of gravitation.

The field equations of the biometric theory of gravitation formulated by Rosen (1973) are

$$N_{ij} - \frac{1}{2} N g_{ij} = -8\pi k T_{ij} \tag{1.1}$$

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where $N^{i}{}_{j} = \frac{1}{2}\gamma^{ab}(g^{hi}g_{hj/a})/b \quad and \quad k = \left(\frac{g}{\gamma}\right)^{1/2}$

in which T_{ij} is the usual stress tensor of the matter and a vertical bar denotes the covariant differentiation with respect to γ_{ij}.

In this paper we have taken up the study of plane symmetric string cosmological models in biometric theory of gravitation. In section 2, we have derived the field equations for plane symmetric metric in biometric theory of gravitation. Section 3 contains vacuum model, section 4 contains string model and section 5 contains some physical and geometrical properties of the model. The last section 6 contains some concluding remarks.

2. Metric and field equations

We consider the Plane symmetric metric in the form

$$ds^2 = e^{2\alpha}(dt^2 - dz^2) - e^{2\beta}(dx^2 + dy^2) \tag{2.1}$$

where α and β are functions of t and z.

The biometric field equations (1.1) can be written as

$$\alpha_{33} - \alpha_{44} = -8\pi k T_1^1 \tag{2.2}$$

$$\alpha_{33} - \alpha_{44} = -8\pi k T_2^2 \tag{2.3}$$

$$\beta_{33} - \beta_{44} = -8\pi k T_3^3 \tag{2.4}$$

$$\beta_{33} - \beta_{44} = -8\pi k T_4^4 \tag{2.5}$$

The energy momentum tensor for a bulk viscous fluid containing one dimensional strings is

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$$T^i{}_j = \rho u^i u_j - \lambda x^i x_j \tag{2.6}$$

where ρ is the rest energy density,

λ is the tension in the string,

u^i is the four velocity vector,

x^i is a space-like vector which represents the anisotropic directions of the string and $\rho_p = \rho - \lambda$

where ρ_p denotes particle energy density.

Moreover the direction of strings satisfy the standard relations

$$g_{ij}u^i u^j = 1,\ g_{ij}x^i x^j = -1, \text{and } u^i x_i = 0. \tag{2.7}$$

$$x^i = (0, 0, e^{-\alpha}, 0)$$

In a commn moving coordinate system, we have

$$T_1^1 = T_2^2 = 0,\ T_3^3 = \lambda,\ T_4^4 = \rho \tag{2.8}$$

and $T_j^i = 0$ for $i \neq j$.

The quantities ρ and λ are functions of time 't' only.

By using (2.8), the field equations (2.2) to (2.5) can be written as

$$\alpha_{33} - \alpha_{44} = 0 \tag{2.9}$$

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$$\beta_{33} - \beta_{44} = -8\pi k\lambda \tag{2.10}$$

$$\beta_{33} - \beta_{44} = -8\pi k\rho \tag{2.11}$$

Here the suffixes 4 and 3 denotes partial differentiation with respect to t and z respectively.

3. Vacuum cosmological model

In this case the above field equations reduce to

$$\beta_{33} - \beta_{44} = 0 \tag{3.1}$$

$$\alpha_{33} - \alpha_{44} = 0 \tag{3.2}$$

From (3.1) and (3.2), we get

$$\alpha = f_1(t+z) + f_2(t-z) \text{ and } \beta = g_1(t+z) + g_2(t-z) \tag{3.3}$$

where f_1, f_2, g_1 *and* g_2 are arbitrary functions of 't' and 'z'.

The corresponding vacuum cosmological model can now be written in the form

$$\begin{aligned} ds^2 &= \exp\{2[f_1(t+z) + f_2(t-z)]\}(dt^2 - dz^2) \\ &\quad - \exp\{[g_1(t+z) + g_2(t-z)]\}(dx^2 + dy^2) \end{aligned} \tag{3.4}$$

4. String cosmological model

From equations (2.10) and (2.11), we get

$$\rho = \lambda \tag{4.1}$$

Field equation (2.9) to (2.11) will reduce to

$$\beta_{33} - \beta_{44} = -8\pi k\rho \tag{4.2}$$

$$\alpha_{33} - \alpha_{44} = 0 \tag{4.3}$$

From the above field equations, we get

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$$\alpha = f_1(t+z) + f_2(t-z) \tag{4.4}$$

$$\beta = g_1(t+z) + g_2(t-z) + \frac{c_1}{2(r+1)} t(t-z)^{r+1} \tag{4.5}$$

$$8\pi k\rho = 8\pi k\lambda = c_1(t-z)^r \tag{4.6}$$

where f_1, f_2, g_1 and g_2 are arbitrary functions of t and z. Here r and c_1 are arbitrary constants.

Since r and c_1 are arbitrary constants they can be chosen in such a way that the density ρ in (4.6) is always non-negative.

The metric (2.1), in this case, can now be written as

$$\begin{aligned} ds^2 &= \exp\{2[f_1(t+z) + f_2(t-z)]\}(dt^2 - dz^2) \\ &\quad - \exp\{[g_1(t+z) + g_2(t-z) + \frac{c_1}{2(r+1)} t(t-z)^{r+1}]\}(dx^2 + dy^2) \end{aligned} \tag{4.7}$$

Thus equation (4.7) together with (4.6) constitutes a Plane symmetric string cosmological model in Bimetric theory.

It is interesting to note that if $c_1=0$, then the above model reduces to the vacuum cosmological model given by (3.4) with $\rho = 0 = \lambda$.

5. Physical and geometrical properties

The model given by (4.7) together with (4.6) constitutes a plane symmetric string cosmological model in Bimetric theory of gravitation.

Volume element of the model is given by

$$V = (-g)^{\frac{1}{2}} = \exp 2(\alpha + \beta) \tag{5.1}$$

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The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i_{;i} = e^{-\alpha}(\alpha_4 + 2\beta_4) \tag{5.2}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{e^{-2\alpha}(\alpha_4 + 2\beta_4)^2}{6} \tag{5.3}$$

where α and β are given by (4.4) and (4.5).

We can observe that the plane symmetric string cosmological model presented here has no initial singularity. Also since, as t tends to infinity, the limit $\left(\frac{\sigma}{\theta}\right)^2$ is not equal to zero, the model will never approach to isotropy.

6. Conclusions

Here we have presented plane symmetric string cosmological models in Bimetric theory of gravitation and discussed some physical and geometrical properties. Finally we have established the existence of plane symmetric string cosmological models, unlike the earlier results of non-existence, in biometric theory of gravitation proposed by Rosen (1973).

References:

Karade, T.M.: *Ind. J. Pure Appl. Math.* **11**, 1202 (1980).

Karade, T.M. and Dhoble, Y.S.: *Letters Nuovo Cimento* **29**, 390 (1980).

Mohanty, G., Sahoo, P.K., and Mishra, B.: Astrophys. Sapce Sci., 281, 609 (2002).

Rao, V.U.M., Vinutha, T., Sireesha, K.V.S.: Astrophys. Sapce Sci.,317, 79 (2008)

Reddy, D.R.K. and Venkateswarlu, R.: *Astrophys. Space Sci.* **158**, 169 (1989).

Rosen, N.: *Gen. Relativ. Gravitation* **4**, 435 (1973).

Rosen, N.: *Gen. Relativ. Gravitation* **6**, 259 (1975).

Yilmaz, H.: *Gen. Relativ. Gravitation* **6**, 269 (1975).